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OUTLINES

OF

NATURAL PHILOSOPHY;

BEING THE

Heads of a Course of Lectures,

DELIVERED IN

COLUMBIA COLLEGE, NEW-YORK,

BY

JAMES RENWICK, A. M.

PROFESSOR OF NATURAL AND EXPERIMENTAL PHILOSOPHY, AND
CHEMISTRY, IN COLUMBIA COLLEGE.

VOL. I.

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OUTLINES

OF

NATURAL PHILOSOPHY.

DEFINITIONS AND INTRODUCTORY REMARKS.

1. Philosophy is the science which inquires into the laws that regulate the phenomena of nature, whether in the intellectual or moral world.

> The term Philosophy was first used by Pythagoras. Cicero, *Tusculanæ Quæstiones*, lib. 5. cap. 3.

2. Philosophy is divided into two branches, corresponding to the two great classes of substances; material and immaterial.

> The material world is the province of Natural Philosophy; the properties and action of the bodies which compose the universe, the objects of its investigations.

3. The essential properties of matter are Extension, Mobility, and Impenetrability.

> Attraction is often classed among the properties of matter, particularly that species of attraction known by the name of Gravitation; but we can conceive matter to exist which neither gravitates nor is attracted in any manner by any other portion of matter, and shall yet possess extension, impenetrability, and mobility.

4. Body is a separate and determinate quantity of matter, contained under some known figure, or existing in some determinate mode.

The same matter, by a change in its mode of existence, may form many different bodies.

5. Extension in three dimensions being a property of matter, it is, mathematically speaking, infinitely divisible; for geometric and arithmetic magnitude are divisible *ad infinitum.*

6. The actual division of matter can be carried to an almost incredible extent, as may be shown: by the great ductility of metals; by the distance at which odours affect the olfactory nerves; by the minuteness of the animalculæ discovered through the aid of the microscope, each of which is an organized being; by the colours of chemical solutions; by the smallness of the particles of light.

Still it may reasonably be doubted whether matter be infinitely divisible; the more probable supposition is, that it may finally be resolved into particles perfectly hard and incapable of further division.

These particles are called atoms; the theory that holds their existence the atomic theory; and it seems to be supported by the strong and conclusive evidence of numerous well ascertained facts. This evidence is, however, rather the object of chemical than of physical investigation.

7. Bodies, being of that class of geometric figures called solids, are inclosed by one or more boundaries.

8. The boundaries of solids are surfaces; the boundaries of surfaces, lines; the terminations of lines, points.

Geometry has been defined by Professor Leslie to be that branch of Natural Philosophy which treats of the property of matter called extension. Leslie's *Elements of Geometry.*

9. By the term Impenetrability of Matter, it is intended to express the fact that no two particles of matter can occupy the same portion of space at one and the same time.

> In this sense the rarest fluid is equally impenetrable with the hardest solid. If matter were penetrable by matter, all the bodies of the universe might be united in any space, however small. Muschenbrock, vol. 1, § 81.

10. The bodies that compose the universe, as regards inhabitants of our earth, are either Terrestrial or Celestial.

> Terrestrial bodies are divided into three kingdoms; the Mineral, the Vegetable, and the Animal.

11. Bodies differ from each other in respect of the ease or difficulty with which the particles they are composed of may be separated. Those in which the particles can be moved among each other by the smallest effort, are called fluid: those, where they adhere more strongly, solid bodies.

12. Motion is continual and successive change of place.

> No body possesses within itself the power of changing its state, whether of motion or of rest. It cannot lose motion in any direction without communicating an equal amount to other bodies in the same direction; neither can it acquire motion in any direction, without diminishing the motion of other bodies by an equal quantity in that same direction.

> This is the annunciation of that principle, called by some authors Inertia, and classed by them as one of the properties of matter. Playfair's *Outlines*, § 21.

13. The foundation of the principles of Natural Philosophy rests upon experiment, and a careful observation of facts. The branch of knowledge that arranges and classifies facts, is called Natural History. Bacon, *Novum Organum*.

14. When from a comparison of a number of facts, known, from experiment or observation, to be true, the existence of a more general fact is inferred, the inference is said to be made by *Induction.* PLAYFAIR, § 3.

It is from induction that all certain knowledge of the laws of nature is derived. This method was first explained, and the rules for pursuing it laid down, by Lord Bacon, in his *Novum Organum.* PLAYFAIR's *Dissertation, Supplement Encyclopedia Britannica.*

15. When general principles have been once established by induction, we can often, by the application of mathematical reasoning, deduce from them conclusions as clear and certain as the principles themselves. PLAYFAIR, § 4.

16. We are said to explain a phenomenon when we show it to be necessarily included in some phenomenon or fact already known, or supposed to be known; and we consider one phenomenon as the cause of another, when we conceive the existence of the latter to depend on some force or power residing in the former. PLAYFAIR, § 6.

17. A fact assumed in order to explain appearances, and which has no other evidence of its reality but the explanation it is supposed to afford, is called an *Hypothesis.*

18. An explanation of any system of appearances or events, founded upon facts known to exist from evidence, independent of the facts themselves, is called a *Theory.* PLAYFAIR, § 8.

19. When one system of events or appearances is similar to another; and when we infer that the causes in the two systems are also similar, we are said to reason from *Analogy.* PLAYFAIR, § 9.

20. A theory discovered from induction, may be employed in

the discovery of new facts, and to predict the result of new combinations. Playfair, § 10.

The order of proceeding is in this case reversed, and is called the method of *Synthesis*, while the inductive method is called *Analysis.*

21. The rules for philosophizing are as follows, viz.:

(1.) More causes of natural things are not to be assigned than are both true and sufficient to account for the phenomena.

(2.) Of natural effects of the same kind, the same causes are, therefore, to be assigned as far as can be done.

(3.) The qualities of bodies which cannot be increased or diminished, and which are found common to all bodies on which experiments can be made, are to be considered qualities of all bodies whatsoever.

(4.) In Experimental Philosophy, propositions collected from the phenomena by induction, are to be considered as either entirely or nearly true, in spite of contrary hypotheses, until other phenomena occur, by which their accuracy may be more fully proved, or by which they may be rendered liable to exceptions. Newton's *Principia.* Vol. 3. p. 2. of Horsley's edition.

The principle of the sufficient reason is also often of great use in philosophizing; it may be announced as follows, viz. Nothing exists in any state that is not determined by some *reason* to be in that state, rather than in any other. Playfair, § 12.

22. Experiment is not only valuable as the foundation of all Natural Philosophy, but is indispensable in a course of lectures upon that subject. It is not however to be expected that all the experiments made by the founders of the science, can be performed before a class. It is therefore proper to explain the purposes for which experiment is introduced.

It verifies the results of our reasoning, and shows if all the circumstances have been taken into account:

It exemplifies how general principles may be applied to explain particular facts :

It impresses the mind more forcibly with the truth of the principles that have been laid down ; and of the inferences deduced from them. PLAYFAIR, § 13.

The definition of motion, which has been given above, involves the consideration of space and time :

23. Space is either absolute or relative. Absolute space is extension without limit, immoveable, but penetrable by matter. Relative space is that part of absolute space which our senses define, by its relation to bodies within it.

24. Place is also either absolute or relative. Absolute Place is the portion of absolute space occupied by a body. Relative Place is the space a body occupies, considered with relation to other bodies.

25. Absolute Time is an abstract idea, of which no satisfactory definition can be given.

Relative time is a portion of duration measured by means of motion.

26. The cause which puts a body in motion, whatever be its nature, is called a Force.

The direction of a force is the straight line in which it tends to cause the point to which it is applied to move.

When several forces are applied to the same body at the same instant of time, they reciprocally modify each other. If they entirely destroy each other, so that the body is kept at rest by their joint action, we say that an equilibrium takes place among them, or that the body is in equilibrio. Mathematicians have succeeded in reducing all considerations of motion to mere questions of equilibrium. POISSON, *Mechanique*, § 2.

27. Mechanics is the branch of Natural Philosophy which treats of the motion and equilibrium of bodies. It also treats of the construction of Machines. The first of these is called by Newton, Rational; the second, Practical Mechanics.

Rational mechanics is divided into two parts; Statics, which treats of equilibrium, and Dynamics, which treats of the motion of bodies.

The first principles of mechanics apply equally to solid and to fluid bodies, but are modified by the individual nature of each. Fluid bodies, however, present peculiar difficulties to the investigation of their mechanical action. We consequently treat of the statics and dynamics of fluids separately, calling the parts of the course devoted to them Hydrostatics and Hydrodynamics.

The action of bodies upon each other necessarily involves motion. Mechanics, in its most extended sense, would therefore include every department of Natural Philosophy, together with Chemistry. We do not, however, carry the definition to this extreme.

28. There is a class of material substances, whose distinctive property is radiation from a centre. The consideration of the action of these bodies, forms a separate department, to which the name of Physics is now restricted.

The radiant substances known, are Heat, Light, Electricity, and Magnetism. Philosophers still dispute whether all these are not modifications of some one principle; and there is with some a strong doubt whether they are material substances or not.

29. Neither Mechanical nor Physical Action, in the restricted sense, produces any change in the internal constitution of bodies. When, however, we place certain bodies in close contact with each other, an action takes place that changes their

nature, character and properties; their mode and state of existence. This action is called Chemical, and the science which considers it, Chemistry.

30. Mechanics and Physics are still farther abridged by the separation of Astronomy from them. It is the science which treats of the motion and appearances of the heavenly bodies.

This division of Natural Philosophy into three great branches, Mechanics, Physics, and Astronomy, (and Chemistry might form a fourth,) affords the means of ranking the objects of study in regular order, and introduces the advantages of philosophical arrangement in the classification and comparison of facts. No one of these departments, however, is entirely distinct and separate from the other. Physical considerations continually arise in treating of mechanical science; mechanical reasoning illustrates physical facts; while the theory of astronomy would fall to the ground, were it not for the powerful aid furnished by mechanical philosophy.

31. The study of Natural Philosophy is attended with many advantages.

It has a tendency to add to the conveniences and comforts of social life; to improve the state of agriculture, commerce, and manufactures; to embellish all the fine and useful arts; to lead us to know and prove the existence of a provident deity; and to enable us to understand his attributes and admire his wisdom.

MECHANICS.

SECTION FIRST.—STATICS.

OF FORCES.

The several circumstances which should be known with regard to a force, are, its point of application, its intensity, and its direction.

When a point is in motion, it will be known by the fact that its perpendicular distance from one or more of three planes mutually intersecting each other in space, is continually changing, wherefore :

32. To determine the position in space of the point of application of a force, we refer it to three planes taken at will perpendicular to each other. The perpendicular distances from the point to these three planes are called Co-ordinates.

The mutual intersections of the planes are called the axes of these co-ordinates, and the axes and co-ordinates are parallel to each other.

The common intersection of the three planes is called the origin of the co-ordinates. (*Leçons des Ecoles Polytechniques.*) Vol. 5.

33. Forces are made commensurable quantities by referring them to some conventional force as the unit; they may then be represented either by lines or numbers.

The intensity of a force is usually represented by a line drawn in the direction of the force from its point of application, and this is the mode of construction adopted by the best writers on statics. POISSON; § 4.

34. The direction of a force in space is determined by a knowledge of the angles it makes with three lines drawn through its point of application parallel to the axes of the co-ordinates. Poisson, § 6.

If we call these three angles a, b, and c, we shall find a constant relation to exist among them determined by the equation. $\cos.^2 a + \cos.^2 b + \cos.^2 c = 1$. Poisson, § 8.

If all the forces are in one plane, the equation becomes $\cos.^2 a + \cos.^2 b = 1$. Poisson, § 9.

If they are parallel, $\cos.^2 a = 1$. and $a = 0°$ or $a = 180°$. Poisson, § 10.

*35. When the direction of a force is once determined, its action will not be changed by transferring the point of application to any other point in its direction, provided we consider this second point as attached to the other by an inflexible straight line, and that the intensity and direction of the force remain the same. Poisson, § 28.

COMPOSITION AND RESOLUTION OF FORCES.

36. When a body is submitted to the simultaneous action of several forces that act upon it in different directions, but are not in equilibrio, it is evident that it must move in a determinate direction, and that there is no impropriety in attributing its motion to a single force acting in that direction. This force is called the Resultant of those that have caused the motion, and they are called Components.

This resultant identically replaces its components, and is in consequence in equilibrio with them when applied to a given point in a direction directly contrary to its true one. Poisson, § 11.

37. The resultant of two forces is represented in magnitude

and direction by the diagonal of a parallelogram whose sides represent the magnitude and direction of these two forces. Poisson, *Additions*. Poisson, § 13, 14, 15, and 16. *Mechanique Celeste*, vol. 1, § 1.

By means of this theorem, every question with regard to the composition of two forces into a single one, and the decomposition of a single one into two others, is reduced to the resolution of a plane triangle.

For instance: if two forces be represented in magnitude by P and Q, the angle contained between their directions by m, the value of the resultant R may be found by the equation.

$$R = P^2 + 2\,P\,Q.\ \cos.\ m. + Q^2,$$

That of the angle which R makes with P by the equation.

$$\text{Sin } x = \frac{Q.\ \sin.\ m.}{R}$$

Poisson, § 17.

38. When three forces are in equilibrio round a point, each of them may be represented by the sine of the angle contained by the directions of the other two.

39. When three forces are in equilibrio round a point, they must be all in the same plane. Poisson, § 18.

40. If three forces applied to the same point are represented in magnitude and direction by the three sides of a triangle, they are in equilibrio round it.

The converse of this is also true; therefore, if three forces be in eqüilibrio, any two of them are greater than the third. Gregory, § 46, 47.

41. To find the resultant of any number of forces applied to the same point, whether they be in the same plane or not: the resultant of two of them must be first taken, and this resultant combined with a third force; the second resultant with a fourth force,

and so on, until all the forces have been employed; the last resultant will be that of all the forces.

This rule gives rise to a remarkable geometrical construction; for if we form a portion of a polygon by lines equal and parallel to those which represent a number of forces applied to one point, the resultant of the whole will be represented by the line which joins the extremity of the last line to the point where the construction was commenced, and completes the polygon. Poisson, § 19.

42. If three forces act at right angles to each other, their resultant will be represented by the diagonal of a rectangular parallelopiped whose sides represent in magnitude and direction the three components.

If we call the resultant R, and the three components X, Y, and Z, $R = X^2 + Y^2 + Z^2$, and if the three angles they form be called a, b, c, $X = R. \cos. a$, $Y = R \cos. b$, $Z = R. \cos. c$. Poisson, § 20.

43. If any number of forces $F\ F'\ F''$, &c. be applied to the same point, and if we represent by a, b, and c, the angles which the force F makes with three rectangular arcs; by a', b', and c', the angles the force F' makes with the same axis, and so on; the whole of these forces may be replaced by three others, X, Y, and Z, parallel to these axes by means of the following equations.

$$X = F. \cos. a + F.' \cos. a' + F.'' \cos. a'' + \&c.$$

$$Y = F. \cos. b + F.' \cos. b' + F.'' \cos. b'' + \&c.$$

$$Z = F. \cos. c + F.' \cos. c' + F.'' \cos. c'' + \&c.$$ Poisson, § 21.

44. If the whole of these forces are in equilibrio, the equations become:

$$F. \cos. a + F.' \cos. a' + F'' \cos. a'' = 0.$$

$$F. \cos. b + F.' \cos. b' + F'' \cos. b'' = 0.$$ Poisson, § 22.

If the point of application of the forces F F′ F″ rests upon a given surface, it is no longer necessary that the resultant should = 0, in order to produce equilibrium ; it will be sufficient that it be a normal to the surface. This normal force will be destroyed by the resistance of the surface, which may in consequence be considered as a force equal and opposite to it. If, therefore, we conceive the surface to be removed, and the point to be situated in free space, the conditions of equilibrium will exist, provided we substitute for the action of the surface a force equal and directly opposite to the resultant of the others.

POISSON, § 39.

OF PARALLEL FORCES.

45. The resultant of two parallel forces acting in the same direction, divides the line of application into parts reciprocally proportionable to the component, is parallel to them, and equal to their sum. POISSON, § 31.

If the points of application and the intensity of the parallel forces remain the same, the resultant will pass through the same point, whatever be the direction of the forces, so that if the forces turn round their respective points of application, the resultant will also revolve around its point of application.

The momentum of a force with respect to any plane, is the product of the force into the perpendicular let fall from its point of application upon the plane. POISSON, § 39.

The momentum of a force with regard to a point, is the product of the force into the perpendicular let fall from the point upon the direction of the force. POISSON, § 52.

46. The momentum of the resultant of any number of parallel forces with respect to a plane, is equal to the sum of the momenta of these forces with respect to the same plane.

These momenta may be either positive or negative: they are positive when the force and the ordinate of its point of application have like signs; negative, when their signs are unlike. Poisson, § 39.

47. The Centre of Parallel forces is the point through which all the successive directions of the resultant pass, while the components turn round their respective points of application without ceasing to be parallel. Poisson, § 37.

It follows from this definition, that if a solid body is acted upon by any number of parallel forces, and if their centre be determined and supposed to be fixed, the body will remain in equilibrio around this point in every possible position, provided the forces remain parallel, and continue to act on the same points of the body.

48. A system of parallel forces is in equilibrio,

(1.) When the sum of the several forces = 0, and

(2.) When the sum of their momenta, in relation to two planes parallel to their direction, is also = 0. Poisson, § 42.

By the word sum we are to understand the aggregate of the momenta incorporated according to their signs, using the affirmative sign for the momenta of those powers that lie on one side of the point of application, and the negative for those that lie on the other. Gregory, *Mechanics*, § 60.

For the theorems by which the centre of parallel forces may be determined, see Poisson, § 39, 40.

CENTRE OF GRAVITY.

49. The force by the action of which bodies are precipitated to the surface of the earth, is called Gravity. Although the d rections of this force in different places would converge towar

the centre of the Earth on account of its spheroidal form; yet, in consequence of the great size of the earth, when compared with the dimensions of the bodies upon its surface, we may, without any sensible error, suppose the action of gravity to be parallel to itself, throughout the whole extent of any one body.

Experience shows, as we shall see in another place, that the force of gravity is not the same on every part of the surface of the earth, yet this variation may also be neglected without any error, and we may consider a heavy body as a collection of a number of particles of matter to which are applied an equal number of equal and parallel forces acting in the direction that is pointed out by the plumb line. POISSON, § 92.

50. These forces have a resultant that is equal to their sum, and parallel to their direction, and this resultant forms what is called the weight of the body. POISSON, § 93.

51. Since every particle in a heavy body is solicited by a parallel force, it follows, that if it take successively any number of different positions, as regards the direction of these forces, their resultant will always pass through one particular point. This point, which, in general terms, we have called the Centre of Parallel forces, is, in this particular case, called the Centre of Gravity. Its characteristic property, in solid bodies, is, that if it be supported, the body to which it belongs will remain in equilibrio around it in every possible position; this follows from the fact that the resultant of the several forces which solicit the body, passes through the point that is thus supported. POISSON, § 95.

We also see that when a solid body is supported by a point that is not the centre of gravity, it will be in equilibrio when the line that joins the point that is supported to the centre of gravity is vertical, and only then. POISSON, § 95.

It is evident likewise that if a body be suspended in equilibrio by a thread, that this thread will be vertical, and the prolongation of its direction will pass through the centre of gravity. This affords a method of determining the centre of gravity of a solid body, however irregular or heterogeneous, by experiment. POISSON, § 96.

When a heavy body is at liberty to move round a fixed point, it will not be at rest until its centre of gravity is either in the highest or lowest possible position.

The first of these positions forms a tottering, the last a stable equilibrium.

A body cannot rest upon a base unless a perpendicular to the surface of the earth from its centre of gravity fall within the base. PLAYFAIR, § 108.

This perpendicular line is called the Line of Direction of the Centre of Gravity.

The motions of animals are regulated by the foregoing principles, and produced by the action of their muscles in throwing forward the line of direction of the centre of gravity. BORELLUS, *De motu animalium.*

The stability of the famous leaning tower of Pisa, as well as many other curious phenomena in mechanics, depends on the properties of the centre of gravity.

52. The position of the centre of gravity may be found by experiment, as we have previously seen: To find it by calculation, we must resolve the following problem.

To find the centre of gravity of a system of bodies whose individual centres of gravity are known.

If we call the co-ordinates of the principal centre of gravity, x, y, and z, the several bodies that compose the system A, B, C, D, &c. the co-ordinates of A, a, a', a'', of B, b, b', b'', &c. we shall have the following equations.

$$x = \frac{Aa + Bb + Cc + Dd, + \&c.}{A + B + C + D, + \&c.}$$

$$y = \frac{Aa' + Bb' + Cc' + Dd', + \&c.}{A + B + C + D, + \&c.}$$

$$z = \frac{Aa'' + Bb'' + Cc'' + Dd'', + \&c.}{A + B + C + D, + \&c.}$$

POISSON, § 99.

If the body be homogeneous, and referred to one plane, we shall have the following equation :

$$d = \frac{A\,a + B\,b + C\,c + D\,d, \&c.}{M}$$

d. being the distance of the centre of gravity from the plane, and M the mass of the body. Hence we have the following rule, that the whole volume of a body multiplied by the distance of its centre of gravity from a given plane, is equal to the sum of all its particles multiplied by their respective distances from the same plane.

Although geometric lines and surfaces have no actual weight, yet it often happens that it is necessary to determine the position of their centres of gravity. In order to do this, we consider all the points in them as loaded with equal weights, or acted upon by equal and parallel forces.

The centre of gravity of a triangle may, by geometric reasoning, be demonstrated to be in the line that joins its vertical angle to the middle of its base and at a distance from the vertex equal to two thirds of this line. The centre of gravity of a triangle may therefore be found by drawing lines from the two angles to points bisecting the opposite sides.

The centre of gravity of a triangle being thus found, it is easy to find that of any polygon by dividing it into triangles, and applying the above formulæ. POISSON, § 103.

A similar chain of reasoning will lead us to discover that the centre of gravity of a solid pyramid may be found by drawing a straight line from the vertex to the centre of gravity of the base, and dividing it into four equal parts; the centre of gravity is at the distance of three-fourths of this line from the vertex. As this applies to a pyramid of any number of sides, the construction may be extended to the case of a cone.

In general, however, the centre of gravity of a mathematical figure can only be found by the application of the fluxional calculus to the formulæ given above. The general method is to refer the figure to three rectangular co-ordinates. POISSON, § 101, 102.

The method of finding the centre of gravity of a plane curve is simplified by supposing one of the planes of the co-ordinates to correspond with that of the curve. POISSON, § 103.

The same formulæ may in like manner be applied to the surface of solids and to the solids themselves.

53. The solid formed by the revolution of a plane curve round a straight line within its plane, is equal to the product of the area of the generating curve into the length of the circular arc described by its centre of gravity in its revolution.

And the surface described by a plane curve revolving in a similar way is equal to the length of the generating curve multiplied into the length of the circular arc described by its centre of gravity.

This theorem was discovered by *Guldin*, the mathematician, whose name it bears. GREGORY, § 125. M'LAURIN, *Fluxions*, § 236.

The knowledge of the position of the centre of gravity is of great importance in almost every part of Mechanics. The following are some of the most useful cases.

The centre of gravity of a straight line bisects it.

The centre of gravity of the perimeter of a polygon may be found when the centre of gravity of a straight line is known by applying the formulæ, § 61.

The distance of the centre of gravity of an arc of a circle from the centre of the circle is a fourth proportional to the radius, the chord, and the arc. Poisson, § 104.

The centre of gravity of an arc of a cycloid, that is divided into two equal parts by the vertex, is in the diameter of the generating circle at one third of the perpendicular height of the arc from the vertex. Poisson, § 105.

The distance of the centre of gravity of a parabola from its vertex is equal to three fifths of the axis. Gregory, § 119.

In a circular sector, the distance of its centre of gravity from the centre of the circle is a fourth proportional to two thirds of the radius, the chord, and the corresponding arc. Poisson, § 110.

The centres of gravity of the surfaces of cylinders, cones, and conic frustrums, are as far from the origin of the co-ordinates as the centres of gravity of the plane figures which are their vertical sections.

The centre of gravity of a hollow and a solid cone do not coincide, for in the one,

$$x = \frac{2a}{3}; \text{ in the other, } x = \frac{3a}{4}$$

The centre of gravity of the surface of a spheric segment bisects its versed sine.

In a conic frustrum, if R be the radius of the greater end, *r* that of the less, and *h* the altitude of the frustrum,

$$x = \tfrac{1}{4}\, h.\frac{3\,\mathrm{R}r + 2\,\mathrm{R}r + r^2}{\mathrm{R}^2 + \mathrm{R}r + r^2}$$

The same formula will serve for the frustrum of any regular pyramid, using the sides of the upper and lower surfaces instead of R and r.

In the solid paraboloid:

$$x = \frac{2a}{3}$$

In a frustrum of a paraboloid:

$$x = \frac{1}{3} h \frac{2\,R^2 + r^2}{R^2 + r^2}$$

GREGORY, § 124.

THE MECHANIC POWERS.

54. A Machine is an instrument, by means of which we change the direction or the intensity of a force, or both.

55. Complex machines are formed by the combination of certain simpler ones, that are called the mechanic powers.

Of these, writers on mechanics formerly enumerated six, viz. 1. The Lever. 2. The Wheel and axle. 3. The Pulley. 4. The Inclined plane. 5. The Wedge, and 6. The Screw. To these modern authors add the Funicular machine.

The Lever.

56. A Lever is an inflexible bar, whether straight or crooked, resting upon a fixed point, that is called the *Fulcrum*, around which it can move freely.

In the lever, and in all other machines, when two forces are applied, one of which is intended to countervail or overcome the resistance of the other; the first of these is called the power, the second the weight.

57. There are three kinds of lever according to the respective positions of the power, the weight, and the fulcrum. In the first, the fulcrum is between the power and the weight. In the second, the weight is between the power and the fulcrum. And in the third, the power is between the fulcrum and the weight.

58. In a straight lever, if the power and weight act in parallel directions, they are in equilibrio when they are to each other inversely as their distances from the fulcrum. Poisson, *Additions*.

59. In any lever whatever, and in any direction of the two forces, they are in equilibrio, when they are to each other inversely as the perpendiculars let fall from the fulcrum upon their directions.

These properties of the lever may be demonstrated directly, but they may also be deduced from the theory of parallel forces, and extended to any number of forces whatsoever; in this case, one must be supposed to act at the fulcrum, in the manner described, when speaking of bodies resting upon a surface; and unless the fulcrum of the lever be also its centre of gravity, it will be necessary to take its weight into consideration; this may easily be done, by considering it as a force applied to the centre of gravity of the lever, and acting in a vertical direction.

The lever is of very extensive use in practical mechanics, both in its simple state and combined. The most usual instances of its application simply are as follows, viz.

First kind of lever. The quarry-crow, handspikes, scissars, pokers, pincers, snuffers, &c.

Second kind of lever. The oars and rudders of vessels, cutting knives fixed at one end, doors moving on their hinges, &c.

Third kind of lever. Tongs, sheep shears, a ladder raising against a wall, &c. In this kind of lever, the intensity of the force is diminished, but the velocity is increased. We shall hereafter see that it is extensively employed by nature, in the organization of animal bodies.

60. A Balance is also one of the applications of the first kind of lever. In it the arms are of equal lengths; hence, as equal weights suspended from its extremities will be in equilibrio, it is made use of to measure unknown weights by means of some given standard.

It sometimes happens that the arms of a balance may be made of unequal lengths for the purpose of fraud. If this be suspected, it is only necessary to weigh the article, first in one scale, and then in the other; if the counterpoising weights in these different positions be not equal, take their geometric mean, and it will be the true weight. GREGORY's *Mechanics*, 133.

A balance, when well constructed, must have the following properties. 1. It should rest in a horizontal position, when loaded with equal weights. 2. It should have great sensibility, so that a very small addition in either scale will disturb the equilibrium. 3. It should soon return to rest, after having been put in motion by a change of the weights.

Of all weighing machines, a good balance is that which can be the most depended upon, in cases where great accuracy is desired.

61. The Steelyard or Statera Romana, is another of the applications of the first kind of lever. Its arms are unequal, and the weight of the substance under experiment is ascertained by the distance of a known weight from the fulcrum.

The steelyard cannot be employed where a great degree of accuracy is desired, but is often convenient from its portability. A very compact and useful form, constructed by Mr. Dearborn of Boston, has been introduced into the customs of the United States.

When the weight to be found, as often happens, is much greater than that of the counterpoise, the centres of the steelyard are not loaded with much more than half the weight borne by those of a balance, employed to weigh the same article; hence, the steelyard may at times be used to weigh very heavy bodies. One has been constructed for this purpose, at the West Point Foundery, which is capable of supporting seven tons, and is sensible of an increase or diminution of 5 pounds.

Other weighing machines have been contrived, to act by a combination of levers. Of these the most elegant and practically useful, is that employed in England for weighing loaded wagons and living cattle; it is called the Platform Balance, and is described by Gregory, in the 2d volume of his mechanics. Likewise by LE SAGE in his *Receuil des Memoires*, Vol. 2, p. 111.

This has also been constructed with great neatness, but on a smaller scale, at the West Point Foundery.

62. When a beam carrying a weight is supported in a horizontal position by two props, the weights which the props sustain are inversely proportioned to their distances from the centre of gravity of the weight. PLAYFAIR, § 128.

63. A compound lever is a combination of two or more simple levers, the first of which is employed to turn the second; the second the third, and so on. In a compound lever, equilibrium exists when the weight is to the power as the product of all the arms on the same side of the fulcrum with the power, is to the product of all the arms on the same side of the fulcrum with the weight.

The Wheel and Axle.

64. The wheel and axle is a machine that consists of a cylinder and a wheel having a common axis, at the two extremities of which are pivots on which the whole may turn. The power is applied at the circumference of the wheel; the weight to the circumference of the axle.

65. The wheel and axle is in equilibrio when the power is to the weight as the radius of the axle to the radius of the wheel.

The wheel and axle may be considered as a lever, so contrived as to have a continual motion round its fulcrum.

It is not necessary that the circumference of the wheel be continuous, the insertion of spokes into the axle is sufficient; windlasses and capstans are, in consequence, modifications of the wheel and axle.

A winch is also a species of the wheel and axle.

The combinations of wheels and pinions, so useful in mechanics, are reducible to the wheel and axle; their effect may be calculated by the following proposition:

66. In a series of wheels and pinions, equilibrium will exist when the power is to the weight as the continual product of the radii of all the pinions is to the continual product of the radii of all the wheels.

Wheels and axles are also sometimes combined together, by cords passing over one wheel and the following axle. The equilibrium is calculated on the same principle as that of the above proposition.

The number of teeth on the circumference of wheels being to each other as the diameters, the proposition is often expressed thus: there is equilibrium in a system of wheels and pinions, when the power is to the weight as the continual product of the teeth in the pinions is to the continual product of the teeth in the wheels.

67. In forming the teeth of wheels it is of great importance to determine their proper curvature, for the motion ought to be communicated equally, and with as little friction as possible. The most advantageous curve for this purpose seems to be the Epicycloid. The Involute of a circle has also been proposed.

See CAMUS on *the Teeth of Wheels*—HACHETTE, *Cours Elementaire des Machines*—BUCHANAN on *the teeth of wheels.*—In order that the same teeth of a wheel and pinion may coincide as seldom as possible, the numbers of them upon each circumference should be prime to each other. The ordinary method of effecting this is to add one to the number of teeth on the wheel.

The Pulley.

68. The pulley is a wheel moveable upon its axis, and having a groove cut upon its circumference through which a cord passes. It is enclosed in a box or case, that is called its block.

A pulley is either fixed or moveable. In the fixed pulley the block remains stationary, in the moveable pulley it rises and falls with the weight.

A fixed pulley does not alter the intensity of the power, but merely changes its direction.

In a moveable pulley, one end of the cord is fastened to a fixed point; the power is applied to the other, either directly or through the intervention of a fixed pulley; the weight is suspended from the block.

This kind of pulley is reducible to a lever of the second kind, and the power is evidently enabled to balance a double weight.

Fixed and moveable pulleys may be combined together in such a way as greatly to increase the intensity of the power. Each different combination will have a different degree of advantage.

To be more conveniently portable, it is usual to place all the moveable pulleys in one block, and sometimes on one axis; one cord only is employed, which is wound round the moveable pulleys, and an equal number of similarly arranged fixed pulleys, alternately. In this combination, equilibrium takes place when the power is to the weight as one to twice the number of moveable pulleys.

The simplest combination of pulleys is that where a number of moveable pulleys hang in a diagonal line above one another, each doubling the power of the one beneath it. In this, while the number of pulleys increases in arithmetical progression, the intensity of the power increases in geometrical progression. The pulley is most extensively used on ship board.

The Inclined Plane.

69. The inclined plane is a mechanic power, consisting, as its name imports, of a plane making any angle, less than a right angle, with the horizontal plane.

70. The general condition of equilibrium in the inclined plane is, that the power should be to the weight as the height of the plane to the product of its length into the sine of the angle which the direction of the power makes with a perpendicular to the plane. Poisson, § 97, 98.

When the direction of the power is parallel to the plane, equilibrium exists when the power is to the weight as the height of the plane to its length, or as the sine of the plane's inclination to the radius.

If the direction of the power is parallel to the base, equilibrium exists when the power is to the weight as the height of the plane to the base. Poisson, § *Additions*.

The Wedge.

71. The ordinary form of a wedge is a triangular prism of wood or metal. Its section is usually an isosceles triangle with a very acute angle at the vertex, but it may be of any triangular figure whatsoever.

72. In an isosceles wedge, equilibrium takes place when e power is to the weight as the thickness of the wedge to its *gth.*

This may be proved by considering the wedge as composed of two inclined planes, joined together at the base.

73. If three forces be applied perpendicularly to the three sides of a scalene wedge, they will be in equilibrio if their directions intersect each other in the same point, and if they are to each other directly as the lengths of the sides on which they act. Poisson, § *Additions.*

This may be demonstrated independently of the consideration of the inclined plane, and the properties of that mechanic power might be deduced as one of the cases.

The mere theory of the equilibrium of the wedge is of little use in practical mechanics. The wedge is generally urged by percussion, and its effects are then often greater than could be anticipated from the theory.

The only instance when a wedge acts in equilibrio, is in the construction of arches.

The wedge is much used in operative mechanics; it is employed to raise heavy weights to a small height; in all cutting instruments, as knives, chissels; in the teeth of animals, &c.

74. A wedge may have the form of a pyramid as well as of a prism; if the faces are inclined at equal angles to the base, the power and weight are in equilibrio when the first is to the last as the cosine of the angle of inclination to the radius.

Piercing instruments of all kinds are applications of this form of the wedge.

The Screw.

75. The screw is a spiral groove or thread winding round the surface of a cylinder, and cutting all lines drawn upon its surface and parallel to its axis at equal angles.

This spiral may be either on the inside of a hollow cylinder or on the outside of a solid one, and the screw is called in consequence the interior or exterior screw, as the case may be. Both must be combined in the practical application, so that the groove of the exterior receives the thread of the interior screw.

The simple screw might properly be referred to the principle of an inclined plane, for it is no more than an inclined plane wrapped round a cylinder; but in using the screw, it is usual to apply the power to the extremity of a lever fixed perpendicularly to the axis of the screw, and it thus becomes a machine compounded of the lever and inclined plane.

76. In the screw, the power and the weight are in equilibrio, when the first is to the last as the distance between any two threads of the screw is to the circumference described by the point to which the power is applied.

The screw is used for compressing bodies ; it thus forms part of the ordinary printing, and of many other presses : It is applied to raise great weights to a small height.

Under the head of hydraulic engines we shall treat of the screw of Archimedes, used for raising water.

In screws constructed of hard materials, the distance between the threads is exactly equal; and as the revolution of the head of the screw corresponds to the passage of it longitudinally through the distance between two of the threads, it may be used for dividing space into very minute parts. On this principle it is used in dividing astronomical instruments, and in *reading off* their divisions. BREWSTER'S *Edinburgh Cyclopædia, article* GRADUATION.

The Funicular Machine.

77. If a cord be attached by one end to a weight, and a force be applied at the other in the direction of its length, no mecha-

nical advantage will be gained. But if the same cord be attached by one extremity to a weight, and by the other to an immoveable point, while a power is made to act at some intermediate point of the cord in a given direction, it will exert a force upon the weight of a different intensity from its own, and draw it in a direction also different. In this case, a rope becomes a machine. POISSON, § *Additions.*

78. If a body be acted upon by forces through the intervention of ropes, the assemblage of ropes is called a Funicular Machine.

If a body be suspended by ropes from fixed points, it is still a funicular machine, for we may suppose powers equivalent in magnitude and direction to be substituted for the resistance of the points. PRONY, *Architecture Hydraulique*, § 248.

79. If a funicular machine, composed of ropes attached to each other, perfectly flexible and inextensible, be in equilibrio under the action of any number of forces whatsoever; and if these forces be transported so as to act parallel to their original directions upon a single point in the system, it will be kept in equilibrio under their joint action. Any one of the forces will also be equal and directly opposed to the resultant of all the rest. PRONY, *Architecture Hydraulique*, § 247.

80. The curve which a chain or flexible cord forms, when suspended from its extremities and acted upon by gravity, either alone or jointly with other forces, is called the Catenaria.

The properties of the Catenaria may be deduced from those of the funicular machine. See POISSON, § 134 to 151.

PRINCIPLE OF VIRTUAL VELOCITIES.

81. If we suppose the equilibrium of a lever to be for an instant disturbed, and motion to take place, the product of the power into the velocity with which its point of application would then move, is equal to the product of the weight into the velocity of its point of application.

As this motion does not actually take place, the velocity is styled *Virtual.*

This same proposition may be extended to the cases of all the other mechanic powers, and may be generalized as follows, viz.

82. When the forces, however numerous, that act upon any machine are in equilibrio, if each force be multiplied into the virtual velocity of its point of application, estimated according to its direction, the sum of all the products will be equal to nothing.

This is not only true in machines, but in every possible system of forces in equilibrio; it may be expressed by the following equation.

$$Pp + P'p' + P''p'' + \&c. = 0.$$

P, P', P'', &c. being the several forces, and p, p', p'' &c. their respective virtual velocities. Poisson.

This proposition may not only be deduced from the consideration of the equilibrium of machines, but may be demonstrated directly, as is done by La Grange in his Mechanique Analytique. In his method of proceeding, this principle becomes the fundamental theorem of mechanics. See La Grange, *Mechanique Analytique.*

OF FRICTION.

83. Whenever the forces, applied to the several points of a solid body, fulfil the conditions of equilibrium that have been laid down, the body will remain at rest. The weight of the body itself must of course have been included among the forces, as if it were applied to the centre of gravity in a direction perpendicular to the horizon.

As far as our investigations have hitherto extended, it would appear that nothing more is necessary to put the body in motion than the slightest change in the direction or intensity of the forces, producing a difference in the result of the equations of equilibrium. In practice, however, it will be found, that if a body rest on a plane surface, or touch a fixed obstacle, a physical circumstance that has not yet been taken into view will prevent motion following an apparent destruction of equilibrium. This is the *friction* which takes place between the body and the obstacle.

> Friction may be regarded as a *passive* force, for it cannot of itself begin motion, and never acts but in opposition to motions communicated by other forces. Poisson, § 126.

84. If a body lie upon a horizontal plane, it will remain at rest, and will press the plane with a force equal to its own weight. If the plane be inclined in a small degree, although the body does not begin to move, the pressure on the plane will be diminished, and a part of the weight of the body will be supported by the friction.

> The exact amount of the pressure and the weight supported by the friction may be found by the theorems of the resolution of forces; if W be the weight and i the angle of plane's inclination, P the pressure and w the part supported by friction,
>
> $P = W . \cos . i.$ $w = W . \sin . i.$ Poisson, § 127.

85. If the inclination of the plane be gradually increased until the body is just about to move, the value of w, at that instant, will be the measure of the friction.

If f be ratio of the friction to the pressure,

$f =$ tang. i. Poisson, § 127.

This furnishes a practical method of ascertaining the quantity of friction. It may also be ascertained by finding the weight which by its descent over a pulley is sufficient to draw the body under experiment along a horizontal plane. The latter appears to be the most accurate method, for it has been found that the force necessary to overcome the friction of a body at rest, and set it in motion, is greater than that which is needed to overcome the friction of a moving body.

The most accurate experiments that have been made on the subject of friction are those of Coulomb and Vince; the former gained the prize of the French academy of sciences in the year 1781. The latter published his researches in the Transactions of the British Royal Society in 1785. See THEORIE DES MACHINES SIMPLES par COULOMB, Paris, 1809, and PHILOSOPHICAL TRANSACTIONS ABRIDGED, vol. 15, p. 654.

A summary of the deductions from Coulomb's experiments is to be found in PRONY, § 1173.

86. The retardation produced by friction is an uniform force. VINCE, § 3.

This also agrees with Coulomb's experiments.

87. Friction is greatest between bodies whose surfaces are rough, and is lessened by polishing them. POISSON, § 128.

88. Friction, *cæteris paribus*, is greater between bodies composed of the same material than between bodies composed of different materials. POISSON, § 128.

89. If the rubbing surfaces remain the same, the friction increases with the pressure. Poisson, § 128.

This deduction appears to agree with the experiments of Coulomb, but is contradicted by those of Vince, who found that friction increases in a less ratio than the pressure. Bossut also mentions the same fact, and adduces the case of a ship when launched in support of his opinion.

90. If the weight continue the same, the quantity of surface has no influence on the friction. Poisson, § 128.

This is also denied by Mr. Vince, who found the friction affected, although in a very small degree, by the magnitude of the surfaces in contact. In practice, however, the increase of the friction with the pressure, and its not being affected by the surface, may be assumed as true, without any risk of error.

91. Friction is very much lessened by making the moving body roll instead of sliding. The cause of friction appears to lie in the porosity of bodies; their surfaces are covered with projections and cavities; when one body lies upon another, the excrescences of the one fall into the pores of the other, and it requires a certain force to disengage them. This must be done either by raising the one body from the other, or by breaking down the projections that interfere with the motion. Coulomb, § 96 to 101.

This hypothesis is sufficient to explain all the facts.

The effect produced by polishing is due to the smoothing of the asperities. Oil, tallow, plumbago, &c. fill up the cavities. In rolling bodies, the asperities do not need to be broken down; nor is it necessary to raise the moving body from the other in order to disengage their parts; this is done by the very action of rolling.

Friction is very much increased when a pressure is applied to the moving body sufficient to prevent the prominent parts of it from being separated from, and raised out of the cavities of the other.

Upon this principle we may account for the very powerful effect of fixed rollers in shaping and flattening bars and plates of metal.

92. These experiments and laws furnish the principles of the practical methods of diminishing friction.

Friction may, in practice, be lessened in various ways.

(1.) The rubbing body may be drawn in a direction inclining upwards from the plane on which it rests. In this manner a part of the force will be applied to raise the projections of the one surface from the cavities of the other.

(2.) The asperities may be smoothed by the mechanical action of polishing.

(3.) The cavities may be filled up with oil, grease, plumbago, &c.

(4.) The body, instead of sliding, may be made to roll, or if this be impossible, it may be laid upon rollers, or on a wheel carriage. The value of wheels and rollers arises partly from the physical circumstance already mentioned, and partly from a mechanical advantage possessed by the power applied to overcome the friction.

Rollers put under a heavy body diminish the friction in the greatest degree possible if they are spheres or cylinders, and not constrained to move round a fixed axis. In wheels, the conversion of sliding into rolling is less complete. PLAYFAIR, § 165.

(5.) Under certain circumstances, friction wheels and rollers may be introduced with great advantage.

Two of the most elegant applications of friction wheels are to be found in the machine of Atwood, and the patent blocks of the late Mr. Garnett, of New-Brunswick, N. J.

(6.) Friction may be lessened by causing heterogeneous bodies to move against one another, instead of homogeneous ones. Besides the above methods, if Mr. Vince's experiments be correct, there are some few instances where advantage is to be derived from a diminution of the surface.

93. Friction is one of the causes which operate to bring bodies moving upon the surface of the earth to rest. A simple knowledge of its action, as a retarding force, is sufficient to show the futility of the hopes of those who seek for the perpetual motion.

94. Although friction destroys motion, and generates none, it is of essential use in mechanics. It is the cause of the stability of machines, and is necessary to the exertion of the force of animals. PLAYFAIR, § 172.

Nails, screws, or bolts, would be of no use in holding the parts of a machine or building together, were it not for their friction. The wedge would be forced back at each interval between the blows that urge it. The pulley would not be moved by the cord passing over it. Arches erected upon the best principles would, if friction did not exist, be forced from their equilibrium, and thrown down by a very trifling weight.

95. When a machine is in equilibrio, under the action of the power and weight, it will not be ready to move until an addition be made to the power that is equal to the friction.

In the lever, if the ratio of the pressure to the friction be as n to 1, if P be the power, W the weight, r the diameter of a cylindrical axis used as a fulcrum, and a the length of the arm to which the power is applied; the expression for the addition required to overcome the friction will be

$$\frac{r}{na}(P+W)$$

PLAYFAIR, § 169.

Of the elementary machines, the lever and inclined plane have the least friction, the screw and the wedge the most.

OF THE STIFFNESS OF ROPES.

96. The resistance which ropes oppose to the effort of a force tending to bend them has a great analogy to friction. It is a constant quantity, being always the same in the same rope under similar circumstances. Prony, § 1206.

The best experiments on the stiffness of ropes were made by Coulomb, and are detailed in the work already quoted; an abstract of them is given by Prony.

97. In ropes the resistance to a power applied to bend them increases with some determinate power of the diameter, which we shall call n.

In new ropes $n = 2$.
In old ropes $n = \frac{3}{2}$. Prony, § 1208.

98. The resistances also are directly as the tensions and inversely as the diameters of the cylinders round which they are wound. Prony, § 1208.

99. The friction of a rope which is wound round a cylinder increases in geometrical progression while the number of turns increases in arithmetical progression.

By means of the above property we may explain the great value of this kind of resistance, when it is wished to oppose a sudden check to the motion of a body, or to make fast the end of a rope that is violently strained. Playfair, § 171.

STRENGTH OF MATERIALS.

No real improvement has been made in the theoretic consideration of the subject of the strength with which materials resist *a strain* applied to break them, since the time of Galileo. In order to subject the resistance of solids to calculation, he supposed them to be composed of a number of fibres parallel to each other; and that when they broke, the fracture commenced at one of the edges and terminated at the other, round which the solid turned as if upon a hinge.

This hypothesis, although not absolutely true, may be made the basis of deductions that agree in a remarkable manner with experiment.

100. If a beam of wood whose section is any given figure having its area $= a^2$, have one end firmly fixed in a wall; if s be the lateral strength of each fibre; d the distance of the centre of gravity of the section from the edge where the fracture terminates, and round which the beam (according to the above hypothesis) turns as a hinge; let l be the length of the beam, and W the weight, that, when hung at the end of the beam, is just sufficient to break it.

$$W = \frac{s a^2 d}{l}$$

PLAYFAIR, § 231.

101. When a beam, instead of projecting from a wall, is supported at both ends, it must break in the middle, and the termination of the fracture will be on the upper side; and hence, when a rectangular beam is supported at both ends, its strength is twice as great as when it is supported at one end only. PLAYFAIR, § 235.

111. If a rectangular beam be supported at the ends by two props, and a weight be laid on the middle to break it; the strength of it will be as the square of the depth, when the breadth is given.

Hence, the strength of beams of the same material are to one another as their breadths and the squares of their depths. And this is true, whether they be all supported at both ends or all at one end only.

In square beams of the same length, the lateral strengths are as the cubes of the sides.

In cylindric beams of equal length, the lateral strengths are as the cubes of the diameters.

If several beams be laid one upon the other, they are not able to bear a greater weight than when laid side by side.

The same rectangular beam is strongest when laid with its narrowest side up. And it is as much stronger in this position than in the other, as the greatest breadth of its section is greater than the least. EMERSON'S *Mechanics*, § 5.

112. If a beam be supported at both ends, and a given weight be laid upon it, the strain upon it will be proportioned to its length.

The strengths, therefore, of beams of the same material, and pressed by equal weights, are inversely as their lengths. But where the power which tends to break a beam is its own weight, its action increases with the square of the length of the beam, for the weight of beams of equal sections is as the length. EMERSON, § 5.

113. A beam whose section is a triangle, and which is supported at both ends, is twice as strong when the edge is uppermost as it is when the opposite side is highest. GREGORY, § 177.

114. The lateral strengths of two cylinders of the same material, and of equal length and weight, one of which is hollow and the other solid, are to each other as the diameters of their ends.

Nature appears to make great use of this principle in her works, where in innumerable instances the strength of substances is greatly augmented without increasing their weight: for instance, in the bones of animals; the feathers of birds; the stalks of reeds, grasses, &c.

In the arts there are very many cases where hollow cylinders may, with great advantage, be substituted for solid ones of equal or even greater weights. This is more especially the case when metallic shafts are employed, for some experiments would appear to show that a hollow metallic beam is stronger than a solid one of the same material and equal diameter. GREGORY, § 172.

106. Upon the same hypothesis from which the above theorems have been deduced, it may be demonstrated that the strength of a beam, in an inclined position, is to its strength in a horizontal position, to resist a vertical pressure, as the square of radius to the square of the cosine of elevation. GREGORY, § 185.

This does not agree with experiment, for, if it were true, the strength of an upright pillar would be infinite. The strength of beams of wood is, however, very much increased, by making the pressure act in the direction of their length.

107. Large beams and bars are in much greater danger of breaking than small ones; and what appears firm and strong in a model, may, when executed on a large scale, fall to pieces by its own weight. Thus we find limits to the extent of the works of art, and even of those of nature. For in all cases whatever, the strength of bodies increases only as the cube of their similar dimensions, while the energy of the power applied to break them increases with the fourth power.

The best experiments on the strength of wood of different sorts are those of Buffon.

Experiments on the strength of wood and iron have been made with much accuracy by Banks in England. They are to be found in Gregory's Mechanics, vol. i. § 191.

Experiments on the strength of white marble, made by Professor Robinson of Edinburgh, are also to be found in the same work.

108. In combining together pieces of wood, or any other materials in machines, or in the erection of buildings; the parts of each piece, and the several pieces, must be so adjusted that the strength may be always proportioned to the strain they are to endure. EMERSON, § 5.

The most important application of this subject is to the theory of architecture. The architecture that is now practised in different parts of the world, or of which remains are to be still seen, appears to have taken its rise from several different sources: Chinese architecture seems to be derived from the tent of their Tartar ancestors; Egyptian architecture, from the caves occupied as dwellings by the first inhabitants of that country; Grecian architecture, from the log hut.

The Pyramid is an improvement upon the earthen tumulus, and the rich ornaments of the Gothic cathedral are copied, in a more costly and lasting material, from the temples of wicker work erected by the Northern nations.

109. When a beam is placed in a perpendicular position, it becomes a pillar, which, if of a circular section and regular proportion, is called a column. By the investigations of La Grange, it would appear that a cylindrical form is the strongest to resist flexure. When, however, we take into view that a column has not only the superincumbent, but its own weight to support, and that the pressure on its lower parts will in consequence be greater than that upon the upper, it is evident that the thickness of the lower parts should be somewhat increased, and this is always done in practice. YOUNG's *Lectures on Nat. Phil.* § 14.

The best outline for a column is a curve that is convex externally. And this is not only the strongest, but the most graceful shape.

When a column is fixed upon a solid base, and without having any weight to support, is exposed to the action of a fluid; the best form for the outline is a curve concave towards the exterior. We find instances of such columns in nature, in the trunks of trees, and at the insertion of their branches. Smeaton, who observed this, has used it with great success in the construction of the Eddystone lighthouse. SMEATON, *History of the Eddystone.*

A wall should also be thinner at top than at bottom; and this is the more necessary, as in most cases the weight is distributed among several floors instead of resting upon the top of the wall. YOUNG, § *Lecture* 14.

OF ARCHES.

The ordinary mode of treating the theory of arches is to consider them as composed of a number of truncated wedges, kept in place by their mutual pressure, and free to move upon each other by the smallest change in the conditions of equilibrium. Friction, as well as the cohesive power of the cement, are left out of the question. The following are some of the deductions from this hypothesis.

110. In any arch composed of a number of wedges, as described above, the wedges will be in equilibrio when they are to each other respectively as the portions cut by the prolongations of their joints from a line passing horizontally through the apex of the arch.

The truncated wedges, of which an arch is composed, are called *Voussoirs*. They are usually of an uneven number.

The central voussoir is called the *Keystone*.

The surfaces where the voussoirs rest upon each other are called *Joints.*

The interior curve of an arch is called the *Intrados,* the exterior the *Extrados.*

Abutments are the masses of masonry that support the arch. The beginning of the arch is called the Spring of the arch, the middle the Crown, and the part between the crown and the spring is termed the Impost. PLAYFAIR, § 224. HUTTON, *Tracts,* vol. i.

111. In a circular arch, the weights of the voussoirs should be as the difference of the tangents of the arcs reckoned from the crown. PLAYFAIR, § 224. HUTTON, *ubi supra.*

The whole mass of masonry is supposed to be cut by prolongations of the joints, so as to extend the voussoirs to its upper surface.

112. If the weights of the voussoirs be all equal, and their lower surfaces plane, the polygon which fulfils the conditions of equilibrium, is the funicular polygon; or if it become a curve, it is the Catenaria.

The equation of the catenaria is

$$y = A \times \text{hyp. log.} \frac{A + x + \sqrt{2ax + x^2}}{A}$$

A being a constant quantity that can be determined by experiment. See PLAYFAIR, § 228.

The most remarkable property of a catenaria is, that the chain or cord, by whose suspension it is formed, has its centre of gravity lower than if it were disposed in any other line.

When it is wished to construct an arch of pieces of timber, that shall be in equilibrio, nothing more is necessary than to hang a model composed of a system of smaller but similar pieces connected together by pivots, from two points, whose distance bears the same relation to the span of the arch to be constructed as the lengths of the pieces of the model do to those which are to be used. The polygon formed in this way will be similar to a larger one, which, if inverted, will have the thrusts of all its parts in equilibrio. HUTTON, vol. i. p. 20.

If the voussoirs be not of equal weights, an arch of equilibration may be found by hanging from a number of points in a catenaria, cords or chains that represent in weight and magnitude the corresponding parts of the arch to be constructed. The new curve thus formed is an arch of equilibration. HUTTON, vol. i. p. 21. ENCYCLOPEDIA BRITANNICA, Supplement.

The chain bridges constructed in this country by Mr. James Finley, of which there are beautiful examples upon the Brandywine and Nishamony, assume naturally the above form, and as their vibrations are unimpeded, are always in equilibrio. See PORT FOLIO for 1810.

A bridge precisely similar to those of Mr. Finley, has been lately erected over the Tweed in Great Britain, by Captain Samuel Brown. EDINBURGH JOURNAL for October, 1821.

Except in the two cases of a chain bridge, and a balanced roof or arch of timber, the above theory is of no value in practice.

113. When the voussoirs that compose an arch are laid upon one another in the process of building, so far from being free to move upon each other, it is found that they will not slide downwards until the inclination of the joint becomes nearly 40 degrees. GAUTHEY, *Construction des Ponts.*

This is still further increased by the action of the cement, and thus it will happen, that an arch which fulfils none of the conditions of equilibrium of the hypothesis shall be stable, while an arch of equilibration may be crushed by its parts acting in a manner so different from the hypothesis.

114. When an arch of any description is loaded with a weight that is nearly sufficient to break it, experience shows that each half has a tendency to divide into two parts; the joints of the voussoirs near the key-stone opening at the arch, while the joints of those nearest the spring open at the opposite side. LESAGE, *Receuil des Memoires*, vol. 2.

The points of division between the two parts are called the points of rupture. A knowledge of their situation is indispensable in ascertaining the proper weight of piers and abutments to support the pressure of arches. Formulæ for finding them are given by GAUTHEY, vol. 1. p. 325. et seq.

115. The thickness of the key-stone of an arch might be no more than is sufficient to prevent it from being crushed by the superincumbent weight; its breadth no more than will include a mass of material sufficient to resist the horizontal pressure of the two halves of the arch.

In practice, a greater thickness should be given in order to sustain sudden shocks; in almost all bridges, however, the thickness of the keystone is made greater than is necessary.

Perronet's rule for the thickness of the arch at the keystone is as follows, viz:

To one twenty-fourth part of the diameter of the arch, add one foot, and from the sum subtract one line for every foot of span; the remainder will be the thickness of the keystone.

This rule of Perronet's, although it gives rise to a construction of arches apparently more bold than of almost any artist who preceded him, does not give as low a value as might be deduced upon the principle we have laid down.

A bridge constructed by Inigo Jones over the Conway, at Llanrost, in North Wales, has the keystone of its centre arch thinner than any of Perronet's, and it has stood for two centuries.

It has been usual to give piers the same thickness as abutments, and where arches are constructed separately this is necessary, but when two arches press horizontally against each other, the pier may be made much smaller, and yet be sufficiently strong. Narrow piers, in bridges, are attended with the advantage of leaving a wider passage for the water.

116. The principal application of the theory of arches is to the construction of bridges. Five different kinds of material are employed in building them, viz. stone, wood, cast iron, wrought iron in the shape of chains, and ropes.

The magnitude of the arches of bridges must be suited to the locality where they are erected, and will be limited by the strength of the material. The largest arches of stone ever constructed were those of the Bridge of Trajan over the Danube; their span was 180 English feet; this bridge is now in ruins. The span of the Waterloo bridge is no more than 120 feet; that of Neuilly over the Seine, 125 feet.

In wood, arches may be of greater magnitude; the bridge erected by Wernwag over the Schuylkill has a span of 340 feet. The length of the famous bridge of Schaffhausen was 365 feet, but it was divided into two arches.

117. The principles of Dome-vaulting have been investigated upon the same hypothesis as those of arches. The results, however, are of little value in practice. It is sufficient to know that the horizontal pressure of the vault must either be counterbalanced by heavy loads of material resting perpendicularly on its spring, or counteracted by some mechanical contrivance. Thus in the dome of the Pantheon, at Rome, the vault is much thicker near its circumference than at its centre, and its exterior figure a less arc of a greater circle than the interior section, which is a semicircle; in gothic vaults the buttresses are loaded with heavy pinnacles, and the domes of St.

Peter's, at Rome, and St. Paul's, in London, are embraced by strong chains.

The construction of domes is easier and less liable to accident than that of arches. In many cases they may be built without centres of carpentry.

Each course keys itself; they may therefore have large openings in the centre for the admission of light.

MECHANICS.

SECTION SECOND.—DYNAMICS.

OF MOTION.

We have seen that the ideas of Space and Time, are necessarily involved in the definition of Motion.

118. Velocity is the relation between the spaces described and the times elapsed since the motion began. Poisson, § 179.

119. The simplest species of motion is where the direction is rectilineal, and the spaces described in equal times are themselves equal; this is called *Uniform Motion*, or motion with an *Uniform Velocity*. Poisson, 179.

Uniform motions differ from one another in their velocities.

In measuring velocities, it is convenient to take some known portion of time for the unit, in terms of which all other portions of time are to be designated. The unit in general use is the second of time.

120. A body if once set in motion will continue to move forward in a straight line with uniform velocity, until it is acted upon by some new impulse.

Although all bodies near the Earth's surface tend to come at last to rest, however violent their original motion may have been, yet, this is no argument against the above proposition, but rather tends to confirm it; for we find the motion of all such bodies to be opposed by retarding forces; among these may be mentioned the resistance of fluid media, and friction; and we also find that by lessening the intensity of the retarding forces, the duration of the motion may be prolonged to such an extent as to show clearly, that if it were possible to remove them altogether, the motion must continue forever. The truth of this proposition is more evident in the motions of the heavenly bodies, that have for ages been known and observed, and yet, in which no diminution is perceptible. Poisson, § 182.

121. If a body, after having described a given space in a certain time shall subsequently describe a greater space in an equal time, we infer the action of some external cause, or new impulse.

A motion of this sort is called *Accelerated*; when the velocity increases by equal increments, in equal times, the motion is said to be *Uniformly* or *Equably Accelerated.*

122. So, also, when a body, after having described a given space in a certain time, describes a less space in an equal time, we again infer the action of new impulses, but in a direction contrary to that of the original force.

A motion, where the velocity in equal times decreases, is said to be *Retarded.*

If the decrements in equal times be equal, it is *Uniformly* or *Equably Retarded.*

123. We measure a force by the quantity of motion it is capable of producing, and this in aggregates of matter will depend upon the velocity of the particles and their number.

The term *Mass* is used in Mechanics to signify the quantity of matter contained in a given body. Poisson, § 310.

124. In homogeneous bodies the mass is proportioned to the bulk; but bodies of different species often contain, under equal bulks, very different quantities of matter.

Density is the comparison between the quantities of matter of different bodies contained under equal bulks.

The difference in density, which is found in different bodies, appears to depend in a great degree upon the interstices that are known to exist between the constituent particles. These interstices are called *Pores*; their number and extent are so great as to render it probable that even in the densest body, the quantity of matter is small when compared with the quantity of empty space. NEWTON's *Optics*, Book 2d.

In applying the principles of Statics to the Dynamics of aggregates of matter, or systems of bodies, the following law, known by the name of the *Principle of D'Alembert*, is of great use.

125. If there be a system of bodies mutually acting in any manner upon each other; and if at any given time we compute the motions that these bodies would have in the succeeding instant, were their mutual action to cease; and if we also compute the motions that, in consequence of their mutual action, they actually have at that instant, the motions that must be compounded with the first of these in order to produce the second, are such as, if they acted upon the system alone, would produce no motion, or would be in equilibrio with each other. PLAYFAIR, § 117. POISSON, § 332. PRONY, § 382.

OF UNIFORM MOTION.

126. The relations between the velocity of a body moving uniformly, the space it describes, and the time of description, may at once be deduced from the definition of Velocity, for if v be the velocity, s the space, and t the time

$$v = \frac{s}{t} \text{ hence } s = t\,v \text{ and } t = \frac{s}{v}$$

A more comprehensive form may be given to this equation, so as to make it capable of comparing the motions of bodies that do not set off from the same point; thus, if s be the distance of a moving body, at some particular period, from a fixed point in the direction of its motion, b the distance of the body from the same point, at the end of the time t, v the velocity as before; then $s-b$ will be the space described in the time t. And therefore

$$v = \frac{s-b}{t} \text{ and } s = v\,t + b. \qquad (1)$$

The variable quantities s and t may be either positive or negative, according as they represent the situation of the moving body in relation to the given point, at periods before or after that where its place coincides with the point.

If another body move upon the same straight line with a velocity $= v'$, if its distance from the given point $= s'$, and if it move during the time t to a distance $= b'$; the equation of its motion will be $s' = v'\,t + b'$ (2)

By combining these two equations we may solve every question in relation to the relative motion of the two bodies; if, for instance, we wish to know the instant they meet, then

$$v\,t + b = v'\,t + b'$$

whence we have

$$t = \frac{b'-b}{v-v'}$$

Poisson, § 180.

127. It has been shown that the resultant of two forces, represented in magnitude and direction by the sides of a parallelogram, is represented by the diagonal; hence it is evident, that if a body be acted upon by two forces that would each impel with it uniform velocity, it will, under their joint action, describe the diagonal of the parallelogram of which the forces are sides.

The same is true if it be acted upon by two variable forces provided the ratio of acceleration be the same in each.

Gregory, § 217, 218.

128. If a number of bodies be moving in any manner whatever,

and if an equal and parallel force act upon each of their particles of matter, the relative motions of the bodies will not be affected.

On this principle we can account for several phenomena; for instance: If a fleet be manœuvring in a current, the evolutions will be performed, and the relative positions of the ships will be the same, as if in still water;

The motions and operations in a ship sailing smoothly and regularly along are performed in the same manner as if the vessel were at rest;

Relative motions upon the Earth's surface are not altered by the rapid motion both of rotation and translation with which our planet is affected in its diurnal and annual course.

OF COLLISION.

The simplest mode in which a uniform motion can be communicated from one body to another, is by impulse.

Their action upon one another is governed by a principle known as the third law of motion, that may be announced as follows, viz,

129. Action and Reaction are equal to each other, and in contrary directions.

This holds good not only when the bodies come into actual contact, but where they act upon one another at any distance whatever. It is nothing more than a different form of the principle of Inertia already laid down, § 12.

130. All the bodies with which we are acquainted are more or less compressible; and when they have been compressed they have a greater or less tendency to recover their primitive form. This tendency goes by the name of *Elasticity*. POISSON, § 421.

131. A Body is said to be perfectly elastic when it recovers its

original form, as soon as the cause that compressed it ceases to act, with a force equal to the compressing force.

Different bodies have different degrees of elasticity. but it does not depend upon their compressibility: the gases are the most easily compressible of all bodies. and are perfectly elastic; while there are bodies easily compressible that have little elasticity, and others that are very difficult to compress, whose elasticity is not withstanding very great.

132. Although no body is absolutely devoid of Elasticity, yet in treating of the collision of bodies we investigate the laws of the communication of motion as if the bodies under consideration were either absolutely non-elastic or perfectly elastic.

Collision of Non-Elastic Bodies.

133. If A and B be the masses of two non-elastic bodies that move in the same straight line with the velocities a and b, they will, after collision, go on together; and if v be the common velocity after the stroke

$$v = \frac{Aa + Bb}{A + B}$$

If b is negative, in which case the bodies meet each other

$$v = \frac{Aa - Bb}{A + B}$$

If $A = B$

$$v = \frac{a + b}{2}$$

If one of the bodies B is at rest

$$v = \frac{Aa}{A + B}$$

If B be an immoveable obstacle

$$v = 0$$

Poisson, § 422, 423, 424. Playfair, § 70.

It may be proper here to observe, that no body in nature is absolutely immoveable; those which are so considered have their masses infinitely great when compared with the bodies that act upon them; so that they destroy their motion without acquiring a velocity that can be measured. It is thus, for instance, with bodies falling upon the surface of the earth.

From the above formulæ, others may be readily formed to show the quantity of motion lost or gained by each body after collision.

The following rule is, however, more general and easy to recollect.

134. If two Non-Elastic bodies moving in the same straight line impinge against each other; and if the *sum* of their motions before the stroke be divided into two such parts as are proportional to the quantity of matter in the two bodies, each of these parts will respectively exhibit the motion of the corresponding body after the stroke. See Helsham. *Lecture* 4.

135. If the two bodies before collision move in lines, making a given angle with each other; as there is no change produced in the absolutely quantity of motion by the mutual actions of the bodies upon each other, their common direction and velocity, after the stroke, may be easily found.

If the forces of the two bodies be respectively r and s; resolve either of them into two others, t and v, one of which (say t) is parallel, the other (v) perpendicular to the direction of s; if from the point of concourse two lines be drawn, one in the direction of s, and equal to the sum of s and t, the other perpendicular to s, and equal to v; the diagonal of the rectangle contructed upon these two lines will represent in magnitude and direction the joint motion after the stroke; divide it by the sum of the masses, and it will give the common velocity.

Collision of Elastic Bodies.

144. If A and B be two bodies perfectly Elastic, moving in the same direction with the velocities a and b, the velocity of A after impact is

$$\frac{(A - B) \times a + 2Bb}{A + B}$$

and that of B

$$\frac{(B - A) \times b + 2Aa}{A + B}$$

Several consequences follow immediately from these formulæ.

(1) When $A = B$, if the velocity of A, after the stroke, be called v, and that of B after the stroke v',

$$v = b \quad v' = a$$

so that in the case when the masses are equal, the velocities are interchanged.

(2) The relative velocity of the bodies, after the stroke, is equal to their relative velocity before the stroke, but with the contrary sign.

(3) In the shock of perfectly elastic bodies, the product of their masses into the squares of their velocities is the same both before and after the stroke. Poisson, § 428.

145. The product of the mass of a body into the square of its velocity is what has been called the *Vis Viva* of the body.

Some philosophers estimate the force of a body in motion in this way, instead of considering it as due to the mass and velocity simply. The third deduction from the action of elastic bodies upon each other, has been alleged by them in confirmation of their views.

138. In order to find the quantity of motion lost or gained by the collision of two Elastic bodies, we have the following rule: Consider them first as non-elastic, and estimate, by the rule in section 134, the motion that each would have after the stroke Find also the quantity and direction of the motion that would be communicated to each of the bodies, which is of course the difference between its original motion, and that just found. If to the quantity of motion that each body would have after the stroke, if non-elastic, be added the motion commmicated to it, upon the same hypothesis, the sum will be the quantity of motion that each body actually has after the impact. HELSHAM, § 5.

139. If an elastic body in motion strike against a greater elastic body at rest, the quiescent body will be carried forward with a quantity of motion greater than that possessed by the striking body; and the striking body will return in a direction contrary to its original one.

It will thus happen, that although there is an increase of the motion, if that communicated to the larger body be alone considered, yet that the *sum* of the two motions, after impact will be exactly equal to what the smaller body had previously.

140. If between two unequal elastic bodies A and C a third, B, be interposed; and if the least A be made to strike on B, with any given velocity, the motion communicated to C. will be the greatest possible when the masses of A, B, and C, are in geometric progression.

If the number of bodies in geometric progression be increased without limit, the quantity of motion communicated to the last may also be increased indefinitely. Notwithstanding this, as all the bodies will move back after collision except the last, the sum of all the motions in the direction of the first motion, is never greater than Aa. The sum of the products of each body into the square of its velocity also remains the same as it was before, namely Aa^2. PLAYFAIR, § 77.

141. If an elastic body strike against an immovable obstacle it will be reflected from it. If the obstacle be a plane, and the body spherical, its direction before and after collision will make equal angles with the plane, but towards opposite sides.

If the bodies be not perfectly elastic, their motions after collision are not such as are determined by § 138. They may however be investigated upon a similar principle.

> When the bodies are not perfectly elastic, the sum of the products of the bodies into the squares of their velocities, does not continue the same after collision that it was before; but the quantity of motion estimated in a given direction, remains the same in every case. PLAYFAIR, § 80. POISSON, § 525, 428.

142. In investigating the laws of collision, whether of elastic or non-elastic bodies, a property of their centre of gravity is discovered, which we shall hereafter find to be common to the centres of gravity of all systems of bodies that act mutually upon each other; this property is as follows, viz. The State of the centre of Gravity of bodies impinging against each other, either as remaining at rest, or moving uniformly forward in a straight line, is not affected by their mutual action.

OF MOTION EQUALLY ACCELERATED, OR RETARDED.

143. There are in nature two distinct species of forces; those of one description act upon bodies during an inappreciable instant of time, and then abandon them to themselves; they consequently cause a rectilineal and uniform motion. Forces of the other species act without interrnption on the moving body during the whole duration of its motion.

> If the action be constantly in the same direction, the motion it produces is Rectilineal, but the spaces described in equal times are not equal. POISSON, § 183.

Those motions where the relation between the spaces described, and the corresponding times is constantly changing, are in general terms called *Variable.*

The forces which by their uninterrupted action cause variable motions, whether accelerated or retarded, are in general terms called *Accelerating Forces.*

Although the motion produced by accelerating forces, is neither uniform, nor an assemblage of uniform motions in the same direction, yet, in order to represent a variable motion with more ease, we consider the time as if it were divided into a number of infinitely small parts, and the motion as uniform during each of these small portions ; variable motion is thus resolved into a succession of uniform motions, the duration of each of which is infinitely small, and all the corresponding velocities different. POISSON, § 184.

The changes of motion, however, are always made gradually, and never *per saltum.* See PLAYFAIR, § 95.

By thus decomposing a variable motion into an infinite number of uniform motions, the demonstrations and calculations are simplified, while the evidence of the one, and the accuracy of the other, are not in any way affected. This mode of proceeding is analogous to that made use of in the higher Geometry, where we suppose polygons of an infinite number of sides to be substituted for curved lines.

144. When a body is moving with variable velocity, the motion may at any instant be rendered uniform by removing the action of the accelerating force ; the velocity of the uniform motion thus produced, is called the final velocity acquired by the moving body, or simply its velocity at that instant of time.

145. A variable Motion, where the increments or decrements of velocity are equal in equal times, is said to be uniformly or equably accelerated or retarded. The accelerating force by which it is produced is said to be *Constant.* POISSON § 187.

146. If s be the space described by the action of an accelerating force, if t be time, v the velocity, and g the measure of the accelerating force then,

$$s = \frac{gt^2}{2} = \frac{v^2}{2g} = \frac{t\,v}{2}$$

whence we have

$$t = \frac{v}{g} = \sqrt{\frac{2\,s}{g}} = \frac{2\,s}{v}$$

$$v = \sqrt{2\,gs} = \frac{2\,s}{t} = g\,t$$

$$g = \frac{v}{t} = \frac{2\,s}{t^2} = \frac{v^2}{2\,s}$$

If the time of the motion's continuance be unity,

$$s = \frac{g}{2} \text{ and } v = 2\,s$$

hence ;

147. The velocity acquired by a body acted upon by a constant accelerating force during a given time, is such as would cause it to describe, with uniform motion, twice the space in an equal time.

g The measure of the accelerating force is equal to twice the space described from rest, in the unit of time. POISSON, § 187.

148. The spaces described are as the squares of the times, and as the squares of the final velocities.

If the times be as the series of natural numbers, the velocities acquired will be as the series of even numbers ; the whole spaces as the series of square numbers ; and the spaces corresponding to each unit of time as the series of odd numbers.

GREGORY, § 243.

The motion of heavy bodies in vacuo, affords an instance of motion with an equably accelerated velocity.

Of the Attraction of Gravitation as an Accelerating Force.

149. The action of the attraction of gravitation is not confined to the surface of the Earth, but pervades the whole Universe.

This great law of nature was first discovered by Sir Isaac Newton, and is confirmed by every fact, and by the result of all calculations.

Newton, considering that the power of gravity acts equally upon all matter upon or near the surface of the Earth, that it is not sensibly less on the tops of the highest mountains, that it affects the atmosphere to its utmost limits ; could not believe that its influence ceased abruptly, but was induced to think it was a more general principle, and extended to the heavens ;

By computing the distance which the moon falls from a tangent to her orbit in a minute of time, and comparing it with the distance a heavy body falls near the surface of the earth, in a second, he found that both observed the same law ; whence he inferred that the cause was the same in both. Having thus shown that the attraction of gravitation extended to the moon, it was easy to infer that it included in its influence all the planets of the Solar System.

M'Laurin, *Philosophical Discoveries of Newton.*

150. The Attraction of Gravitation is common to all ponderable bodies, and mutual between them.

It is proportionable to the quantity of matter in the body.

Its intensity decreases as the square of the distance from the centre of attraction increases.

When a heavy body falls near the surface of the earth, it might at first appear that it did not act upon the earth, for the latter remains apparently immoveable ; nor was it for a long time perceived that large masses of matter projecting above the level of the earth, produced any change in the direction of gravity. These appearances led philosophers, before the time of Newton, to suppose this species of attraction to be confined to the earth, and not to be a general property of matter. When, however, a heavy body falls, the law of the equality of action and re-action, is not less true than in any other case of the communication of motion; but the mass of the earth is so enormous, compared with all the bodies that project from, or move near its surface, that the motion communicated to it by their attracting forces is infinitely small.

It has been found by astronomical observations, taken on opposite sides of lofty mountains, that the direction of gravity, as shown by the plumb-line, was altered by their action.

This phenomenon was first remarked by the French Academicians, who visited South America for the purpose of measuring the length of a degree of the meridian.

Dr. Maskelyne, for many years the British Astronomer Royal, made a series of astronomical observations near Schehallion a mountain of Scotland, by which the amount of the deviation of the plumb-line from the true vertical was noted. An exact survey was, at the same time, made of the mountain, from which its cubic dimensions were ascertained. The specific gravity of the substances composing it were also accurately determined. From these data, Dr. Hutton has inferred the mean density of the earth to be about five times as great as that of water.

Dr. Hutton has more recently proposed to make an experiment of the same nature on the opposite sides of one of the pyramids.

The density of the earth once known, Physical Astronomy furnishes us with the means of determining that of the Sun, and all the other planets of the system.

See HUTTON's *Tracts.* vol. 2.

Bodies falling near the surface of the earth are retarded by the resistance of the air, and thus it will appear that dense bodies *fall more rapidly* than those which are rarer; if, however, a *dense and rare* body fall together in the exhausted receiver of an air pump, they will both reach the plate of the pump at the same instant of time, showing that the action of gravity varies with the quantity of matter.

The law of the decrease of gravity with the increase of the squares of the distances, was deduced *a priori*, by Newton; it is found to agree exactly with observation.

151. If Gravity were a constant force, its action upon a body falling freely near the surface of the earth, would produce an equably accelerated motion.

The theorems that have been laid down to express the relations of the several circumstances attending a motion of this sort, will, therefore, be applicable to the motion of a falling body, upon the hypothesis that the force of gravity is the same in every part of the space described by the falling body, and that it is not acted upon by any other force.

The force of gravity decreases in fact, according to the law just laid down; but this decrease is so slow, that there is no error in supposing it a constant force.

The resistance which bodies moving near the earth meet with from the medium in which the motion is performed, is an error of greater magnitude, but the exact law which this retarding force follows, has been ascertained. It is the usual, and best mode of proceeding to consider the motion of falling bodies, first, as taking place in vacuo, and then to apply a correction for the air's resistance.

152. The distance a heavy body falls in the first second of time is $16\frac{1}{12}$ feet, and it acquires a velocity that would carry it uniformly through $32\frac{1}{6}$ feet in the same space of time: g the measure of the accelerating force in the formulæ of § 146. is therefore equal to $32\frac{1}{6}$ feet.

If it were 32 feet exactly, which is near enough for all cases where no great accuracy is required, these formulæ would become

$$s = 16t^2 = \frac{v^2}{64} = \tfrac{1}{2}\,t\,v$$

$$t = \frac{v}{32} = \frac{1}{4}\sqrt{s} = \frac{2s}{v}$$

$$v = 8\sqrt{s} = \frac{2s}{t} = 32\,t$$

The value of this force varies in different latitudes, as we shall see hereafter.

By the *velocity due to a given height*, is understood the velocity a heavy body would acquire in falling from that height, and by the *height due to a given velocity*, is understood the height from which a body must fall to acquire the velocity.

153. In order that a body projected directly upwards, shall rise to a given height, it must receive an initial velocity equal to that due to the given height.

The velocity will be uniformly retarded.

The times of ascent and descent will be exactly equal.

The formulæ § 146. and § 152. apply equally well to bodies projected upwards, and to falling bodies; only the times must, in the former case, be reckoned from the end of the motion instead of the beginning.

154. If two bodies, A and B, (of which A is the heavier,) be suspended over a pulley, moveable without friction about a fixed

centre, A will descend, and be accelerated by a force proportioned to

$$\frac{A - B}{A + B}$$

or to the difference of the weights divided by their sum.

Playfair, § 88. Poisson, § 335.

If g be the force of Gravity, the formula that will express the velocity of A, at any given time after it begins to descend will be

$$v = \frac{A - B}{A + B}\, gt$$

It is upon this principle that Atwood's machine for investigating the laws of motion of falling bodies is constructed ; although the friction cannot be entirely removed, it is rendered so very small by making the axis of the pulley rest upon friction wheels, that the action of this retarding force may be omitted in the investigation without sensible error.

The resistance of the air may likewise be disregarded, for the experiments are made at such small velocities, that the motion is but little affected by this cause of disturbance.

There are two other ways of ascertaining the value of g : one is by direct experiments upon bodies falling from a great height ; in this way the resistance of the air is of a great magnitude ; the other is by means of the pendulum, the rationale of which shall be explained hereafter.

155. If an arm of an inflexible substance be suspended from its centre of gravity in a horizontal position, by means of a very slender thread, or metallic wire, it will remain in one position, determined by the action of the thread by which it is supported. But any force of sufficient intensity to twi

the thread will be powerful enough to change the direction of the horizontal arm.

There is an instrument invented by Coulomb that acts upon this principle ; it is called The Balance of Torsion, and has, among other uses, been applied to determine the ratio between the attracting power of the Earth, and that of bodies of known density near its surface. Cavendish, an English Philosopher of eminence, conducted a very accurate set of experiments with this view ; from his investigations, it would appear that the mean density of the earth was 5½ times as great as that of water. PHILOSOPHICAL TRANSACTIONS, ABRIDGED, vol. XVIII.

From this, as well as the Observations at Schehallion, it is known that the mean density of the earth is greater than that of any portion of its surface, and that it probably increases in density towards the centre, thus refuting the idea of those persons who have ignorantly imagined the earth to be hollow.

OF BODIES DESCENDING BY THE ACTION OF GRAVITY DOWN INCLINED PLANES.

156. The force that accelerates the motion of a body down an inclined plane is to the force of gravity as the height of the plane is to its length, or as the sine of the angle of the plane's inclination to radius.

The motion of a body down an inclined plane is uniformly accelerated.

The space through which a body descends from rest upon an inclined plane is to the space through which it would have fallen freely by the action of Gravity, as the height of the plane to its length, or as the sine of the plane's inclination to radius.

The velocities which a body would acquire, in descending down an inclined plane, or in falling through its altitude, are equal ; *and, consequently*, the velocity acquired by a body in falling from rest through a given height, is the same whether it fall freely, or descend along a plane of any inclination whatsoever.

The time in which a body descends along an inclined plane is to the time in which it would have fallen freely through the perpendicular altitude, as the length of the plane to its height, or as radius to the sine of inclination; and in planes of equal altitudes but of different inclinations the times are as the lengths of the planes.

In planes similarly inclined, the times of descent are as the square roots of their lengths, or of their heights.

If the chords of a circle terminating at either extremity of a vertical diameter be considered as inclined planes they will all be described by a heavy body in the same space of time, and this will be equal to the time of fall through the vertical diameter.

The velocities which bodies acquire in descending along chords of the same circle are as the lengths of the chords.

157. If a body descend along a system of contiguous planes without suffering any loss of motion at the angles, it will acquire a velocity equal to that it would have acquired in falling freely through the height of the plane, and sufficient to carry it up an equal and similar system of planes to an equal altitude, and in a time equal to that of the fall.

It is not, however, true, that a body can descend along a system of inclined planes, without receiving a shock at each of the angles, and it may be demonstrated that the loss of motion at each angle is proportioned to the versed sine of the inclination of the planes. PLAYFAIR, § 196.

If the number of planes be indefinitely increased, their inclination will be diminished until the sum of the versed sines shall be less than any given magnitude. The above proposition will therefore be true of bodies descending upon curved surfaces.

158. If two heavy bodies descend, without their motion being

checked at the angles, over two similar but unequal systems of inclined planes, the times of descent will be as the square roots of the lengths of the systems; and if they descend over two similar curved surfaces, the times will be proportioned to the square roots of the length of the surfaces.

When bodies decend over similar arcs of circles, the times will be as the square roots of the diameters.

In investigating the above results, the friction of the descending bodies upon the inclined planes, and the resistance of the air, are not taken into account.

If a body be suspended from a fixed point and made to vibrate, its motion will resemble that of a body descending over a curved surface. The retardation produced by friction on the plane will be removed, and another of less magnitude, viz, the friction upon the point of suspension will begin to act. A body suspended in this manner is called a Pendulum.

OF PENDULUMS.

The laws of the motion of Pendulums are deduced from the consideration of the vibrations of a gravitating body of indefinitely small size, and suspended by an inextensible thread supposed void of Gravity. This is called a *Simple Pendulum.*

The motion of such a pendulum in one direction from a state of rest until it begins to return in an opposite direction is called an oscillation.

159. If a Pendulum were constantly to perform equal arcs of the same circle in its oscillations they would be all isochronous. When, however, a pendulum is set to vibrate, the arcs it describes become gradually less in consequence of the air's resistance and the friction upon the point of suspension; nor has any uniform maintaining power been yet applied.

160. In a pendulum vibrating in very small arcs of a circle the accelerating force is nearly proportioned to the distance of the point where the oscillation commences from the lowest point of the arc.

The accelerating force is actually proportioned to the sine of half the arc described in an oscillation; but this ratio finally becomes the same as that of the arcs themselves.

161. The vibrations of the same pendulum in very small circular arcs are nearly isochronous. PLAYFAIR, § 198.

The isochronism of vibrations in small arcs of the same circle has been demonstrated by some upon the principle that the half of the arc finally corresponded with its chord. And as the times of descent along all the chords were equal, it was inferred that the times of vibration in small arcs were equal also. This demonstration is not satisfactory, for it can be shown that the more nearly the chord and arc approach in magnitude, the greater is the difference in the time of describing them.

See § 164.

162. When a pendulum vibrates in very small circular arcs, if T be the time of an oscillation p the ratio of the circumference of a circle to its diameter l the length of the pendulum, and g the measure of the force of gravity, the time of an oscillation may be found nearly by the following formula.

$$T = p \sqrt{\frac{l}{g}}$$

If l and T be given

$$g = \frac{lp^2}{T^2}$$

when $T = 1$

$$g = lp^2$$

POISSON, § 272.

It is by means of the latter formula that the intensity of gravity, or the value of g, may be determined at any point upon the surface of the earth from observations upon the Pendulum.

The times of the vibrations of pendulums of unequal lengths are respectively as the square roots of their lengths.

The time of an oscillation is to the time of the fall of a heavy body through half the length of the pendulum as the circumference of a circle is to its diameter.

The lengths of pendulums are inversely as the squares of the number of their respective vibrations in equal times.

The force of gravity in any place whatsoever is proportioned to, and may be measured by, the length of the pendulum vibrating seconds.

163. If a pendulum be placed between two Cycloidal cheeks, and made to vibrate, it will describe in its oscillations arcs of a cycloid identical with that which would be formed by an union of the two cheeks.

164. A Pendulum being thus made to vibrate in a cycloid is at every moment accelerated by a force proportioned to the arc of the cycloid intercepted between it and the lowest point of the arc.

Hence the vibrations of a pendulum in a cycloidal arc whether great or small are all isochronous. Playfair, § 208.

165. If l be the length of the pendulum vibrating in a cycloid, the time of a vibration whether the arc be great or small will be

$$p\sqrt{\frac{l}{g}}$$

This is the same expression as that deduced for the vibration of a pendulum in a very small circular arc. When, however, the circular arc becomes greater, the value of the time of an oscillation will no longer be the same.

166. The times of vibration in a cycloidal and in a circular arc, and twice the time of descent in the chord of half the circular arc are as

$$1,\ 1+\frac{a}{8r},\ \text{and}\ \frac{1}{.7854}$$

The first and last are constant quantities, but the middle one varies with the extent of the arc of vibration of which a is the versed sine, while r is the length of the pendulum. HUTTON's *Tracts*, vol. 3.

Hence the time in the cycloid is the least, and that in the chord the greatest.

167. There is another singular property of the cycloid, viz, that of being the curve of swiftest descent; for the line in which a heavy body moves by the action of Gravity from one point to another, that is neither in the same vertical nor on the same horizontal plane, in the shortest possible time, is an arc of a cycloid having for its base a horizontal line drawn through one of the given points and intersecting the vertical passing through the other.

POISSON, § 288.

168. The simple pendulum, such as it has been defined above, and from the consideration of whose properties the foregoing laws have been deduced, cannot exist in nature, inasmuch as we cannot abstract from the figure and extent of the body or bodies of which it is composed. The Pendulums on which experiment may actually be made are called compound pendulums.

169. The *Centre of Oscillation* in a compound pendulum is the point in which, if all the matter of the pendulum were supposed to be collected and attached by an inflexible line void of weight to the point of suspension, the vibrations would be isochronous with those of the compound pendulum itself.

The distance, then, between the point of suspension, and centre

of oscillation of a compound pendulum, is equal to the length of an isochronous simple pendulum.

Hence we call the distance between the centre of oscillation and the point of suspension of a compound pendulum, the *Length.*

170. The distance of the centre of oscillation of a pendulum composed of several bodies from its point of suspension may be found by dividing the sum of the products of each body into the square of its distance from its point of suspension. by the sum of the products of each body into its distance from the point of suspension.

If the bodies be A, B, C, &c. the distances of their centres of gravity from the point of suspension a, b, c, &c. and x the distance between the point of suspension and the centre of oscilla-

$$x = \frac{Aa^2 + Bb^2 + Cc^2 + \&c.}{Aa + Bb + Cc + \&c.}$$

x will, of course, be the length of an isochronous simple pendulum. When any of the bodies are situated above the point of suspension their distances from it must be accounted negative quantities. PLAYFAIR, § 201.

171. As we may consider that any compound pendulum whatever is composed of an infinite number of small parts, the above theorem may be applied through the intervention of the fluxional calculus to determine the situation of the centre of oscillation of any regular figure.

In a cylinder suspended from the centre of one of its circular bases, if a be the length of the cylinder, r the radius of the base,

$$x = \frac{2a}{3} + \frac{r}{2a}$$

If the thickness of the cylinder be diminished until it become a rod of insensible diameter, there will be but little error in taking

$$\therefore x = \frac{2a}{3}$$

In a cone suspended from its vertex, if a be its altitude, and r the radius of its base,

$$x = \frac{4a}{5} + \frac{r^2}{5a}$$

if $r = a$, which is the case in a right cone, the expression will become $x = a$
hence, if a right cone be suspended by its vertex, the centre of oscillation is in the middle of its base.

If a homogeneous sphere, whose radius is r, be suspended from a point whose distance from its centre is d,

$$x = d + \frac{2r^2}{5d}$$

If the point of suspension be in the surface of the sphere,

$$x = \frac{7r}{5}$$

GREGORY, § 311. PLAYFAIR, § 202, 203, 204.

172. The centres of suspension and oscillation are convertible points; that is to say, if the centre of oscillation of a compound pendulum be made the point of suspension, the former point of suspension will become the centre of oscillation.

This remarkable proposition, which was demonstrated by Huygens along with the other properties of the centre of oscillation, remained without any practical application until the present time, when Kater has applied it with great success to the measurement of the lengths of pendulums.

173. There are three several purposes to which pendulums

have been either actually applied, or proposed to be applied, viz. to the measure of time, to the determination of the force of gravity, and as a universal standard of measure.

Galileo was the first who observed the isochronism of the motion of pendulous bodies. Huygens, who investigated the subject more closely, discovered, that as a pendulum was gradually brought to rest by friction round the point of suspension, and by the resistance of the air, the time of its oscillations became less. To obviate this inconvenience he proposed to make the pendulum vibrate in the arc of a cycloid. However good this plan may be in theory it was found impossible to reduce it to practice; but the consideration of motion in a cycloid led to the improvement of making the arcs described by pendulums as small as possible.

As a measure of time, pendulums are attached to instruments known by the name of *Clocks*, that serve the two fold purpose of counting the vibrations of the pendulum, and restoring to it at each oscillation as much force as it loses by the retarding forces that act upon it; this subject shall be treated of in another place.

174. The usual form of a pendulum is a lenticular body attached to an inflexible rod of some metallic substance. As it is found that all metallic substances are affected by alternations of heat and cold, the rate of a clock regulated by a pendulum of this sort must be constantly varying. Means have, however, been found of remedying this defect, the general principle of all of which is, to construct the pendulum of two substances combined in such a way that the expansion of the one upwards shall *compensate* that of the other downwards, and vice versa.

The most remarkable of the contrivances for this purpose are the *Mercurial Pendulum* invented by Graham, and the *Gridiron Pendulum* invented by Harrison. The invention of the last is also claimed by French artists.

In the Mercurial pendulum the rod is a simple metallic verge, the bob is composed of a glass vessel containing mercury. The quantity of the latter is so adjusted that its expansion upwards in the containing vessel shall exactly counterbalance that of the rod downwards; the centre of oscillation is thus constantly maintained at the same distance from the point of suspension. Graham investigated with great care the relative expansions of the solid metals, but found them so nearly similar as to prevent his succeeding in a compensation by making the bob of one metal and the rod of another, and appears to have abandoned the idea of employing them. Harrison, on the other hand, after observing that the relative expansions of brass and steel were nearly as 5 to 3 thought of applying the two metals in the *rod* of the pendulum. The rod of his pendulum, instead of being a single metallic verge, is composed of nine bars, alternately of brass and steel, of such proportionate lengths that by their motion in opposite directions the common length of the rod remains constant.

The form of the Mercurial pendulum has been changed, and the instrument improved of late by that eminent maker of philosophical instruments, Troughton. He has also improved the construction of the Gridiron pendulum, and in one of his methods, where the compensating bars are inclosed in a tube, he has made his clocks more fit to be carried from place to place without risk of the adjustment being altered.

Berthoud and Leroy, in France, have also constructed several species of compensation pendulums, the principle of which does not differ from that of Harrison. See *Rees' Cyclopedia, Article Pendulum.*

175. If a sphere of Platina be suspended by the slenderest wire of iron that is capable of supporting it, we may without any great error abstract from the weight of the latter; if the whole Pendulum be then actually measured, the position of the centre of Gravity may be found by the formula.

$$x = d + \frac{2r^2}{5d}$$

176. If a pendulum of this sort, nearly twice the length of a second's pendulum, be set to vibrate, and the number of its vibrations in a given time ascertained, the true length of the second's pendulum may be thence deduced. To ascertain the number of vibrations in a given time, it is by no means necessary to count them; for if the experimental pendulum be suspended in front of the pendulum of a well-regulated clock, if their planes of vibration be parallel, and their axes both in one plane perpendicular to the horizon and to the plane of vibration, and if the pendulum be nearly, but not exactly some known multiple of the length of a second's pendulum, no more will be necessary than to observe the intervals between the times of coincidence, and hence we may infer the number of vibrations without danger of error.

DELAMBRE *Astronomie*, cap. XXXV. § 125.

When the length of the pendulum and its number of vibrations have been thus determined a correction may be applied for the weight of the suspending wire.

DELAMBRE, *ubi suprà.*

It was in this manner that the members of the French Institute determined the length of the pendulum, while measuring an arc of the Meridian. See DELAMBRE, *Base du Systeme Metrique.*

The length of a Pendulum vibrating seconds in the latitude of Paris was found to be 443.296 lines old French measure or 0.993977 metre of the new.

177. If a pendulum be so constructed that it may be suspended at pleasure upon knife-edges either from one extremity, or from a point not far distant from its supposed centre of oscillation; and if a weight slide along the pendulum until its vibrations when suspended by either knife-edge are equal, the position of the second knife-edge will be the centre of oscillation.

The position of the centre of oscillation being thus determined, its distance from the centre of suspension admits of accurate measurement.

This appears to be the best possible method of measuring the length of the pendulum. It was the invention of Captain Kater. His mode of observing the coincidences of the oscillations of the experimental pendulum, and that of the clock although founded on the same principles, appears more likely to be accurate than that of the French Philosophers.

According to his observations, the length of a pendulum vibrating seconds in lat. 51° 31′ 8″, 4 N. when reduced to the level of the sea is 39.1386 English inches.

PHILOSOPHICAL TRANSACTIONS, 1818. EDINBURGH REVIEW, *September*, 1818.

178. From the length of the pendulum as thus found g the measure of the force of Gravity may be at once found by the formula.

$$g = p^2 l$$

If l be taken as $39\frac{1}{6}$ inches, from which it does not sensibly differ

$$g = 386.14 \text{ in.} = 32\tfrac{1}{6} \text{ ft. nearly.}$$

Hence the distance a heavy body falls by the force of gravity in a second of time is $16\frac{1}{12}$ feet.

It is obvious that these numbers are not exactly correct, but approximations only; they are, however, those most commonly used in the English Philosophical Works.

179. When experiments are made upon the length of the second's pendulum with a view of determining the force of gravity in different latitudes, it has been found to vary in such a manner as to show that the Intensity of Gravitation is greater in high than in low latitudes.

The following lengths of pendulums are taken from late observations of Captain Sabine

	Inches.
In Lat. 60° 09′ 42″ N . . .	39.169.
In Lat. 70° 26′ 17″ N . . .	39.198.
In Lat. 74° 47′ 12″ N . . .	39.207.

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The explanation of the cause of this variation in the Gravitating force must be deferred until we treat of *Central Forces*.

180. Since it is possible, without much difficulty, to measure the length of an experimental pendulum, and thus infer the length of one beating seconds, and since this length depends upon the force of Gravity which is invariable at the same place; we might hence deduce an universal standard of linear measure easily accessible, and not liable to change.

Having a standard of linear measure, its square might be taken for the unit of superficial measure, its cube for the unit of measures of capacity; and the weight of its cube filled with pure water as the unit of weight.

181. The properties of a good standard of measure are,

(1.) That it should be easily accessible.

(2.) That it should be invariable.

(3.) That it should be so large that the measures in ordinary use should be deduced from its aliquot parts rather than from itself or from its multiples; for thus any error in the standard would be much lessened.

The pendulum possesses the two first properties in the greatest perfection, but it is far inferior in the last to the unit proposed by the Institute, and actually adopted by the government of France. The unit of the new French measure is a quadrant of the meridian; the standard of length is its ten millioneth part, and is called the metre.

The divisions of the new French measure were all intended to be decimal, but inveterate habits and causes perhaps actually founded in nature have induced the restoration of the old divisions and names in the descending scale, and their application to the new measure.

The measurement of an arc of the Meridian, sufficient to determine its length with accuracy, is an operation that requires much time and labour, and is attended with great expense. For this reason the French Philosophers propose to use the pendulum to recover their unit, in case it should be lost or become changed by use. Thus recurring to the very instrument they had at first rejected, as a standard.

The example of the French teaches us that it would be fruitless to attempt the change of the measures and weights at present in use; all that can reasonably be hoped to be effected is to reduce them to as few denominations as possible, and fix their ratio to some standard in nature, of which the pendulum vibrating in some particular place is the best.

See DELAMBRE *Base du Systeme Metrique.* ADAMS' *Report.* The several REPORTS to the British Parliament. SKENE KEITH's *pamphlet on weights and measures.* JEFFERSON's *Report.*

OF THE MOTION OF PROJECTILES.

182. If a heavy body be projected near the earth's surface in a direction not perpendicular to the horizon, it will describe a parabola, whose plane is vertical and whose axis is perpendicular to the horizon.

In the demonstration of this proposition we suppose the directions of the force of gravity at all points within the range of the projectile to be parallel; and we abstract from the resistance of the air.

This proposition was first discovered by Galileo; for his geometrical demonstration see GREGORY, § 248.

For an analytic demonstration, see POISSON, § 229.

If v be the initial velocity, h the height due to that velocity, g the measure of the force of gravity, d the horizontal range, and E the angle of elevation.

$$h = \frac{v^2}{2g} \quad \text{and} \quad d = \frac{v^2 \text{ sine } 2E}{g}$$

Poisson, § 230.

The horizontal range varies with the sine of twice the angle of elevation, it is of course greatest at 45°; and the ranges with angles equally distant from 45° are equal.

The greatest height to which a projectile rises is as the square of the velocity multiplied into the square of the sine of elevation. The time of flight is as the velocity multiplied into the sine of the angle of elevation. Playfair, § 92.

When the above theory is applied to the motion of military projectiles it is found to differ widely from what actually takes place, for projectiles that according to theory should range in vacuo 20 to 30 miles, do not range in the air more than 2 or 3. This difference between the real and theoretic motion of projectiles arises from the resistance of the air.

183. The resistance of the air to the motion of a projectile increases with the square of the velocity.

With this new element in addition to the two on which the parabolic theory is founded, (viz, the projectile force and that of Gravity,) it has been attempted to investigate the curve that a projectile would actually describe. But the complete integration of its equation has been found beyond the reach of the most subtle calculus. See Poisson, § 234.

The two branches of the curve that would be described by the joint action of the above 3 forces are not similar to each other.

The base of the ascending branch is longer than that of the decending.

The assymptote of the descending branch is a vertical straight line.

The time in the descending branch is greater than in the ascending. The velocity at the end of the descending branch is less than the initial velocity.

POISSON, § 230. LALLEMAND'S *Artillery*.

184. Beside the resistance of the air in front, the ball is farther retarded by a want of support or *non-pressure* from behind. It is known that air rushes into a vacuum with the velocity of 12 or 1300 feet per second; hence, when the initial velocity of the ball is greater than that quantity, it leaves a vacuum behind it.

The greatest velocity that can be given to a military projectile is about 2000 feet per second; but this is not attended with much advantage, for it is speedily reduced to the above limit.

The effects of these resistances vary in every possible manner in relation to the velocity, the diameter, and the weight of the projectile; hence in order to form rules for practical gunnery it would be necesssary to make experiments upon every species of ordnance, and every size and quality of projectile.

HUTTON, *Tract* 37.

The best experiments upon these subjects are those of Hutton, Robins, and Count Rumford.

185. When gunpowder is fired, a permanently elastic fluid is generated, that expands itself both in consequence of its elasticity and of the high temperature at which it is produced. These two causes united appear to give it a force at least 2000 times as great as the pressure of the atmosphere; and it expands itself with a velocity of 5000 feet per second. HUTTON, *Tract* 37.

This holds good in the usual mode of firing ordnance, where the ball is free to move the moment the gunpowder begins to inflame Where the gunpowder is compressed by a heavy weight the experiments of Count Rumford are stated to have shown that it has exerted a force equal to the prodigious amount of 100,000 atmospheres. RUMFORD, *Philosophical Papers.*

This however far exceeds the limits of credibility, but even if the force were much less the effects of small quantities of powder closely confined, as in Military mines, in blasting rocks, &c. might thus be explained. And hence we may see the danger of ill bored cannon and elliptic balls, of obstacles in the barrels of ordnance, and of using wads of an adhesive substance. To the latter cause may perhaps be attributed the bursting of many guns in the naval service of the U. S.

The instrument invented by Mr. Robins to ascertain the velocity with which balls are projected by military ordnance is called the *Ballistic Pendulum.* The same apparatus was also employed by Hutton and Count Rumford. The ball is made to strike a heavy block of wood suspended from a centre ; the velocity is thus changed into one that can be easily measured.

186. If v be the velocity with which the ball strikes the pendulum, b the weight of the ball, p the weight of the pendulum, g, the distance from the point of suspension to its centre of gravity, o the distance between the centres of suspension and oscillation, i the distance from the centre of suspension to the point of impact, c the chord of the arc described and r its radius,

$$v = 5{,}6727\, g\, c\, \frac{p+b}{b\, i\, r} \sqrt{o}$$

HUTTON, *Tract* 34.

The position of the centres of gravity and oscillation are determined experimentally. See HUTTON, *ubi supra.*

Hutton also measured the velocity of the ball by observing the recoil of the gun, on the following principle.

187. If a ball be discharged from a piece of ordnance, the velocity of the ball will be to that of the gun, as the weight of the gun to the weight of the ball. Hutton, *Tract* 34.

This is obviously an immediate deduction from the principle of the equality of action and re-action.

188. The motion produced upon a ball by gunpowder inflamed in a piece of ordnance is accelerated as long as the ball remains within the barrel. But the acceleration is not uniform, for the elasticity of the fluid diminishes with the increase of the space it occupies, and with the diminution o its heat; the elastic fluid also propels the ball by following it, and acts with a force proportioned to the difference of their velocities. From these causes, the effect of gunpowder does not increase in as high a ratio as the space it occupies. Hutton's experiments make the velocity to increase with the 5th root of the length of the piece. Hutton, *Tract* 37.

189. The velocities communicated to balls of equal weights from the same piece and with unequal quantities of powder are as the square roots of the quantities of powder. Hutton.

190. The velocities communicated to balls of different weights by equal quantities of powder are nearly in the reciprocal ratio of the square roots of the weights of the balls. Hutton.

191. The firing of the gunpowder, if of good quality and perfectly dry, takes place within a space of time that is very small, but which is not actually instantaneous. Hutton, *Tract* 37.

This agrees fully with the experiments of Rumford.

192. The deviation of Military projectiles from the path pointed out by the parabolic theory is not confined to the vertical plane, but they are found to be diverted to the right or to the left, and sometimes cross the vertical plane several times in the course

their flight. One cause of this is the quantity of *windage*, in consequence of which the ball is driven from one side to the other of the bore, and thus acquires a motion around an axis that does not coincide with the axis of the piece; another cause is to be found in the want of sphericity in the balls, and of a true bore in the piece. Hutton, *Tract* 37.

193. The depth to which a ball penetrates in wood or earth, is as the density and diameter of the ball and the square of the velocity, divided by the strength or resisting force of the obstacle. Hence,

If equal balls be discharged against the same obstacle the depths of the cavities are as the squares of the velocities.

If unequal balls of the same substance be discharged against the same obstacle with equal velocities, the depths of the cavities are as the diameters of the balls.

An 18 pound ball discharged with a velocity of 1200 feet per second penetrates 34 inches into good dry oak, and a 24 pound ball with a velocity of 1300 feet penetrates 15 feet into dry earth.

If the obstacle be very thin, and the velocity of the ball very great, it may pass through without communicating any motion.
Hutton, *ubi supra.*

194. From the experiments of Hutton and Robins practical inferences of great value have been deduced.

(1.) As great velocities are soon reduced within a certain limit, and as they depend upon the quantity of powder, the maximum service charge has been reduced to $\frac{2}{3}$ds of the weight of the ball.

Even this charge is not necessary except in the case of battering in breach, where the object is very near, and so thick that the ball lodges without passing through.

In close naval engagements great velocities are injurious, for the ball may then pass through both sides of the enemy's vessel without lodging, and the number of splinters produced by a ball in rapid motion is much less than is caused by one moving more slowly.

(2.) The charge being reduced, it is no longer considered necessary that the gun should be as strong; hence the thickness of cannon has been reduced, and the cheaper material of iron substituted for brass.

(3.) As the velocities increase only with the fifth roots of the lengths of bore, cannon have been much shortened without losing in effect. Eighteen calibers is now considered the maximum length of guns for the field service. In the naval service, carronades have been introduced; these are a short species of gun; their bore is smaller than in long guns of the same calibre, the shot must consequently be truly spherical; their charge is only one-twelfth of the weight of the ball; their weight and thickness are proportionably reduced; yet in close action, they produce effects superior to those of long guns.

The advantage of rifling the barrels of small arms, appears to be owing to the circumstance, that in this way a rotatory motion is given to the ball around the axis of the piece; thus the aim is more certain than with a simple cylindric barrel.

195. When a ball is projected from a piece of ordnance at a small angle of elevation, and falls upon water, or on a plane of hard earth, its flight will not cease, but it will rise again and perform a second trajectory, similar to its first, but less; and will continue to rebound until the whole of its projectile velocity is destroyed.

This species of motion, that resembles in some respects the reflection of elastic bodies, is called in Military Projectiles, the *Ricochet.*

Ricochet firing, which is executed at small angles of elevation, and generally with small velocities, is applied with great advantage from sea-coast batteries upon shipping, and in the attack of fortresses. It is to its use in the latter case, that the superiority of the art of attack over the most perfect systems of fortification, is to be attributed.

OF CENTRAL FORCES.

196. If a body be set in motion by two forces that are not in the same direction, and if one of them communicates a determinate velocity and then ceases to act, while the other is an accelerating force and continues to influence it without interruption, the path described by their joint action will be a curved line.

Of this general law, the instances already given of the motion of pendulums and projectiles are no more than particular cases.

The body moving in a curved line may be considered, at any instant whatever, as if it were moving along a tangent to the curve at the point where it then is ; and if the accelerating force were to cease to act, the body would move on with uniform velocity in the direction of the tangent. If the accelerating force tend to solicit the body towards a point, this point is called the centre of force ; and the force itself is called a centripetal force.

The centrifugal force is that by which it would tend to recede from the centre, were it not prevented by the centripetal force.

These two forces are called central forces.

The path, or trajectory, described by a body acted upon by central forces, is called its *orbit.*

The *Radius Vector*, is a line drawn from the centre of force to the point of the orbit at which the body is found.

The *Angular Velocity*, is the velocity with which the angle contained by any two positions of the radius vector is described.

The *Periodic Time*, is that which a body moving in a re-entering orbit employs after passing any given point, until it returns again to the same point. GREGORY, § 279.

197. If a body revolve in an orbit by the joint action of a projectile and centripetal force, the radius vector will describe equal areas in equal times, and in unequal times areas proportional to the times. NEWTON, § *Principia*, lib. 1. sect. 2.

Huygens, in investigating the laws that govern the motion of bodies acted upon by central forces, deduced them from the consideration of circular motions. This mode of proceeding has its simplicity to recommend it, and being elementary, has been made use of in this course. See HUGENIUS, § *de Vi Centrifuga.*

198. If a body move in a circle by the action of central forces, the arcs described in equal times are equal. POISSON, § 259.

199. If f be the measure of the central forces, v the velocity, and r the radius of the circle,

$$f = \frac{v^2}{r}$$

POISSON, § 259.

200. In order to compare central forces in a circle, with the attraction of gravitation, let v be the velocity due to the height h, then

$$\frac{f}{g} = \frac{2h}{r}$$

and if $2h = r$, the central force will be equivalent to that of gravity.

If then a body revolve in a circular orbit, and be acted upon by gravity, the projectile velocity must be equal to what would have been acquired in falling through half the radius.

Poisson, § 260.

201. The central forces of bodies revolving in circular orbits, are directly as the quantities of matter and as the radii of the orbits, inversely as the squares of the periodic times.

Poisson, § 261.

202. If the central forces in circular orbits be inversely as the squares of the radii, the squares of the periodic times are proportioned to the cubes of the radii.

The two last propositions may be extended to the case of bodies revolving in any orbit whatsoever, particularly in elliptic orbits. In the latter instance, half the greater axis of the ellipse is to be substituted for the radius of the circle.

If the centripetal force be gravity, its intensity diminishes as the squares of the distance increase; the above law ought therefore to hold good in the motion of the heavenly bodies, provided gravity be the force which retains them in their orbits.

203. Kepler, the astronomer, deduced the following laws of otion of the planetary bodies from his own observations, e of Tycho Brahé.

e planets move in plane curves, and their *radii vectores* e around the centre of the sun, equal spaces in equal

e orbits of the planets are ellipses, of which the centre e sun occupies one of the foci.

The squares of the times of the planets' revolutions are to other as the cubes of the greater axes of their orbits

equent observations have shown, that the focus is not occupied the centre of magnitude of the sun, but by a point within his body, which is the common centre of gravity of the solar system. If the planets are acted upon by a mutual attraction whose intensity is inversely as the squares of the distances, the above laws are the same as might be deduced *a priori*. The attractive force is that of gravitation.

A farther discussion of this subject would encroach upon the field of *Physical Astronomy.*

204. When a solid body turns around a fixed axis, its several points describe circles, whose planes are perpendicular to, and their centres in, the axis; the centrifugal force that acts upon any one point, is therefore as the radius of the circle which the point describes, and its direction is in the prolongation of the radius. The force of gravity is due to the attraction of the mass of the earth, but because the earth revolves upon its axis, the attractive force is lessened by the action of the centrifugal force, and thus at every part of the earth's surface, except at the poles, the measure of the force of gravity is less than it would have been had the earth remained at rest. POISSON, § 262.

At the equator, the centrifugal force and the attraction of the earth are directly opposed to each other; their resultant is therefore equal to their difference. At other parts of the surface of the earth these forces are oblique to each other.

205. If the earth were a perfect sphere, the centrifugal force at the equator would be to that at any other latitude, as the square of radius to the square of the cosine of the latitude. The centrifugal force at the equator, is to the whole attractive force of the earth, as 1 to 289. POISSON, § 263.

206. The greater intensity of the centrifugal force at the equator, has caused an increase of the equatorial diameter of the earth, and a consequent diminution of its polar axis; the figure of the

earth is therefore more near that of an oblate spheroid than a sphere.

For this reason, the law of the decrease of the centrifugal force from the equator towards the pole, is not exactly what has been stated above; and the proportion of that force at the equator to the whole attractive force of the earth, becomes as great as 1 to 176. POISSON, § 264.

OF THE MOTION OF SYSTEMS OF BODIES.

207. The centre of gravity of a system of bodies, that has no fixed point, moves in space in the same manner that it would if the masses of the several bodies were united in it, and the forces that act upon the several bodies were applied to this centre, in directions parallel to their own, and without any change in their intensities. POISSON, § 458.

If the system have parts, that although not absolutely fixed, are compelled to move upon given curves or surfaces, the same law will still apply; but it will be necessary to include among the forces the resistances exercised by the curves or surfaces. As a corollary, we may deduce from the above proposition, that the motion of the centre of gravity of a system of bodies will be uniform and rectilineal, whenever no accelerating force affects the system, except the mutual action of the several bodies upon each other. This will be true, whether the mutual action be produced by an attractive or a repulsive force, or by actual collision. See §

208. If a number of bodies composing a system, and acted upon by a mutual attraction, but by no other accelerating force, be motion, and if there be no fixed point in the system; the sums the areas described around any given point, are proportioned the times employed in describing them. POISSON, § 460.

If there be any fixed point in the system, the proposition is only true when this point is taken as the origin of the co-ordinates, and as the centre of the areas.

It has already been stated, that change of motion is never effected *per saltum*, but is always gradual; this principle is called the *Law of Continuity*.

209. Whenever the motion of a system of bodies undergoes any sudden change, the sum of the products that arise from multiplying each body into the square of its velocity, becomes less; and the diminution is equal to the sum of the products of each body into the squares of the velocities lost or gained by the moving bodies. POISSON, § 425.

In the case where the bodies that compose the system are free to move in any direction, or where they move in conformity with the law of continuity in known curves, or upon given surfaces, the velocity lost or gained at each instant, by each of the points, is infinitely small; hence the sum of the products of the bodies into the squares of the lost or gained velocities, in a given time, is also infinitely small. The sum of the products of each body, into the square of its velocity, is therefore constant when the bodies are not acted upon by any accelerating force.

210. If the accelerating force that acts upon a body be uniform, the distance to which the body will go before it acquires or loses a given velocity, will be as the square of that velocity.

On this theorem is founded that estimation of the force of moving bodies, which is known by the name of vis viva; and which makes that force proportional to the square of the velocity. This subject has already been referred to, in treating of the shock of elastic bodies. The truth is, that the effect of a body in motion, may be measured either by the distance to which it goes, or by the time that elapses before a resistance of uniform intensity reduce it to rest. If the effect be measured in the first of these ways, it will be found proportioned to the square of the velocity; in the second, to the velocity simply. Both these measures may be considered correct, and are perfectly consistent if rightly understood; but the same term cannot be applied to things so radically different. In the fundamental investigations of dynamics, we conceive the force to be proportioned to the velocity simply; but there are innumerable cases, particularly when investigating the action of machines, where the distance to which a body will go before a given resistance destroys its motion, becomes an object of inquiry. This distance then is the measure of a certain power residing in the moving body, that it has been proposed to distinguish by the term *Impetus*.

PLAYFAIR, § 99.

In the beginning of the eighteenth century, there was a warm discussion among philosophers upon this subject, but it is now well known to have been a mere dispute about words.

In addition to the above general properties of the motion of
tems of bodies, there is another known by the name of th
ciple of least action; for an illustration of which, see F
§ 477. and 304.

This has been more fully proved by our countryman, Mr. Perkins. SILLIMAN'S JOURNAL, Vol. 3.

Elastic fluids are either permanently so, or condensible; the former are called *gases* or aeriform fluids, the latter go by the name of vapour or steam.

214. If a fluid, considered as devoid of might, be contained in a vessel, and if pressure be applied by the intervention of a piston passing through an orifice in the vessel, that pressure will be equally distributed through the fluid in every possible direction. POISSON, § 479.

This pressure is not only distributed equally through the fluid in all directions, but it acts perpendicularly against every point of the vessel that contains the fluid.

A non-elastic fluid may therefore be considered as a machine; for a machine is, by definition, an instrument, by means of which a force is made to act upon points that are not in its direction, and to exert upon them a greater or less effort than if it were directly applied. POISSON, § 480.

215. If a fluid be contained in an open vessel, and remain in equilibrio notwithstanding, it must be solicited by forces that are perpendicular to its surface.

The directions of the force of gravity may be considered as parallel to each other within limited distances, and consequently the surface of a gravitating fluid contained in a vessel, in order to be equilibrio, must be a horizontal plane.

If the extent of surface be great, it assumes a spheroidal figure, corresponding with, or parallel to, the figure of the earth.

The surface of a gravitating fluid in equilibrio is said to be level.

216. If a liquid be placed in a number of vessels communicating with each other at bottom, it will not be in equilibrio unless ll the surfaces be in the same horizontal plane. Water, there-e, rises to equal heights in the opposite arms of a bent tube. SON, § 508, 509.

217. Levelling is the art of drawing a line near the surface of the earth, that shall cut the directions of gravity every where at right angles. Playfair, § 243.

A level line is nearly an arc of a circle, but as the instruments called levels do not give this line, but its tangents only, a reduction is necessary wherever the distance exceeds a certain limit. If L be the length of the arc in English miles, and D its depression in feet below one extremity of the tangent drawn to it at the other extremity:

$$D = \frac{2\,L^2}{3}$$

There are three different kinds of level in common use:

(1) The Water level; in this a fluid is placed in an open tube bent twice at right angles; the sight directed over the surfaces in the two perpendicular branches, gives a tangent to the level line.

(2) The Spirit level consists of a tube hermetically sealed at both ends, enclosing spirit of wine and a small bubble of air. As the air has a tendency to rise to the highest part of the tube, it is inferred that the axis is horizontal when the bubble remains at rest in the middle of the tube; if sights be adapted to this in such a way that the line joining them shall be parallel to the axis of the tube, this line will be a tangent to the level line when the bubble is in the position we have just mentioned.

(3) In the Mason's level, the direction of gravity is pointed out by the plumb line, and a perpendicular to the plumb line is of course a tangent to the level line.

A modification of this instrument, called the American level, has been sometimes used in laying out the directions of canals. It consists of two laths joined to each other, like the sides of an acute angled isosceles triangle; the plumb line is suspended from the vertex. See Puissant, § *Nievellement.* Rees' Cyclopædia, art. *Levelling*

OF THE PRESSURE OF FLUIDS.

218. The pressure of a gravitating fluid upon a **horizontal** base is equal to the weight of a column of the **fluid resting upon** the same base, and whose altitude is equal to the **perpendicular** height of the surface of the fluids, above the **base.**

The pressure on a horizontal plane is therefore proportioned to the base and to the depth of the fluid; whatever be the figure of the vessel in which it is contained, or the extent of the fluid in a horizontal direction.

If three vessels be taken whose bases are equal, if one be perfectly cylindrical, another narrowing towards the top, and the third wider at top than at bottom; and if these three vessels be filled with water, or some other liquid, to the same depth, the pressure upon the three bases will be exactly equal. This result is fully proved by experiment. Poisson, § 493.

The best apparatus for performing this experiment goes by the name of the vases of Pascal. See *Description d'un Cabinet de Physique.*

If the depths of the same fluid, contained in different vessels, be equal, the pressure is as the bases. If the bases be equal, the pressure is as the depths.

219. If one of the sides of the vessel be a plane that is not horizontal, the pressure upon it is proportioned to its area, and to the distance of its centre of gravity below the surface of the fluid. Poisson, § 495.

The proper thickness of walls of a material of known strength, to resist the pressure of a fluid of given depth, may be determined upon this principle. See Belidor, *Architecture Hydralique.* Prony. Gauthey, *Memoires*, vol. 3.

The *Centre of Pressure* is the point through which the resultant of all the pressures on a surface passes, or to which, if a force equal to the whole pressure were applied in a contrary direction, the surface would remain at rest.

If all points in the surface were pressed by equal forces, the centre of pressure would coincide with the centre of gravity; but as the pressure of a gravitating fluid augments with the depth, the position of the centre of pressure is always lower than that of the centre of gravity.

220. Because fluids press in proportion to their altitudes and not their quantities, it follows that any quantity of fluid, however small, may be made to balance any other quantity, however great.

This principle has been called the Hydrostatic paradox, and an apparatus called the Hydrostatic bellows has been contrived for illustrating it.

Upon this principle also has been constructed an apparatus for pressure, called after the name of the inventor, *Bramah's Press*. This will be described when we come to treat of Hydraulic Engines.

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221. If two immiscible fluids are in equilibrio in the opposite arms of a bent tube, their heights above the surface of contact will be inversely as the densities of the fluids. Poisson, § 509.

Density has been defined as the ratio between the weights of equal bulks of different bodies.

222. If a solid body of less density than a liquid be placed upon the liquid, it will sink until it has displaced as much of the fluid as is equal to its own weight, and will then float at the surface.

223. A solid body immersed in a fluid will, when left to itself, sink, if its density be greater than that of the fluid; rise to the surface, and float there, if its density be less; or remain at rest, if its density be equal. Poisson, § 501.

The force with which a body rises or sinks in a fluid, is equal to the difference between the weight of the body and the weight of an equal bulk of the fluid. Hence, if a body be weighed in a fluid, it will lose in weight as much as the weight of an equal bulk of the fluid.

It thus happens that the weight of a fluid contained in a vessel is not sensible while the vessel is immersed in a mass of the same fluid, but becomes evident the moment we attempt to raise the vessel above the surface. From a misapprehension of this experiment, the ancient philosophers asserted that fluids had no weight *in proprio loco;* They supposed the weight to be destroyed, when in fact it is merely supported. Fluids appear devoid of weight when placed in fluids of the same description, not because they do not still gravitate, but because a hydrostatic pressure acts upon them that is equal to their gravity, and in a contrary direction.

224. If the same body float upon the surfaces of two different fluids, the respective bulks of the parts immersed will be inversely as the densities of the fluids; and if a body sink in two different fluids, the weight it loses will be directly as the densities of the fluids.

OF SPECIFIC GRAVITIES.

225. Specific Gravity is the proportion which the weight of a body bears to the weight of an equal bulk of another body.

In this general sense it is the same with density; but it is usual to compare the specific gravity of bodies by means of some common standard, that serves as the unit; and in terms of which the specific gravities of all other bodies are expressed. The standard universally adopted in the specific gravities of liquid and solid bodies is pure water; and this may be obtained by means of distillation, or by making use of rain water newly fallen in places where it is not liable to be contaminated in passing through the atmosphere.

226. It is a general law of nature, that bodies are expanded by heat, and contracted by cold; the bulk, therefore, both of the standard and the bodies to be compared with it will be affected by change of temperature, and their specific gravities will in consequence be changed.

For this reason it is necessary to adopt some given temperature at which experiments shall be made, or to which the results shall be reduced. The French Institute have adopted the point of the greatest density of water, which is from 38° to 40° of Fahrenheit's thermometer. The tables of the English philosophers are constructed for the temperature of 60° of Fahrenheit.

227. From the definition of Density, the following relations between the weights, bulks, and specific gravities of bodies, may be deduced.

(1) If the weights be equal, the specific gravities are inversely as the bulks.

(2) If the bulks be equal, the specific gravities are directly as the weights.

(3) If the specific gravities be equal, the weights are directly as the bulks.

We owe the invention of the method of Specific Gravities to Archimedes. He observed that a body immersed in a vessel containing a fluid, causes the fluid to rise; he thence inferred that the rise was equal to the bulk of the body immersed; and that if two bodies of equal weights, but unequal densities, were plunged in the fluid, the rise would be inversely as their densities. He applied this principle to ascertain whether a crown of gold, made for Hiero, king of Syracuse, was adulterated or not. See Plutarch, *in Archimedem.*

228. The difference between the absolute weight of a body, and its weight when entirely immersed in a fluid, is equal to the weight of an equal bulk of the fluid; and when the bulks of bodies are equal, their specific gravities are directly as their weights. If then s be the specific gravity of the fluid, S that of the solid body, W the absolute weight of the latter, and W' its weight in water:

$$S : s :: W : W - W', \text{ and}$$

$$S = \frac{s W'}{W - W'}$$

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but if $s = 1$, which is the case when the fluid employed is the standard:

$$S = \frac{W}{W - W'}$$

The weight of bodies as determined in the open air, is in some measure affected by the action of the atmosphere, and the specific gravity in consequence; but this it is usual to neglect except in very accurate investigations.

229. The instrument now chiefly employed in determining the specific gravities of bodies proceeds upon the above principle; it is called the Hydrostatic balance and is no more than a very accurate balance, with such additions as will enable the experimenter to weigh bodies with facility in air and water alternately.

230. The general rule for finding specific gravities by means of the Hydrostatic balance, is: weigh the body first in air, and then in water, divide the weight in air by the difference between the weight in air and the weight in water, the quotient is the proximate specific gravity. This rule is liable to modifications in its application, arising from the nature of the substances under experiment.

(1) If the substance be a solid body heavier than water and in one solid mass, it may first be weighed in one of the scales of the balance, and then suspended from beneath it in a vessel of water, when it is again weighed. The body must not only possess the above requisites, but be insoluble in water. If it be soluble in water, it may be weighed in a fluid that does not act upon it, and whose specific gravity, compared with water, is known.

(2) In the case of powders that cannot be tied and suspended to the scale; they are placed in a bucket whose weights in air and in water are known, and thus weighed alternately in air and in water.

(3) If the body be a solid lighter than water, it must be weighed in air, and then attached to a body so heavy as to cause it to sink in water; the compound must then be weighed in water, and after-

wards the heavier body; if now the weight of the light body in air be divided by the difference between the several losses of weight of the compound and of the heavier body alone, the quotient will be the specific gravity.

(4) To find the specific gravity of a liquid, take a phial whose weight when empty and when filled with pure water to a certain mark is known; fill it with the liquid up to the same mark, and weigh it. In this manner you will have the weights of equal bulks of the liquid and of water; divide the first of these by the last, the quotient is the specific gravity. The same method may be employed to ascertain the specific gravity of pulverulent bodies that are soluble in water.

The specific gravity of a liquid may also be obtained by weighing the same solid in air, in water, and in the liquid; the loss of weight of the solid when weighed in the liquid, divided by the loss of weight in water, gives the specific gravity.

231. When the experiments are made at a temperature different from that chosen as the standard, a correction must be applied. This consists in multiplying the specific gravity found by the above methods by the specific gravity of water at the given temperature. To facilitate this operation, accurate tables have been constructed of the density of water at all the temperatures within which experiments are made.

For such a table, see BRANDE's *Chemistry*, § 488. and BIOT, *Traite de Physique*, Vol. I. p. 425.

232. In very accurate investigations, a correction becomes necessary for the loss of weight sustained in consequence of the bodies being weighed in air, and not in vacuo. If s be the specific gravity of the body, as ascertained by weighing it in air and water, m the specific gravity of air at the time of making the experiment, S the absolute specific gravity:

$$S = s + m\,(1 - s)$$

When the body is heavier than water this correction is to be subtracted; when lighter it is to be added. In the mean state of the atmosphere $m = .00122$ nearly, water being the unit. PLAYFAIR, § 251, 252.

The method of finding the specific gravity of air and gases will be afterwards explained.

233. A knowledge of the specific gravities of bodies is of great importance in the physical sciences, in many of the arts, and in commerce.

(1) We can, by ascertaining their specific gravities, determine the difference between two bodies very similar in external appearance, and thus discover their nature, where chemical analysis would be inapplicable.

(2) We are furnished with infallible means of detecting adulterations in drugs, &c. of knowing the value and activity of chemical preparations, and the strength of saline solutions.

(3) We may in this way judge of the purity and value of coin, particularly of gold coin, the richness of metallic ores, &c.

(4) One very important application of specific gravities is to ascertain the strength of spiritous liquors. This cannot be determined as well in any other way.

234. The method of specific gravities furnishes us with the means of determining the bulks of bodies whose specific gravities are known, and of calculating the magnitude of irregular solids.

If B be the bulk of a body in English cubic inches, W its weight in grains, and S its specific gravity:

$$B = \frac{W}{252.576\ S}$$

Where the weight is in Troy ounces:

$$B = \frac{W}{.52746\ S}$$

PLAYFAIR, § 253.

When the bulk is to be determined in cubic feet, and the weight is given in avoirdupois ounces:

$$B = \frac{W}{1000\ S}$$

The above theorems are founded upon the well-known facts that a cubic inch of pure water, at the temperature of 60° of Fahrenheit, weighs 252.576 grs. English Troy weight, and a cubic foot of pure water 1000 oz. avoirdupois.

From the above equations the manner of finding the weight of a body whose specific gravity and magnitude are known may be at once deduced.

OF THE HYDROMETER.

235. The Hydrometer is an instrument for determining the specific gravities of fluids; its theory depends upon the principle already laid down, § 224. That if a body float alternately upon two different fluids, the magnitudes of the parts immersed will be inversely as their specific gravities. The essential parts of the Hydrometer are a bulb or hollow ball, a stem on which a scale is drawn, and a weight by which the instrument is retained in a perpendicular position, and made to sink to a determinate depth in some given fluid.

The common Hydrometer is made of glass; the weight is a second bulb, filled with a proper quantity of mercury or leaden shot. It is, from the brittleness of the material, very liable to accidents; and as the stem must necessarily be slender, when compared with the bulb, the scope of the scale is limited. To determine the various specific gravities of all liquids, whether lighter or heavier than water, a single hydrometer will not suffice. On this account it is not a convenient instrument for general use. On the other hand, as glass is acted upon by but few fluids, it is a valuable instrument for chemical purposes. In consequence of the brittleness of glass, metal has been substituted for it in the construction of hydrometers, and the scope of the scale has been increased by means of moveable weights.

Dicas' Hydrometer is used in the customs of the United States; it is made of silver gilt, has 36 weights, and is capable of measuring all specific gravities from 1 to .825. In the hands of the chemist it would be an instrument of value for nice experiments, but it is not well fitted for the use to which it is applied. One better adapted for ordinary purposes is the Hydrometer of Quin; it is of brass, and although it has no more than four weights, it will determine specific gravities within the same limits as that of Dicas.

Southworth, an ingenious artist of New-York, has constructed one that promises to supersede Dicas' in the customs, and with great advantage. It is made of silver, and has but one weight. Its scale is not less extensive than Dicas' or Quin's.

The Hydrometer is of great use in pharmaceutical preparations, and in the manufacture of sugar and of salts, by determining the exact time at which the process of evaporation by heat is to be stopped.

A variety of it employed to detect the presence of sugar in the processes of the brewer and distiller is called the *Saccharometer*.

Another variety is used to discover adulterations in milk; it is called the *Lactometer*.

236. The Hydrometer is besides applied to weigh and find the specific gravity of solids. The Hydrometer adapted for this purpose is the invention of Mr. Nicholson, and called after his name. See NICHOLSON, *Philosophy*, p. 303. GREGORY, *Mechanics* § 404.

237. Specific Gravities determined by the Hydrometer, are, like those formed by the Hydrostatic balance, affected by change of temperature; a thermometer should therefore be always employed and examined at the same time with the Hydrometer.

In spiritous liquors, the difference between the apparent specific gravity and the true, caused by change of temperature, is very great. To facilitate the reduction of the first of these to the last, tables have been constructed from accurate experiments. See CAVALLO, *Philosophy*, Vol. II.

Dicas' Hydrometer is accompanied with a sliding rule, by which this reduction is rendered very easy.

238. When alcohol and water are mixed together, the compound is not found to possess the same magnitude of the two ingredients, but is less; this diminution in bulk is called concentration. It takes place not only between these two substances, but *most other* cases of the mixture of bodies, whether solid or fluid.

For this reason the famous rule of Archimedes, for determining the proportion of two known ingredients from the specific gravity of their compound, is of no value in practice.

239. The specific gravity of fluids may be ascertained in another way, by means of a set of hollow bubbles of glass of known specific gravities. See PARKE, *Chemical Essays*, Vol. II.

OF THE ATMOSPHERE.

240. We discover, while investigating the properties of fluids, that we are continually immersed in a body of that class. This is the *Air* or Atmosphere that surrounds our globe. The atmosphere being insipid, inodorous, and invisible, its material nature long escaped the attention of philosophers.

The first proof of its materiality may be derived from the action of its currents, or the winds. These are capable of producing considerable effects.

Air also prevents water, or any other liquid, from entering into a close vessel containing it, when the vessel is inverted and plunged into the fluid; it is consequently impenetrable.

241. If a tube, open at both ends, have one of its extremities immersed in a vessel of mercury, and if a piston, fitted air tight into the tube, have previously been depressed to the end immersed; on drawing the piston up, the mercury will follow it to the height of about 30 inches. If the piston be raised to a greater height, the mercury no longer follows it, but its surface remains at rest at the height of 30 inches. If a similar experiment be performed with water instead of mercury, the water will follow the piston to the height of 34 feet, and there stop. DELUC, *Recherches sur l'Atmosphere*.

The ancient philosophers attributed the rise of water in tubes, to the abhorrence they supposed nature to entertain of a vacuum. But it is now known to be caused by the pressure of the atmosphere. The limit to the rise of water was observed and communicated to Galileo by some workmen employed in the service of the Grand Duke of Tuscany. Torricelli, his pupil, imagined the substitution of mercury for water, in the experiment, in the following manner, viz.

242. If a tube upwards of 30 inches in length, hermetically sealed at one end, and open at the other, be filled with mercury, and carefully inverted into a basin of the same liquid, the mercury will fall in the tube until its surface stands at about 30 inches above the level of the mercury in the basin. At this level it will come to rest. Deluc, § 18.

This instrument was at first called the Torricellian Tube, it is now named the Barometer. The space left in the upper part of the tube is called the Torricellian Vacuum.

243. As the proportion between 30 inches and 34 feet, is inversely that of the specific gravities of the respective fluids, and as this is the same with the relative quantity of each other, they would mutually counterbalance; we infer that the force which sustains them in the tubes is the same.

If this force be the pressure of the atmosphere acting in consequence of its gravity, it is manifest that the mercury in the Torricellian Tube must stand highest near the surface of the earth, and that its altitude must diminish as it is raised in the atmosphere.

An experiment of this nature was planned by Pascal, and executed by Perrier, at the Mountain called the Puy de Dome, in the French province of Auvergne. See Pascal, *Œuvres*, Vol. 4. Edinburgh Review, Vol. 20. Deluc, § 14.

Of the Pump.

244. Water rises in the common pump by the action of the pressure of the atmosphere. A common pump is composed of a cylinder containing two valves or *boxes*, opening upwards; the lower valve is fixed in the barrel of the pump, the upper
a piston that works air-tight in the barrel; by alternatel
ising and depressing the piston, the air is drawn out of t

barrel, and the water rises to supply its place ; when all the air is drawn out the water rises to the upper valve, passes through it at the moment of depression, and is then lifted upwards to the spout, or place of discharge.

This species of pump will not raise water above 34 feet.

There are two other kinds of pump, the forcing and the lifting ; but the consideration of their several characteristics must be deferred for the present.

Of the Syphon.

245. If a bent tube, open at both ends, have the end of the shorter leg immersed in water ; and if the air in the tube be sucked out, or otherwise extracted, the water will flow through the tube in a continued stream, provided the height of the upper part of the tube above the surface of the water do not exceed 34 feet. PLAYFAIR, § 343.

A bent tube of this kind is called a *syphon*.

The liquid is supported in the tube by the pressure of the air, and runs with a velocity due to a depth of fluid equal to the difference of altitude between the levels of the surface of the fluid in the vessel and of the discharging end of the tube.

It is not absolutely necessary that the shorter leg should be immersed ; for if the long leg of the syphon be immersed to a depth that will bring the orifice at the other extremity beneath the level of the fluid ; it will still discharge it, until the surface sink to a level with the opening.

The syphon is much used for decanting liquors, and is commonly called a *Crane.*

An amusing philosophical plaything is sometimes made by placing a syphon within a figure in a cup. As soon as the liquor rises within it to a certain height, the syphon is filled, the discharge commences, and continues until it be all exhausted. This apparatus is called Tantalus' cup.

OF THE PRESSURE OF THE ATMOSPHERE.

246. The mean height at which mercury is sustained in the barometer at the level of the sea, is about thirty inches. The force that supports it is the pressure of the air; the weight of a column of mercury is therefore the measure of the pressure of the atmosphere upon a surface equal to the area of the base of the column. If the specific gravity of water be taken as 1000, that of air is 1¼, and that of mercury 13600; from these data, the pressure of the atmosphere at the earth's surface appears to be about 15 lbs. per square inch; and it is equivalent to that of a homogeneous atmosphere, 27600 feet in altitude.

The pressure being 15 lbs. per square inch, a middle-sized man is loaded with a weight of about eleven and a half tons. We are insensible to this enormous burthen, because it presses equally in all directions, and because the air is so perfect a fluid that its particles offer no interruption to our motion through it.

If the air were a non-elastic fluid, and the atmosphere consequently homogeneous, its height would, upon the above hypothesis, be 27600 feet. We shall find occasion to make use of this estimate more than once in our subsequent investigations, although, in consequence of the elasticity of the air, it is far from the truth.

OF THE AIR PUMP.

The mechanical properties of the air are best investigated by the *Air Pump.*

247. The Air Pump is a machine by means of which the air may be extracted from close vessels. It is composed, essentially, of a barrel and two valves like those of the common pump; the
el communicates with a close vessel, called a receiver, and its
iston alternately rises and falls, the air in the receiver is ra-
d.

The air pump was invented by Otto Guericke, a burgomaster of Magdeburg in Germany, in the year 1664. GUERICKE, *Experimenta Nova de vacuo spatio.*

It has successively received many great improvements, particularly from Boyle, Hawksbee, Martin, Smeaton, Cuthbertson, Mendleshom, and Dumoutier.

By combining two barrels together, and working the pistons with rack work turned by a toothed wheel, the operation has been rendered more rapid. By adapting a contrivance that opens the lower valve at each stroke of the piston, the former limit of the rarefaction, which ceased as soon as the spring of the contained air became too weak to open this valve, may now be passed.

The pumps of Boyle and Hawksbee, and many of the later ones, had plates of brass covered with oiled leather on which the receiver was set. In pumps of this construction, an evaporation took place from the surface of the leather that affected the results of experiment; in the best modern pumps, the plate and the bottom of the receiver are ground to fit each other.

The barrels that were formerly of brass, are now made of glass, and the pistons that were also of brass and covered with oiled leather, are made of a soft metal that is forced into the barrel. GREGORY, *Mechanics*, Vol. II.

To measure the exhaustion produced by an air pump, we make use of gauges of several different kinds.

(1) The Barometer gauge; this is a tube open at both ends; the upper communicating with the receiver, the lower plunged in a basin of mercury; as the air is exhausted from the receiver, the mercury will rise in the tube, and the ratio its height bears to that of the mercury in a barometer will mark the rarefaction the pump produces.

(2) The short Barometer gauge; this is a short tube hermetically sealed at top, filled with mercury, and inverted in a vessel of that fluid, the surface of which is enclosed in a receiver communicating with the valves of the pump; when the pump is in operation, the air in this receiver is exhausted in equal measure with that in the principal one,

and when the pressure becomes less than that of a column of mercury of the length of the tube, it will of course begin to descend.

(3) The Syphon gauge resembles in principle the short barometer gauge, but is turned up at the end instead of being inverted in mercury.

(4) The Pear gauge; this is a hollow vessel of glass that takes its name from its shape; it has a small orifice at its larger end, which is immersed in a vessel of mercury; it is thus placed under a separate receiver communicating with the valves of the pump; when the receivers are exhausted, the air contained in the gauge expands itself, and forces its way through the mercury, but when the air of the atmosphere is re-admitted, it forces the mercury into the gauge until the rarefied air left in it is brought to the same density with the atmosphere; the space unoccupied by mercury will therefore be the measure of the rarefaction of the air in the receivers.

In using this instrument in a common air pump, at the same time with the barometer gauge, there is a remarkable discrepancy observed between their respective results; the pear gauge showing a degree of rarefaction far beyond what is pointed out by the other. This is readily explained, when we consider that the oil in the leather that covers the plate of the common pump will evaporate when the pressure of the atmosphere is removed, and fill the cylinder with a rare vapour, which, being elastic, tends to prevent the rise of the mercury in the barometer gauge; no part of this vapour can enter the pear gauge, which is therefore insensible to the existence of it in the receiver. See ENCYCLOPÆDIA BRITANNICA, article *Pneumatics*; and HACHETTE, *Cours Elementaire*.

248. Among the many proofs furnished by the air pump of the pressure of the atmosphere, are the following, viz.

(1) If the hand be placed on the top of an open receiver, it will be pressed down upon it with great violence after a few strokes of the pump.

(2) A receiver is firmly fixed to the plate of the air pump.

(3) A less receiver, placed within a greater, is fixed to the plate of the pump on the re-admission of the air.

(4) A bladder, stretched over the mouth of an open receiver, is burst *with great noise.*

(5) Two hemispheres of brass, ground to fit each other, are so firmly fastened together as to require a great effort to separate them. These hemispheres are called the Magdeburg hemispheres.

The last experiment might, if the exhaustion were perfect, furnish a mode of estimating the quantity of the atmospheric pressure in terms of some known weight and measure; but this may be better ascertained upon the principle already explained. See GUERICKE, *Experimenta Nova.*

249. The pressure of the atmosphere might reasonably be attributed as a cause to its being acted upon by gravity; but to render the fact of the weight and consequent gravity of atmospheric air more certain, a portion of it may be actually weighed; this is effected by adapting a valve opening outwards to a flask; the flask, after being weighed, is screwed upon the plate of an air pump, and the air rarefied as much as possible; if it be now weighed a second time it will be found lighter; the difference in weight is that of the air abstracted.

That air gravitates may also be made manifest from the fact, that the atmosphere continues to accompany the earth in its rapid diurnal and annual revolutions.

The strata of air at the surface of the earth are pressed by the whole weight of the superincumbent strata, and the height of the mercury in the barometer is the measure of this pressure. BIOT, Vol. I.

OF THE ELASTICITY OF AIR.

250. That air is elastic may be proved by many experiments, among which are the following, viz.

(1) If a bladder only partly filled with air be placed under the receiver of an air pump, and the air exhausted from the receiver, the bladder will distend itself, and finally burst.

(2) If a bladder, under similar circumstances, be loaded with heavy weights, on exhausting the air from the receiver, that within the *bladder* acquires force sufficient to raise these weights.

(3) The elasticity of the atmosphere is also shown by the manner in which the air pump itself works; for it would be impossible to exhaust the air by means of that machine, were it not elastic.

(4) The elasticity of the air may be likewise proved by merely inverting a close vessel full of air in water: the resistance it offers to farther immersion, and the partial entrance of the water into it, are proofs of the elasticity of the air it contains.

251. In consequence of the elasticity of air, we can at once infer, that if it were pressed by a weight in addition to that of the superincumbent strata of the atmosphere, it must occupy a less space, while, if the pressure of the atmosphere were lessened, it must dilate itself, and occupy a greater. But the exact law of condensation, and the relation between the space occupied by a given quantity of air, and the pressure by which it is acted upon, remain to be investigated. This may be done by means of the following experiments, viz.

(1) If an open tube be screwed into the top of a vessel partly filled with mercury, and if it dip into the mercury in such a way that the air contained in the upper part of the vessel shall be entirely enclosed; and if the whole be placed beneath a tall receiver upon the plate of an air pump; the mercury in the vessel will be forced up into the tube when the air is exhausted from the receiver, and it will be found to rise *pari passu*, and to equal heights with the mercury of the long barometer gauge. We may hence conclude that the elasticity of air of equal or less density than the air of the atmosphere, is exactly equivalent to its pressure.

(2) Take a cylindrical glass tube, bent into the form of a syphon, open at the extremity of the longer branch and closed at the other; pour in at first a small quantity of mercury sufficient to fill the bent part of the tube, and thus enclose the air in the shorter branch without compressing it; pour in next at the open end an additional quantity of mercury, it will compress the air in the shorter branch, and rise into it, but not as fast as it rises in the open branch; and when the air enclosed in the shorter branch shall have thus been condensed into half the space it had formerly occupied, it will be found that the mercury in the open branch will stand as high above the common plane of the mercury and air as the barometer stands at the moment. In this case the original pressure is evidently doubled. When the altit

of the mercury shall have reached to twice that of the barometer, at the instant the original pressure is tripled, and the enclosed air will be found to occupy exactly one third of the original space. Biot, *Traite de Physique*, Vol. I.

In order that the air contained may be perfectly dry, it is usual to invert the tube, and insert the open extremity through a cork into a receiver containing deliquescent salts for some time before the experiment is performed; by adapting the tube in the same way to a receiver, and filling it with gas, we may make experiments on the force of the gases.

(3) Take a very slender tube of glass, hermetically sealed at one end; introduce into it a small column of mercury; this column in a small tube will not permit the air to pass it, and if the sealed end of the tube be held downwards, it will compress the contained air by its weight; on the other hand, if the open end of the tube be held downwards, the mercury will descend a certain distance, and there stop. The quantity of compression in the one case, and of dilatation in the other, will furnish means of computing the relation between the pressures and the corresponding spaces. Biot, Vol. II.

(4) If a portion of air be left in the tube of a barometer, and the space it occupies observed before it is inverted in a basin of mercury, this space will be found to be very considerably increased after the inversion of the tube is effected; if the tube be inclined in various degrees, the space occupied by the air will be found to lessen until the tube be brought into a horizontal position, when the air will be of the same bulk as it was originally. Biot, Vol. II.

The second and fourth of these experiments were contrived by Mariotte, the third by Dalton. They fully confirm the following law, called, after the inventor, that of *Mariotte*.

252. The air of the atmosphere resists the action of forces that tend to compress it, with an elasticity that is directly as its density, and inversely as the space it occupies.

This law is also true of all other permanently elastic fluids.

In all the above experiments and deductions, the temperature is supposed to remain unchanged.

253. Air resists pressure equally in all directions, and if all pressure were removed, there is no assignable limit to the space a given quantity of air would occupy. PLAYFAIR, § 333.

254. It is usual in performing a course of experiments with the air pump, to show the necessity of the presence of air for the support of animal and vegetable life, and of combustion.

(1) An animal under the receiver of an air pump, becomes restless and uneasy as the air is exhausted, then falls into convulsions, and finally dies.

(2) A plant kept for some days in an exhausted receiver withers and dies.

(3) A candle is extinguished when the air is drawn from the receiver under which it is placed.

These experiments are no longer of much value, in consequence of the great light that has been thrown upon these subjects in another way by modern chemistry.

OF THE BAROMETER.

255. When the basin and tube of the Torricellian apparatus are firmly fastened to a common support, on which a scale is drawn, it forms the instrument known by the name of the barometer.

The altitude of the mercury in the barometer is reckoned by us in inches and decimals, and by the French in decimal parts of the metre.

The barometer, besides its original purpose of showing the pressure of the atmosphere, is now applied to the mensuration of heights, and to predict changes in the weather.

256. For both of these purposes, it was at one time consider- *essential* that the scale of the barometer should be lengthen-

ed; to accomplish this, a number of changes were made at different times in its form; nearly all of these have been laid aside, in consequence of their being much more remarkable for ingenuity of contrivance, than for their practical utility.

The wheel barometer of Hooke is the only one that remains in common use; Professor Leslie of Edinburgh has proposed to restore the use of the conical or pendent barometer as a portable one.

The lengthening of the scale of the barometer has been rendered unnecessary, by the great improvement that has taken place of late years in the graduation of instruments; barometers are now constructed with terniers, that give the height of mercury to the nearest thousandth part of an inch.

257. When the barometer is used in the mensuration of heights it must be rendered portable; this can be effected in a variety of ways.

(1) The mercury may be enclosed in a leather bag or vessel whose softness and pliability permit the action of the air; the capacity of this vessel may be diminished by the pressure of a screw upon its lower extremity, and the mercury forced up until it fills the tube, and is held firmly without the power of oscillation.

(2) In Deluc's barometer, the general form resembles a syphon, but the upper part of the open branch is a separate tube of glass, joined to the other by a joint of ivory, in which a stop cock is inserted. By inclining the instrument the mercury is brought to strike against the top of the tube, and is fixed firmly in that position by closing the stop cock. In using this barometer it is necessary to employ a scale upon each branch of the syphon. DELUC, *sur l'Atmosphere.*

(3) A very complete portable barometer was planned by Sir George Sharpsburgh, and subsequently improved by Ramsden and Troughton. This instrument, when in use, is suspended by gembals from a tripod stand; its own weight is then sufficient to keep it steady when exposed to the weather. The legs are hollowed out, and, when put together, form a case, in which the tube, cistern, &c. are packed. See REES' CYCLOPÆDIA, article *Barometer.*

(4) Another, still more portable, has been planned by Sir H. Englefield. It, with all its appendages, is enclosed in the space of a walking cane.

Measurement of Altitudes by the Barometer.

259. As we ascend from the surface of the earth the density of the air diminishes, for each stratum of air is compressed only by the weight of those above it; the upper strata are of course less compressed, and consequently less dense than those below them. Playfair, § 334.

> Were the atmosphere homogeneous, the height of the mercury in the barometer would decrease in the same ratio with the distance from the surface; but as this is not the case, some other law remains to be investigated. This is done upon the hypothesis that the temperature and the force of gravitation are uniform throughout; and the following result is obtained.

259. If the heights from the surface be taken, increasing in arithmetical progression, the densities of the corresponding strata of air will decrease in geometrical progression; and if the heights from the surface be taken in arithmetical progression, the corresponding columns of mercury will decrease in geometric progression. Playfair, § 336.

> As logarithms have to their numbers a relation, the converse of the above, it is evident that they might be employed in this calculation. If b be the height of the column of mercury at the surface, b' its height at any altitude, h above the surface, and m a constant co-efficient, to be determined by experiment:
>
> $$h = m\,(\log. b - \log. b')$$
>
> m may be determined by ascertaining the value of h, in some particular case where b and b' are known, by levelling or by trigonometric calculation. If the temperature be 32° of Fahrenheit's thermometer; if b and b' be taken in English inches, and h in English fathoms, $m = 10{,}000$; if then the difference between the tabular logarithms of the heights of mercury be taken, and if the mean temperature be that of freezing, it will be only necessary to remove the decimal point four places back, and it will give the height in fathoms.

260. This rule is true only in this one particular case; for the density of the air is affected by its temperature, and the column

of mercury in the Barometer will be lengthened by heat, and contracted by cold; corrections must therefore be applied to the result of the above rule, for both these causes of irregularity.

> Air has been found by the experiments of Sir Geo. Shucksburg and Gen. Roy, to expand .00244 of its own bulk for each degree of Fahrenheit's thermometer. The expansion of mercury for each degree of Fahrenheit's thermometer, is about .000101 of its bulk. See REES' CYCLOPÆDIA, article *Barometer.*

261. If, therefore, b be the height of the mercury at the lowest station, and b' at the highest; t and t' the temperatures of the air at these stations, q and q' the temperatures of the quicksilver in the two barometers; h the perpendicular height in fathoms:

$$h = 10.000. \left(1 + .00244 \left(\frac{t+t'}{2} - 32^\circ\right)\right) \log. \frac{b}{b'(1 + .000101 (q\, q')}$$

262. The following are the rules for the practical application of this problem, viz.

(1) In order that the observations may be carefully and accurately made, two portable barometers must be procured of the best construction, and filled with mercury of the same specific gravity; into the wooden frame of each, must be sunk a thermometer called its *attached* thermometer; two other thermometers must also be provided, *detached* from the barometers. Of the two barometers, one with its attached thermometer is to be placed in the shade, at the top of the eminence whose height is to be measured; the other remains at a station below; let them continue a sufficient space of time at each station for each detached thermometer to acquire the temperature of the air.

(2) An observer on the eminence must note down the height of the mercury in the barometer, and the temperature pointed out by both the attached and detached thermometers; *at the same time*, another observer at the station below, will make similar observations upon the instruments there placed. If, in this manner, two or three sets of observations be taken at each station at short intervals of time, the mean of their results may be taken as the true altitude.

(3) The calculation is performed as follows, viz.

Correct the height of the mercury in the barometer at the upper station, for the difference of temperature; this correction is found by multiplying the number of inches and decimals by the constant number .000101, and the product by the difference of the attached thermometers. If the temperature at the upper station is the least, the correction is additive, if greater, it is subtractive.

The difference between the logarithms of the observed height of the mercury at the lower station, and the corrected height at the upper, is the approximate elevation.

To this remains to be applied a correction for the temperature of the air; this correction is found by multiplying the approximate elevation by the constant number .000244, and the product by the difference between the mean of the two detached thermometers and 32°. If the mean is greater than 32°, this correction is to be added to the approximate elevation; if less, it is to be subtracted.

It may often happen that but one barometer can be procured, and that in consequence the observation cannot be performed at the same time at both stations. The result in this case is by no means as accurate as in the other. Were we acquainted with the effects that the presence of moisture produces on the pressure of the air, or knew the law of the decrease of temperature in rising from the surface of the earth, a single barometer, and successive observations, would suffice.

Professor Leslie has laid down a formula for estimating the decrease of temperature, but experiments are still wanting to test its accuracy. See Leslie, *Geometry.*

With regard to the effect of moisture contained in the air but little is known, for we are as yet without good instruments for even measuring its quantity.

When the process we have just described is conducted with care and attention, barometric measure is found to give results quite as accurate as the best trigonometric operations.

The remarkable coincidence of m in the barometric formula with the number 10,000, when the height is to be estimated in English fathoms, facilitates the calculation extremely.

When the altitudes of the mercury are observed in parts of the French metre, and the height is to be found in that measure, the calculation is by no means so simple, for in that case $m = 18{,}393$. To do away this difficulty, Biot has constructed tables that by a simble subtraction give the heights in metres; these tables are accompanied by others for the two corrections. See BIOT, *Astronomie Physique*, Vol. 4. MALORTI DE MARTEMONT, *Military Topography*.

In elevations not greater than a mile the following rule of Professor Leslie's will be found to give results very near the truth; as the sum of the two mercurial columns at the two different elevations is to their difference, so is the constant number 52,000 to the approximate height.

It has been proposed to substitute the altitudes of mountains measured by the barometer for horizontal bases in geodotic operations; and this method has actually been put in practice by Humboldt, in Mexico, with great success.

263. The boiling point of water, which, under the mean pressure of the atmosphere at the surface of the Earth is 212° of Fahrenheit's thermometer, diminishes with the decrease of this pressure. It has, for this reason, been proposed to substitute for barometric measurement, an observation on the temperature of boiling water.

Although the apparatus in this experiment is both cheaper and more portable than the barometer with is accessaries; the experiment itself is liable to fallacies from which that with the barometer is free.

Use of the Barometer to predict changes in the weather.

264. The height of the mercury in the barometer, even at the same place, is by no means constant, but is found to vary with heat and cold, and with the changes of the weather; the corresponding change in the barometer is known to precede those that take place in the atmosphere; hence, if carefully watched, it may enable us to foretell the phenomena of the weather.

For this purpose, Dr. Halley has proposed the following rules:

(1) In calm weather, when the air is inclined to rain, the mercury is commonly low.

(2) In serene and settled weather, and also in calm and frosty weather, the mercury is generally high.

(3) Upon very high winds, although not accompanied with rain, the mercury sinks lowest; regard being had to the point of the compass from which the wind blows.

(4) The greatest heights of the barometer, are found upon easterly and north easterly winds; to which it may be added, that under a southerly wind it is commonly low.

(The above four observations hold good in every part of Europe; but in North America they are not entirely correct; for instance, the greatest heights of the barometer in New-York take place under westerly and north west winds.)

(5) After very great storms of wind, when the mercury has been for some time low, it generally rises very fast again.

(6) The more northerly places have greater alterations of the barometer, than the more southerly.

(7) Within the tropics and near them, there is little or no variation of the mercury in all weathers. At St. Helena it is little or nothing; at Jamaica 3-10ths of an inch; whereas in England, it is 2 1-2 inches; and at Petersburgh, nearly 3 1-3d. See Rees' Cyclopædia, article *Barometer.*

We shall account for these changes, and their connexion with the weather, when we come to treat of the air as *a vehicle of moisture,* and of the *winds.*

OF THE SPECIFIC GRAVITY OF ELASTIC FLUIDS.

265. As the densities of elastic fluids are small, when compared with water, experimental philosophers, in order to render the difference between them more sensible, make use of some other substance as the unit. Some have for this purpose em-

ployed the air of the atmosphere, others oxygen, and others again hydrogen.

As the last is the lightest of all known substances, it possesses advantages of a peculiar kind over the other two; the numbers too that represent the specific gravities of bodies in terms of hydrogen, as the unit, have a most remarkable coincidence with those that mark the proportions in which bodies combine chemically with each other. On the other hand, atmospheric air has this to recommend it, that it has been found to be uniform in composition in every part of the globe.

266. The comparative weight of equal volumes of atmospheric air, and of any other gas, may be found as follows:

Take a globular flask of thin glass, whose capacity is at least five or six quarts; it must not be less than this, otherwise the errors that necessarily take place in weighing, would have too great an influence on the result. This flask must be provided with a stop cock that will effectually cut off all communication with the external air. If the stop cock be opened, and the flask screwed upon the plate of an air pump, the air may be exhausted, and a partial vacuum formed within the flask. Let the stopcock be now closed, the flask detached and weighed by means of an accurate balance. Call the weight thus found P.

If the stop cock be opened, the air of the atmosphere will enter the flask, and the arm of the balance at which it hangs will preponderate; let it be again accurately weighed, and call its weight P' It is evident that the augmentation of weight is due to the air that has entered the flask; if we suppose the vacuum to have been perfect, $P'—P$ will be the weight of the bulk of atmospheric air contained in the flask.

In the same way the weight of an equal bulk of some other elastic fluid may be found. The flask is first to be weighed empty; let p be its weight; this is not essentially equal to P, for it will depend upon the density of the atmosphere at the time this last experiment is made. It is then to be filled with the gas, and again weighed; if the weight, when full of gas, be p' $p'-p$, will be the weight of the contained gas; and if s be the specific gravity in terms of atmospheric air:

$$s = \frac{p' - p}{P' - P}$$

Biot, *Physique*, Vol. I.

The same formula may be employed to find s in terms of an equal bulk of hydrogen, by substituting for P′ the weight of the flask filled with that gas.

267. If this process be repeated with the same gas, atmospheric air from the same place, the same air pump, the same flask, and the same balance, the result will be found different; this shows, that however exact the experiments may be, they cannot be compared together until they have corrected for errors arising from the following circumstances, viz.

(1) The pressure of the atmosphere is not constantly the same; the air contained in the flask, when it is weighed full of air, is acted upon by this pressure, and its mass will be of greater or less weight, according as the pressure of the atmosphere is, at the moment, a greater or less quantity.

(2) The pressure of the atmosphere is not the sole cause of a change in the weight of a given volume of air; change of temperature produces a similar effect.

(3) These two causes have a proportionate influence upon the weights of all the other gases, when they are introduced into a flask whence the air has been exhausted. The temperature and pressure at which they are introduced into the flask must therefore be noted.

(4) The flask itself does not always continue of the same capacity; for the glass of which it is made is contracted or dilated by change of temperature.

(5) Atmospheric air and the gases are capable of containing a certain quantity of moisture; this varies with the temperature, and with the care that has been taken to free them from water; the same quantity of gas weighs either more or less, as it contains more or less vapour. In order, then, to compare the results of experiments on this subject, we must know the quantity of vapour contained in the gas, as well as in the air of the atmosphere.

(6) All these causes influence the experiment in another manner, by changing the density of the exterior air in which the flask, when it is weighed, is, as it were, immersed. The loss of weight of the flask, whether empty or full, when weighed in the air, will vary, with its own bulk, with the pressure of the atmosphere, with the temperature and hygrometric state of the exterior air.

(7) We have supposed in these experiments that the vacuum produced by the air pump is perfect; this, as we have shown, is never the case; for whatever pains be taken to exhaust the flask, there must always remain in it a small quantity of elastic fluid, whose existence is shown by its action on the gauge of the pump. The quantity of this elastic residuum must be measured; it must also be ascertained whether it be entirely gaseous, or entirely aqueous vapour, or whether it is composed of a mixture of the two, and in what proportions. Biot, *Traite de Physique*, Vol. I.

OF THE ATTRACTION OF COHESION.

The laws of the equilibrium of fluids, deduced *à priori*, are in some measure influenced by a species of attraction that frequently takes place between their particles and those of solid bodies. This is called the attraction of cohesion, or from one of its most remarkable cases *Capillary Attraction*. The motion of fluids is affected even more than their equilibrium, by this cause, it is in consequence necessary to consider its phenomena before we can pass to the subject of Hydrodynamics.

268. If a polished plate of some solid substance be suspended horizontally from one of the arms of a balance, and if, after having counterpoised it by weights in the opposite scale, it is brought in contact with a liquid, it will be found to adhere to it. This is made manifest by the fact that they cannot be separated without the evident exertion of a determinate force. Biot, Vol. I.

This adhesion is not caused by the pressure of the atmosphere, for it takes place even in vacuo.

When, by the action of weights applied to the opposite arm of the balance, the plate is raised, it will be found to carry with it a portion of the liquid. It is evident, therefore, that the added weight is not the measure of the attraction between the plate and the fluid, but of the attraction existing between the particles of the fluid body itself. Biot, Vol. I.

269. When capillary glass tubes are dipped into water, it will be found to rise in them. When two plates of glass are immersed in the same liquid, parrallel to, and at small distances from each other, a portion of the fluid will also rise between them.

When the altitude of the column of water is measured, it will be found to have risen highest in the tubes of smallest bore. This phenomenon is found to occur in a greater or less degree with all liquids that are capable of moistening glass, and although the rise is most perceptible in small tubes, it appears in some degree in all vessels whatsoever, as is shown by a ring formed around the sides with a concavity on its upper side.

Tubes of other materials than glass are capable of exerting capillary attraction on water, as are also the natural pores of many bodies, both organic and inorganic. The capillary suspension takes place, whether the tube remain immersed in the liquid or not, provided there be a drop of the liquid adhering to the lower end of it.

If the liquid and the solid body be of such a nature that they do not adhere, the liquid, instead of rising in the tube, is depressed; thus, mercury does not rise as high in a glass tube as its own original level.

270. The surface of those liquids that rise in capillary tubes, will be found on examination to be concave upwards; and in cylindrical tubes, where the liquid moves with freedom, the concavity is found to be nearly a hemisphere, touching the interior of the tube. The surface of a liquid that is suspended by capillary attraction between two parallel plates, is also concave upwards, and of the form of the half of a cylinder. The following are some of the more remarkable phenomena of capillary attraction, as observed by experiment:

(1) The surface of those fluids that are depressed in capillary tubes is convex upwards. The heights to which the same fluid rises in

cylindrical capillary tubes of different diameters, are inversely as the diameters.

(2) The height to which a fluid rises between two concentric tubes, is half that to which it rises in a tube of the same material, and whose diameter is equal to the distance between the concentric tubes.

(3) The height to which a fluid rises between two parallel glass plates is half that to which it would rise in a tube whose diameter is equal to the distance of the plates.

(4) If the plates, instead of being parallel, are inclined to each other, the surface in contact with the plates, instead of being a horizontal straight line, becomes curved, and this curve is an hyperbola.

(5) Whether a fluid rise or fall between two vertical and parallel planes, immersed in the fluid, the planes tend to approach each other. It is for this reason that two small vessels of glass, floating on mercury or water, tend to come together whenever they are placed near to each other.

271. All these circumstances prove that an attraction exists between glass and water, and between certain other solid and fluid bodies. In others, as between mercury and glass, there is an apparent repulsion. If, however, a small globule of mercury be placed upon dry paper, and then touched with a tube of glass, the mercury will adhere to the glass, and may, by means of it, be lifted from the paper.

As this experiment shows there is no actual repulsion between mercury and glass, philosophers infer that the reason mercury is depressed in capillary tubes is, that its particles have an attraction for each other, much more powerful than they have for the particles of the glass.

272. The attraction of cohesion is also observed between solid bodies; but they must be brought into close contact with each other; in other cases, they may be made to adhere closely by the intervention of a fluid.

On this last principle, we can explain the process of soldering metals.

273. The sphere of the attraction of cohesion is different in

different bodies, but is at most small; the law of its decrease in terms of the distance, has not yet been ascertained.

274. Clairaut has proved, that if the law by which the particles of a tube attract a liquid be the same as that by virtue of which the particles of the fluid attract one another, the fluid will rise above the level, whenever the intensity of the first of these attractions exceeds half the intensity of the second. If it is exactly half, the surface of the interior column will be a plane, and level with the surface without. If it is less than half, the surface will be depressed and convex upwards. Laplace, *Mecanique Celeste*, Supplement au X Livre. Clairaut, *Figure de la Terre.*

275. If thus, in consequence of the joint action of these two attractions, the surface of the fluid in the tube is rendered either concave or convex, the equilibrium between the outer and inner masses of fluid is destroyed; in the first case, the pressure of the enclosed fluid is rendered less than that of the outer mass, it will therefore rise in the tube until the weight of the column becomes equal to the difference of the two pressures; in the second, the pressure of the enclosed fluid is increased, it therefore falls in the tube.

This is the theory of Laplace; it is illustrated at length, and applied to the explanation of the phenomena by him, in the supplement to the 10th book of his *Mecanique Celeste.* This theory not only accounts for all the phenomena, but is very remarkable for its clearness and scientific precision.

The theory usually given in the English books is Dr. Jurin's; but it is very inferior to that of Laplace.

The analytic expression deduced by Laplace, is composed of two terms; the first of these is much greater than the second, and expresses the attraction of the mass of fluid that is terminated by a plain surface. Upon this term, in his opinion, depends the refracting power of transparent bodies, the attraction of aggregation, determining the three several states in which bodies exist, and the phenomena of chemical affinity. The second term expresses that part of the action which is due to the sphericity of the surface. On this term capillary action depends. Laplace, ubi supra.

MECHANICS.

SECTION FOURTH.—HYDRODYNAMICS.

OF THE MOTION OF FLUIDS.

The investigation of the laws that govern the motion of fluids is attended with difficulty, in consequence of our ignorance of the nature of the particles of which this species of bodies is composed, and of the manner in which they act upon each other. We are in consequence compelled to deduce the fundamental proposition of Hydrodynamics from an hypothesis.

This hypothesis considers a mass of fluid to be contained in a vessel whose sides and bottom are indefinitely thin; an orifice is supposed to be pierced in the bottom or side of this vessel, through which the fluid is propelled by the action of its gravity; we then assume that the fluid is divided into a number of horizontal *laminæ*, each of which descends perpendicularly downwards. Every particle is therefore supposed to descend in a vertical line. Upon this hypothesis,

276. The velocity with which a fluid issues from a very small orifice, pierced in the side or bottom of a vessel, is the same that a heavy body would acquire in falling through the perpendicular altitude of the surface of the fluid above the level of the orifice. Bossut, *Hydrodynamique*, § 218.

If g be $32\frac{1}{6}$ feet, h the altitude of the fluid, and v the velocity:

$$v = \sqrt{2gh}$$

De Buat, *Hydraulique*, § 3.

If the vessel be of such a figure, and the orifice so placed that the fluid may spout directly upwards, it will rise to the level of the surface of the fluid in the vessel.

If the vessel be kept constantly full, and if a be the area of the orifice, t the time of discharge, g and h as before; g, the quantity discharge may be found by the equation

$$y = a\,t\,\sqrt{2\,gh}$$

if $t = 1$ second

$$q = a\,\sqrt{2\,gh}$$

hence,

$$a = \frac{q}{\sqrt{2\,gh}}$$

And because $v = \sqrt{2\,gh}$, the quantity of fluid discharged in a given time, is equal to the contents of a cylinder or prism, whose base is equal to the area of the orifice, and whose altitude is equal to the space that would be described in that time, with the velocity due to the height of the fluid.

If orifices be pierced in the side of the vessel at different altitudes, the velocities and quantities discharged are as the square roots of the depths of the orifices beneath the surface of the fluid.

277. If a vessel be permitted to empty itself through an orifice pierced in the side or bottom, the velocity with which the fluid is discharged is uniformly retarded. The time in which it will empty itself is double that in which an equal quantity of fluid would be discharged from an equal and similarly situated orifice in an equal and similar vessel; and the quantities discharged decrease in equal times as the series of odd numbers.

Upon this last principle the Clepsytra or water clock may be constructed; thus, if the fluid contents of a prismatic vessel that discharges itself in 12 hours, be divided into 144 equal parts; 23 of these will flow out during the first hour, 21 during the second, and so on.

278. If a fluid issue through a horizontal or oblique orifice, in the side of a vessel that is kept constantly full, it will describe a parabola in a vertical plane, passing through the axis of the orifice, and whose directrix is the intersection of this plane with the surface of the fluid produced.

This is a necessary consequence of the theory of projectiles, laid dow in Section I.

279. If a vessel be filled with water, and placed upon a
ane; and if, upon the altitude of the fluid as a di
cribe the circle, the distance to which the flui

along the horizontal plane, from a horizontal orifice pierced in the side of the vessel, is proportioned to the ordinate of the curve, drawn through that point.

If the orifice be at the centre of the circle, the horizontal range will be the greatest possible; and at equal distances above and below the centre, the ranges will be equal.

OF THE DISCHARGE OF LIQUIDS THROUGH ORIFICES AND PIPES.

When the foregoing theory is submitted to the test of experiment, it is not found to agree exactly with observation, for,

280. When a liquid descends in a vessel towards an orifice, pierced in its bottom, the thread of fluid immediately above the centre of the orifice, is the only one that descends perpendicularly. Other portions of the liquid describe curved lines, and reach the orifice in directions approaching more or less to horizontal. In the same manner there is a deviation from the right line direction when a fluid approaches an orifice in the side of a vessel.

This motion may be rendered evident, by throwing powders into the fluid whose specific gravity is a little greater than its own. Bossut, § 312.

281. The section of a jet that issues from an orifice, does not continue of the same magnitude with the area of the orifice, but is contracted. The actual discharge is not in consequence equal to that which is deduced from the hypothesis.

Sir Isaac Newton first discovered a contraction in the jet, and thought it to be in the proportion of 5 to 7, or as the square roots of 1 and 2. If it were of this amount, the quantity discharged might be represented by $a \sqrt{gh}$. From this, it was erroneously concluded, that the velocity was due not to the height of the fluid, but only to its half. This is not the case, for the decrease is in the area of the vein, and not in its velocity.

282. The narrowest part of the jet is called the Vena *Contracta*; its distance from the orifice, as determined by the ex-

periments of Bossut, is equal to the radius of the orifice, and its area is to that of the orifice, as 62 to 100, or as 5 to 8. Bossut, § 359.

283. Beyond the vena contracta, the fluid again spreads in the form of a frustum of a cone, the angle of whose vertex is 32°.

284. From the researches of Bossut, the following deductions are obtained, with regard to the discharge of fluids, from vessels kept constantly full.

(1) The actual discharges are less than the theoretic in the proportion of 62 to 100, but are *nearly* proportioned to the square roots of the depths of the fluid. Bossut, § 355.

(2) If the altitude of the fluid be constant, but the area of the orifice variable, the discharges are *nearly* proportioned to the areas of the orifices. Bossut, § 354.

(3) In general, the quantities discharged, during a given time, through different orifices, and at different depths, are *nearly* in the compound ratio of the areas of the orifices and the square roots of the depths. Bossut, § 356.

(4) These laws are not exactly, but only *nearly* true, in consequence of friction. To this cause it is owing, that small apertures discharge rather less than the calculated quantity; and that in orifices of equal areas, but different figures, those with the smallest circumferences will, under the same circumstances, discharge the greatest quantity of fluid.

285. If the fluid, instead of passing through an aperture pierced in a thin plate, be made to pass through a tube or *adjutage*, the discharge will be much increased. If the tube be cylindrical, and of the same diameter with the orifice, the hypothetical discharge, the discharge through an additional tube, and the discharge through a thin plate, are to one another as the numbers 16, 13 and 10. Bossut, § 384.

Other adjutages are attended with different degrees of advantage. A full set of experiments has been made upon them, by Professor *Venturi*. See Cavallo's *Philosophy*, Vol. II. chap. 7.

From them it appears, that where water is to be admitted from a reservoir into a pipe, through which it is to discharge itself, the part of the pipe adjacent to the reservoir should be made of the form of the *vena contracta;* while the opposite extremity of the pipe should be also conical, its sides diverge from the axis at angles of 16°.

286. When a prismatic, or cylindrical vessel, is permitted to empty itself through an orifice ; if the quantity that would be discharged in a given time be calculated upon the hypothesis in section 276, and reduced in the proportion of 5 to 8 ; it will afford results that will be accurate, until the fluid shall have descended nearly to the bottom ; at this time a funnel shaped cavity is formed at the surface, and a whirling motion takes place, that renders the discrepancy still greater ; for this no adequate correction has yet been found. BOSSUT, § 433.

On this account, in the clepsydra, no dependence can be placed on the regularity of the discharge of the last portion of fluid.

OF SPOUTING FLUIDS.

287. A jet of fluid whose direction is perpendicularly upwards, should, according to theory, rise to the same height with the surface of the mass contained in the reservoir. This is far from being the case in practice; for the rising fluid is subject to the action of several retarding forces.

The most obvious of these are the friction on the sides of the orifice, and the resistance of the air. But the particles of the fluid have, in addition, to push before them the particles that have preceded them whose velocity is diminished, and to overcome the resistance of those that after having risen to the height of the jet, and lost their whole initial velocity, are again descending.

It is evident that there must be a limit both to the diameter and the altitude of the jet. Cavallo states the former to be 1¼ inches, the latter 100 feet.

that is constant. The mean velocity of the current is that which, multiplied by each point in the section of the bed, gives a product that is equal to the sum of the products of each point, in the section, into its own velocity. Du Buat, § 64.

295. The relation between V the mean velocity, v the velocity at the surface, and U the velocity at bottom, may be expressed by the following equations, viz.

$$v = (\sqrt{V - 0.25} + 0.5)^2 = (\sqrt{U} + 1)^2$$

$$V = (\sqrt{v} - 05)^2 + 0.25 = (\sqrt{U} + .5)^2 + 0.25$$

$$U = (\sqrt{V} - 1)^2 = (\sqrt{V - 0.25} - 0.5)^2$$

In order to facilitate the calculation, by means of these formula, Du Buat has constructed a table, by means of which v and U may be found when V is known. See Du Buat, § 65.

The velocity at the surface of a stream, in an open channel, may be found by means of an instrument invented by *Pitot*, and called after his name, *Pitot's* tube; for a description, See Du Buat, § 572, and Bossut, § 657.

Another and better instrument has been contrived by Gauthey. See his *Memoires*, Vol. III.

296. When the channel in which a fluid runs is not straight, the resistance to its motion is increased; when the channel is composed of a number of straight parts joined by elbows, the increase of the resistance may be calculated on the following principles.

(1) The resistance caused by elbows in the channel, increases with the square of the velocity of the fluid.

(2) It is proportioned to the square of the sine of the complement of the angle, that the prolongations of the parts of the channel make with each other, provided it do not exceed 36°; beyond this limit, the resistance increases more rapidly.

(3) In the same channel, the resistance increases with the number of *elbows or* bends in its course. Du Buat, § 104.

Of Rivers.

297. Rivers are natural channels, through which the water proceeding from springs, or from the rain that falls and runs over the surface of the ground, is conveyed to the sea.

The beds of rivers appear to have been formed by natural causes, operating for a long series of years, and we generally find them in a permanent state, subject neither to increase or diminution, except within limits that do not affect the magnitude of their channels, or the stability of their banks. The tendency of nature, to produce an equilibrium among active and passive forces, is no where more strongly marked, than in the case of the beds of rivers; for we find that a river always runs over ground exactly suited to the velocity of its current. Thus, where it is very rapid, the bottom is solid rock, as the rapidity diminishes, we find large rolled stones, then gravel, sand, and lastly mud and clay. The water, by its own action, carries off the finer particles from those places where it is most rapid, to deposit them where it is less so.

It sometimes happens, however, that a river is not in train, or that it is subject to a periodical increase of its waters; and owing to either of these causes, it may be a source of ruin and devastation to its borders, instead of being their ornament, the vehicle of their commerce, and the cause of their fertility. In such cases, we must call in the aid of art, and art can only be successful, by copying the means that nature has herself made use of, in similar cases: For instance,

298. When a river that occasionally overflows its banks has many sinuosities, as we know that the velocity and consequent discharge will be increased with the quantitity $\frac{1}{b}$; we may cut through a number of the points, and thus, by shortening the course, render the rapidity and consequent discharge greater.

299. When this will not suffice, or is inapplicable, we erect dikes, thus imitating the course of nature, in which we always find the banks of those rivers that are subject to inundations higher than the surrounding country.

300. One of the most important purposes to which rivers are subservient, is that of navigation. Some rivers are to be found fitted for this purpose without any aid from art; but there are also many, where the intervention of human ingenuity is necessary to overcome certain obstacles, or obviate difficulties that naturally exist.

(1) Streams running in mountainous countries, frequently pass abruptly from one level to another, forming *Falls.*

(2) Their beds are often traversed by rocks, or other strata, too hard to be removed by the action of their current; in this case, an accumulation of water takes place above the obstacle, until the quantity $\frac{1}{6}$ becomes sufficiently great to cause a change in the velocity sufficient to discharge the whole stream. Such obstacles are called *Rapids.*

(3) Rivers, in descending from a mountainous country, carry with them large quantities of sand, gravel, &c.; when they reach the plains, this is deposited; the stream spreads itself over a great extent of ground, and becomes too shallow for navigation.

(4) Where a river meets the waters of the ocean, it is affected by their tides, and at the alternation of ebb and flow, it becomes for a short space of time absolutely quiescent. At this moment it will deposit any remains of earthy matter it may carry with it, and thus form a bar at the mouth, or even islands. In process of time these islands increase in magnitude, and finally join the main land. In this way the deltas of rivers are formed, of which we have instances in the Nile, the Mississippi, &c. The latter river, in the short space of time that has elapsed since its first discovery, has extended a deposit 50 miles into the Gulf of Mexico.

(5) A river may be unobstructed, and deep enough for the purposes of navigation, but so rapid as not to admit of an ascending trade.

301. Each of these obstacles admits of an appropriate remedy, except in cases where its magnitude is beyond the reach of human exertion, or where the cost would exceed the value of the benefit.

(1) Where the navigation of a river is interrupted by a fall, we may make a lateral channel or *canal,* and apply locks or sluices.

(2) The same remedy may be applied to the case of rapids.

(3) As shallows arise from a diminution of the velocity of the stream, they are to be remedied by making it greater; this is to be done by contracting the dimensions of the channel. Piers may be built at intervals, and alternately from each bank; the whole deposit will take place between the pier heads, and the stream will not only maintain its former depth, but will clear itself a deeper channel.

(4) When a river is of moderate size, bars at the mouth may be removed upon a similar principle: two piers are to be built into the sea from the two shores, inclining to each other at a small angle, until they approach near enough to give the river a velocity in flowing through them that will overcome the action of the tide.

(5) When a river is too rapid for the ascending trade, dams may be built across it at intervals; the whole of the fall will thus be transferred to them; in each of these dams a gate or sluice is made; when vessels are to descend this is opened, and they pass with great velocity; when the water has run out so far that the upper and lower ponds are nearly of the same level, the ascending trade may be drawn by main force through the sluice.

This contrivance, however simple and rude it may appear, was unknown to the ancients.

In most cases, however, a double sluice, or gates enclosing a chamber called a lock, has now been substituted. It has very great advantages, even in rivers, over the old sluice; and without it the canals of modern times, that frequently pass through great differences of level, would have been impracticable. Sluices of the original form are used in China, where locks are unknown, and may still be seen in Italy, in Flanders, and in Holland, where there is no great change between the levels of continuous navigable waters, whether natural or artificial.

Of Canals.

302. Navigable Canals are sometimes made use of, as has been already stated, to form a communication between different levels of a stream, whose navigation is interrupted by falls; at other times, they are formed along the bank of rapid rivers, as the least expensive mode of improving their navigation; at others they receive the waters of a navigable stream, and convey its trade to one whose level is lower; or, finally, they may

be made channels of communication between navigations that are lower than the canals themselves.

Canals were constructed by several of the ancient nations; but as they were ignorant of the use of sluices and locks, they were confined to plain countries, and were carried along the same level for their whole course. Those which are carried over levels higher than either of the streams with which they communicate, and that receive their waters from feeders and reservoirs, at their summit level, are justly considered among the noblest triumphs of human genius. Sluices were invented in the 11th century, and the first are supposed to have been erected on the canal of Martizana in Italy.

303. A lock is a chamber, interposed between two contiguous levels of a natural or artificial navigation; it is enclosed on the lower side by a gate, and its bottom is on a level with the bottom of the lower part of the canal; on the upper side, it is enclosed by a wall rising to the height of the bottom of the upper level, and this is surmounted by a gate. The opening or shutting of these two gates, establishes or cuts off the communication with either level at will. When a boat is to rise from the lower level, the lock, if full, is emptied; the boat is then introduced into the basin, and the lower gate closed behind it; water is now admitted through channels contrived for the purpose in the walls, or by wickets in the upper gate; when it shall have risen to a level with the upper portion of the navigation, the upper gate is opened, and the boat is drawn forwards.

The size of a lock will depend upon the magnitude of the navigation, and the size of the vessels that are used upon it. Where the boats are not intended for any other purpose than the navigation of the canal itself, those which are drawn by a single horse are the most economical; a lock for the passage of these need not exceed 60 feet in length, and 5 in breadth. The locks on the great canal in the state of New-York, are 90 feet long, and 16 feet wide; the canal itself is 4 feet deep.

Canals should be at least a foot deeper than the greatest draught of water of the vessels intended to navigate them. This will give room for partial deposits of mud, allow for evaporation and waste, *that it may not* be possible to supply instantaneously; and what, in a well planned canal, is of more importance, will do away a resistance analogous to friction, that becomes very visible when boats nearly fill up the channel in which they move.

In locks of the above dimensions, the upper gate may be made in a single leaf, but in larger locks, the upper gates are made of two parts, and so are the lower gates in all cases. By making the gates to fold, the two parts meeting at an angle, whose vertex is opposed to the accumulation of fluid, great additional strength is obtained.

There is an analytic investigation of the proper angle, at which the gates of a sluice should meet each other, to be found in the second Vol. of the American edition of Hutton's Course. It does not, however, appear to carry conviction with it. The best engineers make the vertex of the angle to project towards the head of water, from $\frac{1}{3}$ to $\frac{1}{4}$ of the whole breadth of the lock. GAUTHEY, *Memoires*, Vol. III.

The fall of a single lock should not exceed 10 feet; where the change of level is greater than this, it should be divided among two or more locks. The limit to the height of a lock, is found in the additional expense of construction in masonry and wood, and the greater risk of destruction in deep, than in shallow locks; the thickness of a wall that will sustain the pressure of a given head of water, is equal to half the depth of the water. GAUTHEY, Vol. III.

For the dimensions of the timber of the gates, in order to resist given pressures, See GAUTHEY, Vol. III, p. 75, et seq.

Deep locks also cause a much greater expenditure of water, than where the whole fall is divided among several; for this reason, where circumstances have compelled engineers to make use of a single lock, they have endeavoured to save the water by several ingenious contrivances. For a lock of this description, See BELIDOR, Vol. IV.

Wherever it is possible, a good engineer will endeavour to make all his locks of nearly equal fall. Or a better rule is, that they shall diminish in depth in a small degree, as they recede from the point where the supply of water is admitted. Where an additional supply of water is brought in at a lower level, the locks may again be constructed of the maximum depth. By an attention to this rule, the water used in the first lock will carry a boat from the highest to the lowest level, without requiring any increase, or lessening the general depth of the canal. GAUTHEY, Vol. III.

Locks of the fall of 8 feet, should never be placed nearer to each other than from 1 to 200 yards, (according to their superficial dimensions,) unless the width of the intermediate canal be proportionably increased. Where they are nearer than this to one another, they cause a very great waste of water.

Where locks are 200 yards apart, and the boats pass upwards and downwards alternately, each boat will require no more than a single lock-full of water; where they are at the same distance, and the boats follow each other, the expenditure will be equal to two locks for each boat; the nearer the locks are brought to each other, the greater will be the expenditure, until when they are adjacent, and the lower gate of one lock forms the upper gate of the next, the expenditure in boats passing alternately upwards and downwards, will be equal for two boats to the fall of as many locks as are combined in the system. GAUTHEY, Vol. III.

A want of attention to this last circumstance, has led many engineers into great errors. For it has been usual to combine as many locks together, at one place, as possible, and yet, in calculation, no more than two locks of water have been allowed for each boat.

304. One of the chief difficulties which attends the construction of a canal, is that of procuring an ample supply of water for the purpose of lockage, and to meet the waste occasioned by leakage, and by evaporation from the surface.

The quantity of evaporation is about 3 feet per annum; the leakage will depend upon the nature of the banks, and the lockage on the number of boats that ascend and descend, and a more or less favourable construction and location of the locks. It is usual in Canal estimates to allow twice as much water for leakage as for evaporation.

305. Where the canal, from its superior elevation, or from any other cause, cannot be furnished with water from either of the navigations in which it terminates; *Feeders* must be drawn from some source, either natural or artificial.

The quantity of water that a given feeder will furnish, or the magnitude and slope of a feeder, that will furnish a required quantity of water, may be calculated by an application of the formula of Du Buat, given at § 293.

306. Where there are no sufficient natural sources of supply, or where they are subject to failure at certain seasons of the year, ***Reservoirs*** are constructed; in these the waters of springs, small streams, &c. that would otherwise run to waste, are collected in the first case; and in the second the excessive supply of some seasons is treasured up against a time of drought.

Reservoirs are also of use to receive the water of rivers, &c. before it is delivered into a canal, and retain it until it shall have deposited the mud and sediment which they have carried with them. It is extremely injurious to a canal to permit water to be introduced into it, until it shall have undergone this process of purification.

307. For the same reason, where a stream intersects the course of a canal, and their several waters are nearly at the same level; instead of admitting it into the canal, it is often necessary to carry it beneath its bed; this is done by means of a contrivance called a culvert. BELIDOR, Vol. IV.

308. When the level of the intersecting stream is higher than that of the canal, it may be carried over it, by means of an aqueduct bridge; in like manner, if the level of the canal be the highest, a similar bridge may be used to support its waters.

Some of these aqueduct bridges cross streams of considerable size. In the case of the Duke of Bridgewater's canal, that crosses the Irwell, near Manchester, vessels may be seen navigating the river and the aqueduct at the same moment; one passing directly over the course of the other.

Cast iron has, of late years, been used in Great-Britain, for the construction of aqueducts, and with much advantage. The most magnificent work of this sort is on the Ellesmere canal, in the vale of Llangollen, in Wales.

It has also been proposed to use the same material in locks.

309. Where feeders, or adventitious supplies, bring into a canal more water than can be used, *Waste-gates must be formed at intervals* along it, and in places whence the superfluous *liquid may* be readily carried off, either by natural or artificial

channels. A waste-gate is a wedge-shaped mound, whose edge is at the level, where the water in the canal should stand.

310. There are countries where water is not to be found in quantities sufficient for the supply of the locks of a canal. Various contrivances have been proposed to meet this difficulty; among them, may be mentioned the hydraulic locks of Betançourt; the inclined plane, and the perpendicular lift. See LESAGE, *Recueil des Memoires.* REES' *Cyclopædia.* FULTON *on Inland Navigation.* REPERTORY OF ARTS, Vol. I and II.

Of the Motion of Water in Pipes.

311. Where a pipe is adapted to a reservoir of water, the fluid will fill and flow through it, whatever be its figure or the number of its sinuosities, provided no part of the pipe is higher than the level of the water in the reservoir.

312. Where the water from a reservoir is conveyed in long horizontal pipes of the same aperture, the discharges, in equal times, are nearly in the inverse ratio of the square roots of the lengths. PLAYFAIR, § 285.

313. The principle of Du Buat, that we have seen applied to rivers and canals, also holds good with regard to conduit pipes, viz. The sum of all the retarding forces is equal to the accelerating force. By the application of this principle, we obtain a solution of the two chief problems of the motion of water.

(1) To determine the discharge from a conduit pipe, where the height of the reservoir and the diameter of the pipe are given; we have, as before:

$$V = \frac{307\,(\sqrt{r} - 0{,}1)}{\sqrt{b} - \mathrm{L}\,\sqrt{b + 1.6}} - 0.3\,(\sqrt{r} - 0.1)$$

and V the mean velocity being given, the quantity is found by multiplying it by the area of the tube.

In this equation, the value of b is determined by dividing the fer the pipe by h, a fictitious height of the reservoir a little less than the true. If the mean velocity v, calculated by this hypothesis, is such that

$$h + \frac{v^2}{2g} = \text{H the true height,}$$

we shall have calculated the true mean velocity; but if this is not the case, we assume another value for h, until, by repeated trials, we shall find a number that will answer the purpose. Du Buat, § 228.

If the pipe has a number of sinuosities, the angles of which are known, the resistance that each offers may be found by the formula

$$\frac{\text{V}^2 \sin.^2 o.}{3000}$$

where o is the supplement of the angle formed | by the parts of the pipe | at | the bend. Du Buat, § 229.

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(2) To determine the diameter of a pipe that will discharge a given quantity of water, where its length and the height of the reservoir are known.

If r be the mean depth, equal to half the radius, the given quantity, p the ratio of the circumference of a circle to its diameter:

$$r = \left(\frac{2(\sqrt{b} - \text{L}\sqrt{b + 1.6})}{4p\sqrt{307}}\right)^{\frac{2}{5}}$$

Du Buat, § 230.

The motion of water in pipes has of late attracted much attention, in consequence of its great importance to the health and comfort of large cities. It is well known that the regular sources of the springs and wells that exist in cities are either entirely cut of, or contaminated by such impurities as render them offensive and unwholesome. No other resource remains but to draw supplies of water from a distance. The ancients effected this by means of open aqueducts; the moderns, where the distance is not great, make use of pipes; where the distance is great, the main stream is brought in an aqueduct, and the distribution effected by pipes.

314. Conduit pipes are liable to be obstructed by lodgments of air in the upper sinuosities of the tubes, and by deposits of sediment at the lower ones. The first of these is prevented by a valve that floats on water, and is thus kept closed while the pipe is filled with that fluid, but falls and opens as soon as air lodges, permitting it to escape, followed by the water that raises and closes the valve. Deposits of sediment in the pipe itself are prevented by making its lower part communicate with an air tight box; this receives the sediment, and when full it is removed and emptied.

OF THE PERCUSSION AND RESISTANCE OF FLUIDS.

315. The several circumstances of the percussion of fluids against planes may be represented by the following formulæ:

$$f = m.\ s.\ a^2\ v^2\ \sin^2 i \qquad \text{or}$$

$$f = 2\,m\,s\,a\ {}^2gh\ \sin{}^2 i$$

Where f is the force with which the fluid strikes, m a constant co-efficient, determined by experiment in the case of water, s the specific gravity of the fluid, v its velocity, h the height due to that velocity, g the measure of the force of gravity, and i the angle of the inclination of the plane to the course of the fluid. Bossut, § 719, et seq.

Where the extent of the plane is indefinite $m = 1$; where the extent of the plane is not greater than that of the section, the stream $m = \frac{1}{2}$.

In introducing s into the formula, regard is had merely to the inertia of the particles; there is, however, a cause of error arising from the different cohesive force of the particles of different fluids; but it is only perceived in slow motions, and is very small, except where the fluid under consideration has much viscidity. Playfair, § 293.

Experiment has shown that the force does not vary in the ratio of *sin ²i, except* where the angle i is between 60° and 90°. Bossut found *that where a prow,* of the form of a wedge, is drawn through a fluid, if *the resistance to its* base, = 10,000, the resistance to the wed *self will be*

$$10.000 \cos.^2 i + 3.153 \left(\frac{i}{6^\circ}\right)^{3\frac{1}{4}}$$

PLAYFAIR, § 294.

316. If the plane struck by the stream, be itself in motion, the impulse is as the square of the difference of their velocities. And

If the fluid be at rest, and the plane alone in motion, the resistance is as the square of the velocity of the plane.

Hence it would appear, that the resistance of a fluid to a body in motion, is the same with the percussion of a fluid moving with the same velocity against a body at rest; but although this is founded on reasoning very evident, it does not agree with the fact, for the resistance is less than the percussion in the proportion of 5 to 6. Playfair supposes this difference to arise from the action of the fluid on the hinder part of the body moving through it, by which the resistance is in some measure counteracted. It is, however, the absolute quantity, and not the ratios of the resistances that are affected, for they are still among one another, as the squares of the velocities. PLAYFAIR, § 293.

317. Supposing the resistance to oblique surfaces to vary with the square of the sum of the angle of inclination; the resistance to a cylinder, placed vertically in a stream, is two thirds of that which would be encountered by the plane, which is its section; and the resistance to a hemisphere, is equal to one half of that opposed to a circular plane of the same diameter. BOSSUT, § 734, § 735.

OF THE MOTION OF WAVES.

318. Where pressure is applied to any portion of the surface of a liquid, the column pressed upon is shortened, and sinks below the natural level of the surface; while the contiguous *columns rise* above that level. As soon as the last have attained *a certain height*, they fall and sink below the natural level of the *surface*, raising not only the first column, but communicating

similar motion to those which are contiguous to them on the opposite side. These, again, communicate a similar oscillatory motion to columns situated beyond them, and thus the impulse is propagated over a wide expanse, but decreasing as it recedes from the place where the action first began.

The alternate elevations and depressions thus produced, are called waves.

In the propagation of waves, there is not necessarily any progressive motion in the fluid itself.

319. Newton compared the motion of waves to the oscillations of a bent tube, whose branches were vertical; and upon this principle deduced expressions for the velocity of waves, and the time of an undulation.

The time of an undulation, is the time from the wave being highest at any point, until it is highest at that point again; the velocity of a wave is the rate at which the points of greatest elevation change their position.

320. The time of the oscillation of a fluid in a bent tube, is equal to the time of the vibration of a cycloidal pendulum, whose length is half the length of the column of fluid. Poisson, § 559.

If a be the altitude of a wave, b half the breadth, p the circumference of a circle whose diameter is one, the time of an undulation is

$$\frac{p}{8}(a+b)^{\frac{1}{2}},$$

and the space which a wave appears to pass over in a second, is

$$\frac{8\,b}{p\,(a+b)^{\frac{1}{2}}}.$$

Playfair, § 298.

321. **Where** a wave meets an obstacle it is reflected; if the obstacle is a vertical plane, the reflection takes place in a series of waves similar to those which would have occurred had the obstacle not been interposed, but in an opposite direction. If the obstacle have an opening in it, a series of waves proceeds from that opening, in the form of concentric circles.

Where two or more series of waves cross each other, they do it without confusion.

Where two series of waves cross each other, the greatest altitude of the ridges is equal to the sum of the altitudes that would have been produced by each separately, and the depression of the hollows or trough is increased in like manner.

Hence, where a wave is reflected by a vertical obstacle, the water rises higher immediately against the obstacle than in any other place.

Where the original impulse, instead of ceasing to act, continues to exert its force, one series of waves is raised after another, and in crossing each other, the altitudes are continually increased; we may thus account for the great height to which waves are sometimes raised by the long continuance of violent winds.

322. The action of the wind upon water may be much lessened by interposing a viscid fluid between them. Oil spilt upon the surface of water, spreads over it to a great distance, and it is found that, by preventing the succession of new impulses, it tends to still the waves. See FRANKLIN, *Works*, Vol. III.

323. Where the depth of water is so great, that the oscillation proceeds without being disturbed, its columns, that form the waves, have no progressive motion, but each is kept in its place by the hydrostatic pressure of the contiguous columns. But if an inclined obstacle be presented to them, such as a rock, or beach, the columns of water in the deeper part are no longer balanced by those in the shallower, and therefore acquire a progressive motion.

In this manner breakers and surf are formed; and hence the waves break and dash against the shore, whatever be the direction of the wind. PLAYFAIR, § 299.

OF THE MOTION OF ELASTIC FLUIDS.

The general hypothesis, by means of which the theory of the motion of fluids is investigated, is applicable to elastic as well as non-elastic fluids. Of elastic fluids, that whose motion is most important, is the air of the atmosphere.

324. Air will rush into a vacuum at the surface of the earth, with the velocity a heavy body would acquire in falling through the height of a homogeneous atmosphere.

If V be this velocity, H the height of a homogeneous atmosphere,

$$V = 8\sqrt{H}$$

$$\text{if } H = 26000 \text{ feet}$$

$$V = 1328 \text{ feet.}$$

325. The velocity with which air rushes from a vessel into one exhausted of air, is uniformly retarded, and becomes 0, when the air in the two vessels becomes of equal density.

OF THE WINDS.

326. Where the air of the atmosphere is in motion, its currents are called *Winds*. The principal cause of the winds, is the disturbance of the equilibrium of the atmosphere by the unequal distribution of heat.

This unequal distribution of heat, is partly due to the figure of the earth and its circumambient atmosphere, partly to the alternation of seasons caused by the obliquity of the equator, and partly to local causes.

(1) *The* intensity of the sun's rays is as the quantity that falls on any *given space*; hence, if a small portion of the earth's surface be con*sidered* as a plane, the intensity of the sun's rays will be as the sine *of the sun's* altitude above the horizon. Those places where the

sun is vertical, are of course most heated. The atmosphere being a spherical shell concentric with the earth, the rays of the sun must pass through it for a greater distance when they fall obliquely, and a greater quantity must be lost and dispersed by refraction and reflection.

(2) The meridian altitude of the sun above the horizon, is not the same two days successively at the same place ; the difference in the heat received from the sun at any given place, on different days, is therefore great. According to Prevost, the proportion between the quantities of heat that reach the surface of the earth, at Geneva, in July and December, are as 7 or 8 to 1. YOUNG, § 56.

The earth does not appear to grow either warmer or cooler ; the whole of the heat, therefore, which is received by the earth, radiates from it again. In summer, any given portion of the earth receives more heat than it radiates, and hence there is an accumulation of heat after the solstice ; this does not reach its maximum until about a month later. The maximum of cold is, for the converse reason, a month later than the winter solstice.

(3) The temperature of any particular place, and even of whole countries, is affected by the vicinity of seas, great lakes, mountains, morasses, forests, and by the greater or less degree of cultivation it exhibits.

Seas and great lakes are more slowly heated than the surface of the earth ; for the rays of the sun are not absorbed by the upper strata, but pass through, and are not entirely lost until they reach a great depth ; the heat derived from the sun is therefore distributed throughout the whole mass. They are likewise more slowly cooled by radiation ; for when the upper stratum has given out its heat by radiation, it becomes heavier than the one immediately beneath, and descends to give it the place due to its difference of specific gravity.

As we rise above the level of the sea, the heat is found to decrease at a rate that is stated by Playfair, as 1° of Fahrenheit for 270 feet. On ascending into the atmosphere, therefore, at any latitude, to a certain height, we may reach a point where the mean height of the thermometer is 32°. This altitude is called the *height of perpetual congelation.* It is elevated 15,000 feet above the level of the sea, *at the equator* ; and at the latitude of 80° no more than 1200 feet. The cause of this diminution of heat, appears to be owing to the diminution in the density of the atmosphere, and the superior capacity of rare air for heat.

High mountains, therefore, affect the climate, and may not only temper a warm one, but increase the rigour of a cold one. We find in the torrid zone, mountains covered with perpetual snow, and plains affected with no greater heat than the most temperate climates.

Forests tend to produce cold, by preventing the sun's rays from reaching the ground. Morasses and small lakes increase the cold of a climate, by the absorption of heat the evaporation from their surfaces causes.

Cultivation affects the mean temperature, by exposing the surface of the earth to the rays of the sun. It produces an effect not only on the country where it takes place, but upon those adjacent; thus, the climate of Italy has been influenced by the destruction of the forests of ancient Gaul and Germany.

The mean temperature of some places is affected by another cause. When they are near the line of perpetual congelation, and foreign masses of ice are introduced by any accidental circumstance, they will gradually become colder; for in the alternations of the one side of the freezing point, heat is suddenly let loose, and radiates to a distance, while thaw is a slow and gradual process; the adventitious ice will therefore increase rather than diminish. We may in this way account for the accumulation of ice on the shores of Greenland, the consequent change in the climate of Iceland, and the advance of the glaciers on the mountains of Switzerland. See PLAYFAIR, § 387, et seq. LESLIE, *Elements of Geometry.*

The northern hemisphere is found, by observation, to be warmer than the southern; this is supposed to arise from the greater length of the summer on the northern side of the equator; the interval between the vernal and autumnal equinox of the same year, being 7 days longer than the interval between the autumnal and the vernal of the succeeding year.

327. The several winds whose course may be explained upon the foregoing principles, may be divided into the following *classes:*

1. The trade winds.
2. Monsoons.
3. Local modifications of these two.
4. The westerly winds that are found to prevail near the northern and southern limits of the trade winds.
5. The variable winds of the temperate and frigid zones.
6. Land and sea breezes.

The air over that zone of the earth which is most heated by the solar rays, is expanded, becomes lighter in consequence, and rises; the adjacent columns, to the north and south, rush in to supply the place, and thus a current is produced, that, in the absence of any other cause, would be from the north, on the north side of the heated zone, and from the south, on its south side. But in consequence of the rotation of the earth upon its axis, another motion is combined with that just mentioned; for the air acquires a velocity corresponding to that of the portion of the earth immediately beneath it, and as it comes from a latitude where the velocity is less, to one where it is greater, it will have an apparent motion in a direction contrary to the diurnal revolution of the earth, or from east to west; this motion, combined with the other, will make the direction of the trade wind at its northern limit, N. E. and at its southern limit, S. E. As it approaches towards the zone that is most heated, the velocity from N. to S. and from S. to N. is destroyed by the counteracting effect of the current flowing in from the opposite direction, and the apparent course of the wind becomes due east.

This is the cause of the trade wind, which, with certain exceptions, blows continually between the tropics, and even reaches the parallel of 30° on both sides of the equator. In summer it is to be found on the coast of America as far north as 32°.

The zone where the trade wind is due east, varies but little, and is at a mean on the north side of the equator; this is owing to the greater heat of the northern hemisphere. This theory of the trade winds, that supposes them to be caused by the air in its motion southward, *falling* back towards the west, is mentioned by Halley, but rejected *by him*; it has since been espoused by Franklin, Laplace, and Playfair, and is fully entitled to be received as true; for it not only e plains the phenomena, but is derived from causes that are known to exist in nature. Playfair, § 410. Franklin, *Works*, Vol. III. Laplace, *Mechanique Celeste.*

328. When the air has risen over the central zone of the trade winds, to a certain height, it there forms a counter current, that carries it back from the equator, towards the poles. The general direction of this upper current must incline to the westward. The zone of the trade winds being limited by the parallels of 30° of north and south latitude, the counter current descends to the earth there; hence we find, that in the north and south Pacific, and in the Indian Oceans, westerly winds are almost constant between the parallels of 30° and 40°.

Not only the trade, but all other winds, are accompanied by counter currents in the higher regions of the atmosphere; and the quantity of air in those opposite currents is so nearly equal, that the average weight of the air, as measured by the barometer, is nearly the same at all places on the earth's surface. Playfair, § 410.

The surface that would separate two opposite currents of equal pressure, is elevated about 18,000 feet above the level of the earth. Playfair, § 412.

329. The Indian Ocean is bounded by land on the north. When the sun is on the north side of the equator, this land is more heated than the water, the air adjacent to it rises, and that from the ocean rushes in to supply its place; but as it comes from a portion of the earth, whose velocity of rotation is greater than that it passes over, its apparent direction becomes S. W. When the sun returns to the south side of the equator, the land cools, and the regular trade wind from the N. E. again prevails.

These periodical winds, that are met with in the Indian Ocean, are called Monsoons.

330. Local circumferences produce many remarkable modifications of the trade winds, and monsoons. Thus, in the gulf *of Guinea,* the wind blows from the south and south west con*tinually. This* is explained by supposing that the S. E. trade wind, *after passing* the equator, inclines towards the shore, in consequence *of* that being much heated by the rays of a tropical sun.

Adjacent to the region of this westerly wind, is a tract of sea where perpetual calms appear to exist, attended with thunder, and almost continual rain. In this tract, the counter current of the wind, just spoken of, appears to descend again to the earth's surface.

331. The remaining portion of the earth's surface, is the seat of variable winds. In it, the changes of temperature are more rapid, and subject to less certain laws; hence, the currents are less constant in their direction, and of no definite duration. Yet, notwithstanding these irregularities, there is a tendency in most countries to periodical winds.

In Europe, a north east wind prevails in summer, to a very great degree; this is supposed to arise from the same cause as the trade winds. The Elesian wind of the ancients is this, modified by the action of the adjacent countries.

In the United States, a north east wind is likewise frequently felt; it also arises from the same great cause, and becoming yearly more frequent, during the summer months, promises to be the prevailing wind of that season.

A remarkable current, called the Gulf Stream, is to be found in the Atlantic Ocean, near the coast of the United States; a constant trade wind blowing into the Gulf of Mexico, raises the water therein above the general level, and it discharges itself with great rapidity between Cuba and Cape Florida; although it spreads, and its velocity diminishes, it continues to be felt as far as the banks of Newfoundland to the north. This current, drawing its supplies from tropical regions, is more heated than the adjacent ocean; and in winter is considerably warmer than the land of the United States; for this reason, a wind is frequently found blowing towards the Gulf Stream over the continent of North America. The direction of this is N. W. and it is the prevailing winter wind of our climate.

A continuous ridge of lofty mountains extends the whole length of both Americas; within the zone of the trade winds, the plain of Mexico is 8,000 feet above the level of the sea, and the peaks of the ridge are more than double that height; the trade wind, therefore, has not a continuous current around the earth at this place, but would pass along the sides of these mountains to the north and south, were it not impeded in the former of these directions by currents from the N. W. and N. E. as we have just explained; the accumulation ments of Saussure, it would appear, that while the temperature increases in arithmetical progression, the quantity of moisture it is capable of holding in solution increases in geometrical.

Hence, if two portions of air, of different temperatures, and both saturated with humidity, be mixed together, a precipitation of moisture must take place. PLAYFAIR, § 419, § 420.

339. Upon this fact, Dr. Hutton of Edinburgh was led to found a very ingenious theory of the formation of clouds. If large portions of the atmosphere, saturated or nearly saturated with humidity, be driven against each other by contrary winds, he infers that a precipitation of humidity, or the formation of clouds, must take place.

This theory may be illustrated by a geometrical diagram. See PLAYFAIR, § 420, and 421. HUTTON, *Dissertations.*

340. Although clouds may be formed, and precipitation ensue, at any altitude, yet these phenomena are likely to occur most frequently near the plane which separates the two great currents spoken of in § 328. This stratum of air, elevated about 18,000 feet above the level of the sea, is hence justly called the region of the clouds.

When clouds are formed, they descend by the action of their gravity. If in descending, they meet with strata of dry and warm air, they are again dissolved and disappear; thus clouds are frequently seen to roll from the summits, down the sides of mountains, to a certain level, and then disappear. This phenomenon is considered by the neighbouring inhabitants as a sign of dry weather. When, instead of meeting with dry, they pass through moist strata of air, they are no longer absorbed; and as they come from high and cold regions of the atmosphere, they absorb heat from the lower strata to such an extent as to cause precipitation, and the moisture therein contained joins the descending cloud to increase the quantity of rain it discharges. For this reason, we often find the same rain more violent in valleys than on the neighbouring mountains.

341. *Where two* masses of air, of high, but different temperatures, *are mixed together*, the precipitation is greater than when two

equal masses, at lower temperatures, are mixed, although the difference of temperature is in both cases equal.

We may in this way account for the great quantity of rain that often falls in a small space of time in tropical climates; and for the violence of the showers that accompany thunder gusts in summer.

342. When precipitation of humidity takes place, where the temperature is lower than the freezing point, the particles are frozen, and, arranging themselves in crystalline masses, form flakes of snow.

These flakes sometimes exhibit themselves in the form of six-pointed stars.

343. It has been said, that when drops have once been formed, and are afterwards frozen in their descent, they constitute hail. See PLAYFAIR, § 429.

But as hail often takes place in the warmest weather, this cannot be considered as a sufficient explanation. The following theory of hail, taught for many years in Columbia College by my predecessor Dr. Kemp, is more consistent with the phenomena.

It is known that when water is frozen in a Torricellian vacuum, it granulates and assumes the form of hail; hail also reaches the ground with a very great velocity: hence we may conclude, that it is formed in very rare air, and in a high region of the atmosphere. The decomposition of organic substances, is constantly giving out hydrogen gas, and this, from its specific levity, rises to the higher regions of the atmosphere; here, as no gas can remain long over another unmixed, it mingles with atmospheric air, and becomes susceptible of being inflamed by electricity. Should it be thus acted upon, it forms water, will be condensed into a space much less than it formerly occupied, and would leave a vacuum, did not the adjacent portions of air rush in to fill the void. The sudden rarefaction of this air, will produce an intense cold; the newly formed water will be frozen, and under circumstances that will cause it to granulate; descending from a lofty region, it will have great velocity; formed from hydrogen gas, and by the electric discharge, it will occur most frequently during the summer months, and accompany lightning.

The same theory was also extended by its author, to the explanation of the cause of meteoric stones. In the absence of all certain knowledge, it would be unphilosophical to insist upon it; yet if water is held in solution by the air, it may carry with it such earthy substances and metallic oxides as are soluble in it; and if the metals, their combinations or their ultimate elements, be capable of existing in an ærial form, they may float in the atmosphere, and may be united into solid masses by electric or galvanic action.

A similar theory, having reference also to the agency of terrestrial volcanoes, is mentioned by PLAYFAIR, § 431.

344. In the absence of the sun, the earth becomes cooler than the superincumbent air. That stratum of air which is adjacent, will communicate to it a portion of its heat; the moisture it contains will be separated, form a cloud, and finally precipitate; the same action will be propagated by it to the next higher stratum; and thus a cloudiness or mist will be formed, extending to some distance from the earth. When the sun again appears, such part of the moisture as has not actually reached the ground, will be again rapidly absorbed.

In this manner, we may account for the formation of dew; which appears only in calm weather, and during the night. We also see why the mist that attends dew appears to rise, although the moisture actually falls.

345. The pressure of moisture in the atmosphere, is known by the deliquescence of certain salts; by the torsion of hempen and flaxen cords, of cat-gut, whalebone, and hair. These substances are called Hygrometric substances, and have been applied in various ways to the construction of instruments called Hygrometers. The most ingenious of these, are the Hygrometers of SAUSSURE, DELUC, and KATER.

Professor Leslie of Edinburgh, has proposed the use of a modification of his differential thermometer, as a Hygrometer; and this, of all others, is the least liable to exception.

346. The quantity of rain that falls in any one place, may be measured by means of an instrument that is called a rain-gauge.

The principle of this gauge, consists merely in its collecting all the water that falls upon a determinate surface, and then measuring its depth in inches and decimals. The best rain-gauge is that of Howard.

The quantity of rain varies in different places very considerably, and is subject to no fixed law. It is consequently needless to cite any particular instances.

347. That phenomenon, the water-spout, is supposed to be caused by the action of conflicting currents of wind, that form a whirl, in the interior of which is a vacuum, where the water rises, as in the common pump. Others suppose the phenomenon to be electrical. See FRANKLIN, *Works*, Vol. III. MUSCHENBROCK, Vol. II.

348. The intimate connexion between rain and the winds, is evident from many appearances.

(1) Where trade or other winds blow uniformly, hardly any rain falls; when the monsoons change, heavy rains take place.

(2) In tropical climates, the rainy season always takes place when the sun approaches the zenith, and at this time the winds are most variable.

(3) There are some spots of continual rain, and in them there seem to be opposite currents of air continually meeting.

(4) In other places, it hardly ever rains. They are usually far inland, and without any of those inequalities of surface that promote the mixture of air.

(5) In countries near the ocean, where the air is much loaded with moisture, every change of wind is accompanied by rain; this is remarkably the case in Scotland, and on the north western coast of England.

(6) Lowness of the barometer is a sign of rain; it indicates that the equilibrium of the atmosphere is destroyed, and it is therefore probable that, before it is restored, winds from different quarters, and at different temperatures, must come into collision with each other. See PLAYFAIR, § 423.

OF AIR AS THE VEHICLE OF SOUND.

349. When an elastic body is drawn from its natural position, and then permitted to return, it regains a state of rest by a series of isochronous oscillations. These vibrations being communicated to the air, that is also a compressible and elastic body, produce therein alternate condensations and dilations, that are gradually propagated throughout the whole mass, in the same way that waves are formed upon the surface of a fluid. When these waves succeed each other rapidly, they cause in the organ of hearing a sensation that is called SOUND. BIOT, Vol. II.

In order to show that sound may be caused by the rapid oscillation of an elastic body, take a long and thin plate of steel, one of whose extremities is fixed in a vice, if it be drawn to one side and then left to itself, a series of vibrations will begin; if the plate be long enough, these vibrations may be counted, but in that case, they will produce no sound; if the plate be now shortened, the vibrations will become more frequent, and after they can no longer be counted, a sound begins to be heard. If we have counted the vibrations of the longer plate, we may now calculate the number of vibrations that the sounding body makes, on the principle that the number of vibrations are inversely as the squares of the lengths of the vibrating bodies. BIOT, liv. II.

To show that air is a vehicle of sound, we have recourse to the experiment of the bell in an exhausted receiver.

350. Air is not the only vehicle of sound; the other elastic fluids, whether gases or vapour, conduct it also. Liquids also convey sound, and so do solid bodies. BIOT, liv. II.

That water is capable of transmitting sound, might be inferred from the well-known fact, that fishes are furnished with organs of hearing. We may however prove it fully, by ringing a bell beneath the surface of the water.

351. The transmission of sound through the a
tantaneous, but it requires a certain time to tr

place to another. By the investigations of Newton, it would appear that the velocity of sound is 968 feet per second, while actual experiment makes it 1142 feet.

The correctness and accuracy of Newton's investigation, as far as mathematics is alone concerned, has been fully tested by succeeding analysts. It, therefore, is important to ascertain the circumstances that have not been taken into account, and by the omission of which, the mathematical process is rendered erroneous. Newton himself supposes the atmosphere to contain sulphureous and saline particles, but modern chemistry has shown that these do not exist. Neither does the rapidity of the oscillation bring the particles of the air itself into contact. Some have supposed, that the presence of moisture in the atmosphere, would be sufficient to account for the acceleration of sound ; but if the air were saturated with moisture, the increased velocity would not be 8 feet per second.

Another probable cause, was pointed out by Laplace, viz. the action of the heat that is let loose by the compression of the air. This has been fully investigated by Biot, and is shown by him to be sufficient to explain the phenomenon. Biot, liv. II.

352. The velocity with which sound is propagated in a liquid, is greater than in a gaseous body ; its intensity is also greater ; but this is much diminished when the sound passes from the water into the air, and vice versa.

The motion of sound is much more rapid in solid than in fluid bodies ; but although experiments have been contrived for the purpose of determining the actual rate, they have not been entirely successful. Biot however states, that sound travels more than ten times as fast through cast iron as it does through the air. Biot, liv. II.

The best experiments on these subjects, have been made by Perolle and Chladni. Chladni, *Acoustique*, § 217.

353. The great variety of sounds that the human ear is capable of distinguishing arises from three causes. 1. The greater *or less frequency* of the vibrations of sonorous bodies ; 2. From

the greater or less intensity ; and, 3. From the greater or less simplicity of the sound.

Sounds are consequently distinguished, as high or low, acute or grave, flat or sharp, as loud or weak, and as clear or confused, &c.

354. Sound being conveyed through the air in a succession of waves, is capable of being reflected by obstacles, in which, every point of incidence becomes a new point of propagation. From a knowledge of this fact, we may derive the explanation of many curious phenomena.

Where the obstacle is near the sonorous body, the reflected sound reaches the ear at so short an interval after the original one, that they appear as one, but increased in intensity. In this way, we see the use of sounding boards in musical instruments. In the Italian Theatres, the side of the orchestra is made one great sounding board.

Where the obstacle is more remote, the sound first becomes confused, and finally we hear the reflected sound separate and distinct from the other. It is then called an echo. The ear is capable of distinguishing between 8 successive sounds in a second of time ; hence, if the reflected follow the original sound at an interval of more than 140 feet an echo is produced.

The same voice appears louder in a close room than in the open air; but if the apartment be large, confusion will often arise from the reflected sounds. This confusion may be much increased by an improper construction.

It is of the utmost importance that Churches, the Halls of Legislative Assemblies, Theatres, Concert Rooms, &c. should be constructed upon proper principles ; so that the voice may be as much aided as possible, by the reflection of the sound, and yet no disturbance or echo take place.

355. Where the waves that proceed from a sonorous body strike against continuous obstacles, and are reflected in such a manner as to pass through one or more points, either together, or at small intervals of time, a sound will be heard by an ear placed there, that will often be louder than the original sound.

In an ellipsoid, for instance, if a sound be uttered in one of the foci, all the reflected waves will cross one another at the same instant in the other focus. A conversation in a low tone, may therefore be kept up in a room of this shape, between two persons placed in the two foci, that will be inaudible to any other persons.

To produce this effect, the ellipsoid need neither be very large nor continuous.

In a spherical figure, the same effect will be produced in relation to two points equi-distant from the centre, and situated in the same diameter; for the interval will not be sufficient to render the sound confused. In this way, we may explain the phenomena of whispering galleries, and account for many deceptions that have been practised on the credulity of mankind. Others have, however, been practised, by making use of the rapidity with which solid bodies conduct sound.

The forms that are best fitted for producing such phenomena, are the worst possible for those of distinct hearing; such forms should therefore be always avoided. Mere theory would have shown that the representative chamber at Washington, and the Theatre at New-York, would be, as experience has proved them, almost totally unfit for their intended objects. In both these buildings, the roofs are vaulted and reverberate the sounds; in the latter, there is actually a good whispering gallery. The reader who wishes to pursue this subject farther, may consult CHLADNI, *Acoustique*, § 210.

356. The human voice may be rendered audible at great distances, by means of the speaking trumpet. To produce this effect, the waves that would otherwise be dispersed through the air, must be reflected from the sides of the tube, and reinforce the sound in the direction of the axis. A cylindrical tube will not do this; but it is necessary that the tube should enlarge at the extremity most distant from the mouth. CHLADNI, § 201.

Some have supposed, that the speaking trumpet acted by the communication of motion to a series of elastic bodies; as the quantity of motion communicated by this means, is greatest, when the bodies are in geometric progression, the logarithmic curve has been recommended as a proper one to form the outline of the speaking trumpet. HELSHAM, *Lecture*.

357. The hearing trumpet is the reverse of the speaking trumpet, and is so contrived, that all the sound that falls upon a large surface, may be concentered in the auditory canal of persons whose hearing is defective. A variety of figures has been given to this instrument, hollow cones, paraboloids, and ellipsoids, likewise figures the concavity of whose section is turned towards the axis. In some of them, for convenience sake, the instrument is twisted.

Of Musical Sounds.

A musical sound is one whose vibrations can be estimated.

An interval is the relation which the number of vibrations of two musical sounds have to each other. We usually take a grave sound as the standard with which we compare one that is more acute. Melody is a succession of musical sounds.

Two musical sounds expressed at the same time constitute an accord. Harmony is a succession of accords, or the co-existence of several melodies. CHLADNI, § 4.

358. In the lowest sounds that are perceptible to the human ear, the sounding body makes at least 30 vibrations per second; and it is still capable of receiving the impression where the sounding body makes from 8 to 12000 vibrations.

Chladni takes for his *ut* or fundamental note, one whose number of vibrations per second, is some power of the number 2. The lowest note of the violoncello or piano forte, he considers as caused by 128 vibrations of the sounding body per second. From this, the number of vibrations that cause any other musical sound whatsoever may be determined by calculation. CHLADNI, § 5.

359. The greater or less simplicity that exists, in the relation between the numbers of the vibrations of sounding bodies, is the source of all harmony.

An interval is a concord where this relation is simple, a discord where *it is complex.* The ear, although unable to count the number *of vibrations,* is capable of observing the effect of such vibrations *as return together.* CHLADNI, § 6.

The nicety with which an ear perceives the coincidence, causes it sometimes to be called a musical ear, or an ear for music.

360. The simplest of all relations, is that of 1 : 1, or where all the vibrations of two sounding bodies are performed in equal times. This relation is called the *Unison.*

361. The interval where the number of the vibrations of one body is double that of the other is called the octave. It is so named, because it is the eighth note of the musical scale, and every other note in the scale takes its name from the number of the place it occupies.

It is well known from experiment, that a sound and its octave have such a resemblance, that one may be considered as a repetition of the other. Hence,

(1) The nature of interval is not affected by taking the sounds that compose it one or more octaves higher or lower :

(2) All possible intervals may be regarded as comprised within a single octave, and may therefore be expressed by improper fractions, not less than 1, nor larger than 2.

362. The interval where the number of vibrations of one body is triple that of the other, is the next in simplicity to the octave. Where it is regarded as belonging to the first octave, on the above principle, its fractional form $\frac{3}{1}$ becomes $\frac{3}{2}$. It is then the fifth to the fundamental note. The next relation in simplicity, is that of $\frac{4}{1}$; but this is the second octave to the fundamental note, and does not extend the scale. $\frac{5}{1}$ is the next interval, and when reduced to the first octave, it becomes $\frac{5}{4}$. This expresses the interval that is called the third.

A note with its third and fifth, is called a perfect accord.

At the origin of music, its scale appears to have been limited to these three notes ; but they do not constitute a scale in consequence of the great extent and inequality of the intervals. The scale is completed by adding,

(*1*) Two notes that constitute a perfect accord with the fifth, as follows, viz.

$$\frac{3}{2} \times \frac{5}{4} = \frac{15}{8} \quad \frac{3}{2} \times \frac{3}{4} = \frac{9}{8}$$

and

(2) Two notes, such that they shall form another perfect accord, of which the octave becomes the fifth, viz.

$$1 : \frac{5}{4} : \frac{3}{2} : : \frac{4}{3} : \frac{5}{3} : 2$$

In this manner, we obtain the musical scale of 7 notes. Besides the names drawn from their numbers, they are known by particular appellations in some countries, and by letters in others; the intervals, with their several distinctive epithets, are as follows, viz.

1	$\frac{9}{8}$,	$\frac{5}{4}$,	$\frac{4}{3}$,	$\frac{3}{2}$,	$\frac{5}{3}$,	$\frac{15}{8}$,	2
1st.	2d.	3d.	4th.	5th.	6th.	7th.	8th.
C	D	E	F	G	A	B	C
ut	*re*	*mi*	*fa*	*sol*	*la*	*si*	*ut*

This scale is called the diatonic scale. Chladni, § 12.

It is also called the scale of the major mode, because, in addition to the perfect accord we have made use of, that is called *Major*, there is another composed of the intervals $1 : \frac{6}{5} : \frac{3}{2}$ that is called *Minor*. Chladni, § 9.

363. The ratios of the numbers and fractions, that express the intervals of the scale of the major mode, are of three different magnitudes, viz.

of 9 : 8, of 10 : 9 and of 16 : 15.

The first of these is called a major tone, and designated by the letter T; the second is called a minor tone, and designated *t*; the third a semitone, and designated H. They succeed one another in the following order:

T, *t*, H, T, *t*, T, H.

Cavallo, Vol. II.

364. In the scale of the minor mode, they succ
nother in a different order, say,

T, H, *t*, H, T, *t*, T,

If, then, the sixth be considered as the fundamental note, this order of intervals may be expressed by the same sounds as the other; but in this different order of arrangement:

A	B	C	D	E	F	G
la	*si*	*ut*	*re*	*mi*	*fa*	*sol*

365. Nature appears to admit no other order of intervals that are fit for music. Therefore, in the diatonic scale, no other note than A or C, can be taken for the key or fundamental note. But the ingenuity of musicians has contrived to make every note of the diatonic scale capable of being the key note, either of the major or minor mode. CAVALLO, Vol. III.

This object has been accomplished by the interposition of certain intervals or half notes between those of the diatonic scale. These are used occasionally, and have no appropriate individual name or letter, but derive their appellation from the neighbouring principal notes, with the addition of *Sharp* or *Flat*, according as their number of vibrations is greater or less than of those that produce the principal sound itself.

These half notes are interposed, to divide the intervals of the major and minor tones; they are consequently five in number:

C* D* F* G* A*

D^b E^b G^b A^b B^b

This method is not however perfect, as may be ascertained by taking D for the key note, and comparing its number of vibrations with the notes that become its 3d and 5th. It is, however, well known by experience, that if an interval occur, that differs but little from one that may be expressed by simple numbers, it sounds to our ear almost exactly like the simpler. This is a happy circumstance, for it affords the means of obviating the difficulty, by dividing the error among several notes, instead of accumulating it upon one. Thus, in tuning instruments, we do not make either the major or minor mode of any one key perfect.

This mode of obviating the disadvantage, is called *Temperament*. For a more full view of this subject, see Chladni, § 21, et seq. and Biot, Vol. II.

Of the Vibrations of Sounding Bodies.

366. The elasticity of a sonorous body is the cause of its vibrations; and this elasticity may arise either from tension, compression, or from the hardness of the body itself.

The direction of the vibrations of a sonorous body may be either longitudinal, transverse, or circular. Chladni, § 34.

In its transverse vibrations, the parts of a sounding body pass alternately backwards and forwards, in lines perpendicular to the axis.

The longitudinal vibrations of a sounding body, consist in contractions and dilatations of the sounding body, or its parts, in the direction of its axis.

Circular vibrations are performed in a reciprocating manner. In cylindrical or prismatic rods, the sounds produced by this species of vibration are always a fifth lower than the sound produced by the longitudinal vibration of the same body.

The intensity of sound depends on the greater or less arc described for the vibrations, on the size of the sonorous body, and on the velocity of the vibrations. Chladni, § 35.

367. A musical string may either vibrate entire, or divided into a number of equal parts separated from each other, by points that are called *Nodes*.

The lowest sound that can be produced in this way, is that of the entire string. If it be divided into two parts, the sound is its octave; if into three, the 5th above its octave, &c.

The sounds of the aliquot parts of a string, may be produced by laying the finger lightly upon the proper node, and then applying the bow to the middle of the part that is to vibrate. Sounds of this sort are called harmonic sounds.

368. The vibrations of membranes stretched lengthwise differ but little from those of strings; the nodes of their vibra-

tions are, in this case, fixed transverse lines. Where the membrane is stretched in more than one direction, the investigation of the law of its vibration becomes more complex. CHLADNI, § 46.

369. The vibrations of the air that can be most readily investigated are such as are produced by its passage through narrow openings and passages. The velocity of these vibrations depends on the velocity of the current of air, and on the magnitude of the opening. CHLADNI, § 49.

370. If a rapid current of air, passing through a narrow opening, or pipe, strike against a membrane, and causes it to vibrate, the sound is much increased. But it also becomes rougher.

The voice of men and animals is produced upon the above principle. There are in the larynx two membranes of a figure nearly semicircular, that, when applied to each other, form a complete circular surface. The circumference of these membranes, that are called the ligaments of the glottis, is fastened to the sides of the larynx, and their straight edges are capable of either closing completely, or leaving a lenticular opening that is called the glottis.

Where this opening is large, the air passes without causing sound; this is the case in our ordinary respiration. But if the membranes be drawn together, the air that passes from the lungs by the larynx, acts upon the membranes; they are thus set in motion, and communicate their vibrations to the current of air that is passing out. The other organs of the mouth act upon this current, and each of them produces an effect upon the voice. The closer the glottis is drawn by the action of the ligaments, the more acute the sound.

Every possible variety of sound is produced by a change of the magnitude of the opening, within limits that do not exceed $\frac{1}{10}$ of an inch.

Those who wish to pursue the subject of the vibrations of sonorous bodies, may consult CHLADNI.

Of the Organ of Hearing.

Our organs of hearing are situated on the two opposite sides of the head, in cavities of the temporal bones. The outer ear is a cartilage of a form nearly that of half an ellipsis. It appears to be intended to strengthen the sound.

The auditory passage is partly cartilaginous, partly bony; it is closed by the tympanum, a membrane that receives the impression of the waves of air, in order to communicate them to the inner ear.

Between the tympanum and the labyrinth, is the cavity of the tympanum, an irregular space that differs but little from a hemisphere; it is filled with air, and communicates with the mouth, by means of the eustachian tube.

Beyond this, we find the two openings of the labyrinth, called the round and oval windows from their figure; the round window is closed by a membrane; through the oval, the impressions made upon the tympanum are conveyed to the labyrinth by means of a series of small bones; these are four in number, and are styled the anvil, the hammer, the lenticular bone, and the stirrup. The base of the latter closes the oval window. The labyrinth is so called, from its complicated figure and parts; it is the inner portion of the organ of hearing, and is closely enveloped in the temporal bone; it contains the substance of the auditory nerve, spread in membranes and fibres in a gelatinous liquid. The parts of which the labyrinth is composed, are three semicircular canals, the cochlea, and the vestibule.

371. The impression of sound is communicated in the following manner:

The vibrations of the air, caused by those of the sonorous body, strike upon the tympanum; this sets in motion the four small bones that act upon one another, like a combination of levers, and the base of the stirrup communicates the impression to the gelatinous liquid contained in the labyrinth. The vibrations of the tympanum are also communicated to the air in its cavity, and this transmits them to the round window; thus the impression is made at once, in two different ways. The auditory nerve, whose substance is spread *throughout* the labyrinth, transmits these impressions to the brain, *the common* centre of sensation. CHLADNI, § 232.

372. We also hear sounds propagated through liquid or solid substances, especially when the vibration is communicated to the solid parts of the head, that transmit it to the auditory nerve.

The impression received in this way is stronger than that communicated through the air, and conveyed to the ear in the common way. CHLADNI, § 233.

373. The sound of our own voice is conveyed to the inner ear by means of the eustachian tube ; and when the usual channel of sound is closed, we may often hear by opening the mouth.

MECHANICS.

APPENDIX I.—OF MACHINES.

374. Machines may be considered as tools interposed between the natural agent, or worker, and the task to be performed, to render that work capable of being executed, which would have been difficult, if not impossible, without the intervention of some such contrivance.

375. We interpose machines between the agent, and the work to be performed, for the following reasons, viz.

(1) To accommodate the direction of the moving power to that of the resistance.

(2) To render a power, that has a fixed and determinate velocity, efficacious in performing work with a given velocity.

(3) To enable a natural agent, having a given intensity, to overcome another power or obstacle, whose intensity of resistance is greater.

376. Compound machines consist of the mechanic powers combined with each other in various ways, and modified in various manners. Hachette divides the elementary parts, of which machines are composed, into ten series, having relation to the change they produce in the direction or nature of motion.

Of all the lines that a point can describe, upon a plane, the two simplest are the circle and the straight line. If the point describe an entire circle, turning continually in one direction, this species of motion is said to be *continuous circular;* if, after having described the entire circumference of the circle, or any portion of it, in one direction, the point shall return over the same path to its original position, the motion is said to be *reciprocating circular;* if it describe a straight line without changing its direction, the motion is *continuous rectilineal;* if it change its direction, and return over the same straight line, to the point whence it first set off, the motion is *reciprocating rectilineal.*

Among these four kinds of motion, there may take place ten different combinations; but of two of these, there is no practical instance; of the application of the remaining eight, examples will occur in the machines hereafter considered. See HACHETTE, *Traite Elementaire*, p. 5.

377. In every elementary machine, there are three motions to be considered, viz.

(1) That which results from the action of the moving power itself.

(2) The motion, whether circular or rectilineal, of that part of the machine which is immediately acted upon by the moving power.

(3) The motion that is transmitted, by means of this part of the machine. HACHETTE, p. 6.

378. Of the moving powers employed by mechanicians, the most important are, the force of gravity, the elasticity of springs, the strength of men and animals in motion, water, wind, the force of the elastic vapour of water or steam, and the expansive energy of the gases generated by the detonation of gunpowder.

The action of the three first may be investigated and explained upon the principles of dynamics; water, air, and steam, act according to laws that are determined by the principles of hydrodynamics; whence we also draw the laws that govern the action of the gases generated by fired gunpowder. This last, however, is limited in practice to one particular case, that has already been fully treated of.

Those machines, the investigation of whose action depends upon the properties of fluids, are called *Hydraulic Engines.*

To the above powers or mechanic agents may be added, the attractions of electricity, magnetism, and chemical affinity, and the variation in the pressure of the atmosphere, that is marked by the changes in the altitude of the mercury of the barometer.

OF THE MOTION OF MACHINES.

379. Where the momentum of the power applied
ine is greater than the momentum of the resistan

chine is put in motion; if the power be one that act more forcibly on bodies at rest than on bodies in motion, its action is of course diminished, and the acceleration of the machine is less in the second instance than in the first. The diminution of the accelerating force continues, till the instant when that force becomes equivalent to the resistance; the acceleration then ceases, and the motion of the machine becomes uniform. Playfair, § 185.

Gravity is the only force that is known to act with equal intensity on bodies whether at rest or in motion. In machines propelled by all other mechanic agents, a *permanent state* of working is soon induced.

By *Resistance*, we understand the resultant of all those forces that oppose themselves to the action of the moving power, whether arising from the action of gravity, the friction of the parts of the machine, the resistance of the medium, the work to be performed, or from any other cause whatsoever.

380. Where a machine attains its state of uniform motion, the momentum of the power is equal to that of the resistance, and is the same that would be in equilibrio with the resistance, were there no motion at all. Playfair, § 186.

This species of equilibrium is called dynamical.

381. In all machines, the work done is to be estimated, not from the quantity of resistance overcome, but from the quantity overcome in a given time; or it is as the momentum of resistance multiplied into its velocity. Playfair, § 187.

382. The strength of men and of animals is most powerful when directed against a fixed resistance; when the resistance is overcome, and the animal is in motion, its force is diminished; and with a certain velocity, the animal can do no work, but can only keep up the motion of its own body. Playfair, § 176.

If f represent the absolute effort of any moving force when it has no velocity, and W its velocity when it can do no work; and if F represent an effort corresponding to a given velocity, V, then, if the force be uniform :

$$F = f\left(1 - \frac{V}{W}\right)^2$$

Gregory, § 376.

383. In all machines that work with a uniform motion, there is a certain velocity, and a certain load, that give the greatest effect, and which are, therefore, more advantageous than any other. Playfair, § 188.

If the force be uniform, it will perform the greatest quantity of work, when the impelled point moves with one third of the greatest velocity of which it is capable. Gregory, § 377. Playfair, § 188.

If, therefore, the load that is just able to keep a machine acted upon by uniform force at rest, be found by experiment, it must be reduced to $\frac{4}{9}$ of this quantity, in order that the effect of the machine may be the greatest possible. Playfair, § 189. Gregory, § 377.

Although the force exerted by animals is not absolutely uniform, yet no great error will arise from considering it as such ; the above theorems are therefore applicable to all machines moved by the strength of animals and of men.

384. It is of the utmost importance that the motion of every machine should be rendered as uniform as possible. To obtain this end, a fly or regulator is most commonly used.

385. A Fly is, generally speaking, a circular disk of metal, moving with great velocity. When the action of the accelerating force is more than equal to the resistance, the fly opposes its inertia, but gradually acquires an increased velocity, and corresponding momentum. When the action of the accelerating force becomes less, the fly does not at once lose its velocity, but parts with it gradually, distributing throughout the other parts of the excess of momentum it had previously acquired.

Although a fly adds in fact to the momentum of the resistance, yet it will frequently enable an irregular force to do work that it would otherwise be incapable of performing ; thus, although a man who turns a winch is capable of exerting during a portion of its revolution a force equal to seventy pounds, yet as during another portion he can exert a force equal to no more than twenty-five, he cannot be employed to overcome a resistance greater than the latter; and thus the whole excess of his force is lost. By the application of a fly, he is enabled to work with a force equivalent to the raising of a weight of thirty pounds. GREGORY, Vol. II.

Two of the most remarkable instances of the use of the fly, are to be found in the steam engine, and in Vauloue's pile engine. The effect of a fly is proportioned to its velocity, when its weight and diameter are given.

Flies are sometimes constructed with leaves, but they are disadvantageous in working machines, for they waste the moving power unprofitably. When, however, the moving power is great, compared to the work to be performed; and where it acts with an accelerated force, as in the case of gravity, they may not be without their use. Thus they were formerly employed to regulate clocks, and are still employed in kitchen jacks.

386. Where the resistance is variable, as in the case where the same machine may be employed to perform very different quantities of work, the action of the moving power itself is regulated by an instrument called a *Governor*.

A governor consists of two heavy balls, suspended by a system of levers, forming a conical pendulum. Where the velocity is small, the balls hang close to the axis; as the velocity increases, they are carried out, their centrifugal force counteracting their gravity. The levers act upon a system of wheels and pinions; to which, where the balls are more distant than a certain quantity from the axis, they communicate motion in one direction; and when they are nearer, in the opposite. These wheels move machinery that acts to lessen the intensity of the moving power; thus, in water wheels, they close or open the aperture of the penstock; in wind mills, they increase or diminish the quantity of sail; and in steam engines, they open or shut a valve placed in the steam pipe.

387. A contrivance, acting upon the same principle as the fly, is sometimes used to accumulate or condense force received from a

long succession of impulses, and discharge it in an instant of time upon a given object.

We have an instance of this in the coining mill.

OF MACHINES MOVED BY THE FORCE OF GRAVITY.

388. When a weight in its descent causes the motion of a machine, it may sometimes be rendered uniform, by employing a resistance that increases with the velocity ; when the velocity has reached a certain quantity, the retarding becomes equal to the accelerating force, and the velocity of the machine remains uniform.

A fly with leaves, such as has already been described, is a regulator of this sort.

389. Of those machines where gravity is the moving power, clocks are the most important. In them, a pendulum is employed as the regulator ; this, by forcing the *Escapement* at each vibration against the last of a train of wheels and pinions put in motion by a weight, prevents their being accelerated by its descent, and, at the same time, receives an impulse just sufficient to continue its motion.

The exact adjustment of these two forces is a nice problem in practical mechanics. See REES' *Cyclopædia, Maintaining Power.*

390. In the clock, the velocity of the maintaining power is small, being no greater than that acquired by a falling body in a very short space of time ; hence wheels are in it made to turn pinions, and velocity is gained at the expense of intensity of force. The calculation of the number of teeth on the wheels and pinions, and the diameters proper to transmit the maintaining power to the escapement, and thence to the pendulum, is abstruse. Modern artists, however, find this done to their hand, and content themselves with imitating the works of their predecessors.

As an instance of the manner in which this is effected, we subjoin an account of the parts of a common eight day clock. The first or *great wheel* carries the hour hand, and of course should revolve in twelve hours; it is attached by means of a ratchet wheel, to a barrel, around which a spiral groove of sixteen threads is cut to receive the cord by which the weight is suspended. The action of the ratchet wheel is such, that the weight in its descent moving the barrel, must carry the great wheel round with it, but when the clock is wound up by raising the weight, the barrel moves freely, leaving the great wheel at rest. The great wheel is cut into ninety-six teeth, and turns a pinion of eight teeth, on the same axis with which is the hour wheel, of sixty-four teeth, that turns a pinion of eight teeth; on the same axis with this, is the second wheel, that has sixty teeth, and turns a pinion of eight teeth; this pinion is upon the same axis with the swing wheel, and its plane is, in the common clock, perpendicular to that of the three other wheels; the swing wheel, in this case, is of that sort called a crown wheel; it has sixty teeth, and acts upon two pallets, by means of which a series of alternating impulses are communicated to the pendulum. The number of these impulses during a revolution of the great wheel will be, by § 66.

$$\frac{96}{8} \times \frac{64}{8} \times \frac{60}{8} \times 60 \times 2 = 84600$$

And in one hour $\dfrac{84600}{12} = 7200,$

or one in every half second. This clock may therefore be regulated by a half second pendulum.

A crown wheel and pallets, are among the earliest escapements, and by no means the best; they are however continually to be met with even in modern clocks. In the best clocks, particularly when intended for astronomical purposes, we find a variety of escapements, contrived with much ingenuity. For an account of which, see Rees' *Cyclopædia*, article *Escapement.*

The other improvements consist in applying a compensation pendulum, and a spring to continue the motion of the train while the clock is wound up. Much, too, appears to be gained in accuracy of workmanship, by the great division of labour. The several parts of a clock require sixteen different persons for their manufacture. See Rees' *Cyclopædia*, article Clock-making.

Most clocks are furnished with an additional part, composed of machinery that strikes the hours. Much ingenuity has in many instances been shown in the contrivance of this, particularly when accompanied by chimes, &c. In general, however, the accuracy of a time-keeper is impaired by any load beyond the train we have already described.

An ingenious and very simple clock was contrived by Franklin, and this has been modified by Ferguson. See FERGUSON, *Select Exercises*, p. 1, et seq.

OF MACHINES PROPELLED BY SPRINGS.

391. After a spring has been coiled up, it begins to expand itself, and continues to do so with a retarded velocity, until its elasticity ceases to act. The retardation of the velocity is nearly uniform, or, in other words, the elasticity of a spring is proportional to its distance from a state of quiescence.

This property of springs appears to have been discovered by Dr. Hooke, but the discovery is also claimed by Huygens.

392. A spring may be made to act with uniform force, by lengthening the lever to which it is applied, in the same proportion as its tension becomes less; in this manner, the spring is applied in the common watch to produce a motion nearly isochronous; the spring is wound up in a cylindrical barrel, to which the end of a chain coiled around another barrel is attached; this last, which is called the fusee, is of the figure of a conoid generated by the revolution of an hyperbola around its assymptote.

393. A watch is a portable machine, that measures the portions of time. It differs from a clock in this respect, that the one can only be used when at rest, while the other may be carried from place to place. For this reason, the pendulum is not applied to it as a regulator; its construction is more difficult, therefore, than that of the clock, and has for nearly four centuries exercised the skill of the most ingenious mechanics; nor has it yet reached the ultimate point of perfection.

In the common watch, there are five wheels, viz. The great wheel upon the same axis with the fusee, and connected with it by a ratchet that allows the spring to be wound up without interfering with the motion of the train; the great wheel drives a pinion fixed to the axis of the centre wheel; the centre wheel revolves in an hour, and drives a pinion fixed upon the axis of the third wheel; the third wheel drives a pinion fixed to the axis of the contrate wheel; the contrate wheel, whose teeth in the common watch are perpendicular to its plane, drives a pinion whose axis carries the balance wheel; the balance wheel acts upon two pallets, and thus gives a reciprocating motion to the balance.

The action of the balance as a regulator is aided by a small spring, that accelerates its motion during the first part of its vibration, and retards it during the last.

The number of teeth in the wheels and pinions, varies in watches constructed by different makers; as an example of the mode of the adjusting it, take the following:

Time of going, 28 hours.

Number of turns of the spiral on the fusee, 7.

Great wheel 48 teeth,
Pin. 12 — 60 centre wheel.
Pin. 8 — 56 third wheel.
Pin. 7 — 80 contrate wheel.
Pin. 6 — 9 balance wheel.
2 pallets.

Number of beats per hour 14,400, or 4 per second.

Time of revolution of the contrate wheel, 1 minute.

The dial work, by which the hands that show the hours and minutes are carried, is as follows, viz.

A pinion, called the cannon pinion, is fitted tight upon the arbour of the centre wheel, so as to be carried around with it in the space of an hour. This pinion has a hollow cylindrical shaft, with a square head on which the minute hand is fitted, and drives a small wheel, on whose axis there is a pinion that drives a second wheel; this second wheel has the cylindrical shaft of the cannon pinion for its axle; the number of teeth in the wheels and pinions are so adjusted to each other, that this last wheel shall be twelve times as long in its revolution as the cannon pinion, and the hour hand is fitted to it.

In a watch whose train is planned like that we have described above, a hand that will mark seconds may be adapted to the axis of the contrate wheel, and the beats of the watch will divide each of them into four equal parts. REES' *Cyclopædia*, article WATCH.

394. The elasticity of a spring is affected by alternations of heat and cold, hence the form of the fusee will not alone be sufficient to render each impulse communicated to the balance equal. The balance spring is also affected by the same causes. To compensate the irregularity arising from these sources, we have two several contrivances.

(1) A compensating curb, by which the effective length of the balance spring is increased or diminished, according to the temperature; this curb is composed of slips of two different metals, and acts upon the same principle with the gridiron pendulum. It was used by Harrison in his chronometer; but he himself pointed out a source of error in it, which led to the adoption of

(2) A compensation balance. In this, the balance itself is composed of two different metals; it is now employed in all known chronometers. The best compensation balances are those of Mudge and Arnold.

395. Even the best springs are not homogeneous throughout, and thus the law discovered by Hooke is not physically true; the irregularity of action arising from this cause, is met by adjusting the general shape of the fusee to each particular case, for which purpose a tool has been contrived. The friction, another source of irregularity, is diminished by bushing with jewels the pivots of those wheels whose motion is the most rapid.

396. It might be fairly expected, that a watch, whose movement was aided by jewelled pivots, the fusee adjusted to the slightest irregularities of the main spring, the action of heat and cold *compensated* by a curb applied to the balance spring, or by the *counteracting* influence of the parts of the balance itself, would be a *perfect* time-keeper. Another source of irregularity, how-

ever, remains in the unequal transmission of power, arising from there being alternately favourable and unfavourable positions of the teeth of the wheels and pinions; to this is to be added the irregular action of the old scapement, composed of a crown wheel and pallets.

A remedy for the unequal transmission of power was introduced by Harrison; it is called the *Remontoir ;* this is a slender spring that is wound up eight times in a minute, and is applied to turn the contrate wheel ; the preceding parts of the train (increased by the introduction of two additional wheels and pinions, together with a fly) having no other use but to wind up this spring.

The crown wheel has been entirely abandoned in the best watches, and instead of it, a variety of escapements have been introduced. The general principle of these, is, that they shall communicate no more than a momentary impulse to the balance, and that this shall be applied at that point where the momentum of the balance is a maximum. The balance itself is made so heavy, that the maintaining power is not sufficient to set it in motion, but merely, as in the pendulum clock, to restore the quantity that is lost at each vibration. It would be impossible even to mention the names of the several escapements. For a description of them, see Rees, article *Escapement.*

397. A watch whose motion is rendered nearly regular by the above-mentioned contrivance, is called a Chronometer ; it may be employed for a variety of purposes in Astronomy, but is particularly useful in determining the longitude at sea.

The watch in its most improved state is one of the most noble instances of human ingenuity; it has for this reason been chosen by Paley as an apt illustration of his arguments in his *Natural Theology.*

The best chronometers that have yet been constructed, are those of Parkinson and Frodsham.

The principle of the division of labour is carried to a great length in the manufacture of watches ; it is computed that the several parts of the works of a watch are the production of twelve different workmen, before they come into the hands of the person whose name is impressed on them as the maker ; and that he employs twenty-one more in finishing and preparing it for sale. Rees, article *Watch.*

OF MACHINES MOVED BY THE STRENGTH OF ANIMALS.

398. The effect of a descending weight would be to raise an equal weight to the height whence itself fell; this moving power has, therefore, an independent and definite value in itself; in the case of all other mechanical agents, we estimate their value in some conventional manner, in the same way that we compare forces with one another, by the intervention of some conventional force that is taken as the unit.

Thus, the powers of men and horses have been compared with each other; and we estimate the force of water, wind, and steam, in terms of the latter.

399. The action of a man and horse can only be compared together, when their powers are exerted to produce the same effect, or when applied to draught. The greatest force of draught that a man can exert, may be taken at seventy pounds; his greatest velocity in walking, six feet per second, or a little more than four miles per hour.

If we apply to this the theorem of § 383, a man should work to the greatest advantage in drawing up a weight of 31$\frac{1}{3}$ pounds, and walking with a velocity of two feet per second. GREGORY, § 378.

Euler and Playfair make the maximum weight only sixty pounds, the most advantageous application of human force to draw twenty-seven pounds, with a velocity of two feet per second. PLAYFAIR, § 178. EULER on *Vessels*, Book IV.

400. The utmost strength of a horse is estimated as about equivalent to a weight of 420 pounds; his utmost velocity in walking, 10 feet per second; his maximum of power will therefore be to raise a weight of 186$\frac{2}{3}$ pounds, with a velocity of 3$\frac{1}{3}$ feet per second. GREGORY, § 378.

The strength of a horse is usually estimated as seven times that of a man.

The estimate is made by Hachette in another manner, viz.

401. A man working 10 hours per day, and continuing his labour throughout the year, can raise 10,000 French cubic feet of water to the height of a foot ; or one hundred and eleven cubic metres of water, to the height of one metre. He therefore proposes to represent the strength of a man by the number 111. The strength of a horse he represents by the number 777. HACHETTE, § 14, and § 21.

402. When a man walks, his muscular strength is exerted to produce two different effects ; to move his weight forward with a certain degree of velocity ; and to raise it to a certain height. The first of these is applied to draught, the second to the support and carriage of burthens.

From the erect position of man, and the columnar structure of his body, his power of producing the second effect is much the greatest. Seventy pounds is the greatest weight he can draw over a pully, while he is capable of moving under any weight less than twice his own.

According to Playfair, the greatest effort of a man's strength in raising a weight, is when it is to that of his body as 3 to 4. A Roman soldier was loaded with a weight of 60 Roman pounds, and travelled at the rate of 5 miles per hour.

A French Grenadier is loaded with 56 French pounds, and the rate of his march is 3 miles per hour.

403. A horse, on the other hand, can support much less weight on his back than he is capable of drawing ; and while a man can ascend nearly perpendicularly upwards, a horse cannot mount a plane inclined more than 45 degrees to the horizon.

Of the Crane.

404. The crane is a machine made use of to raise heavy weights, and to remove them to a small distance from the place whence they are raised. The principal parts of a crane, are the *arbour*, the *gib*, and a *wheel and axle*. A rope, that is wound

around the axle, passes over a pulley at the extremity of the gib to the weight; this pulley is generally single, but is in some cases compound. The whole machine has a horizontal motion around the arbour as an axis, and in some cranes, the upper block of the pulley has a motion along the gib; by the combination of these two motions, a weight may be set down at any point within the area of circle, whose centre is in the axis of the arbour, and whose radius is the projection of the gib.

The axis of the wheel and axle of cranes is sometimes horizontal; they may then be moved by the muscular action of men, taking hold of pins placed on the circumference of the wheel, or by the weight of men walking within them; the wheels are sometimes cut into teeth, and propelled by pinions worked by a winch. At other times, the axis of the wheel and axle is vertical, and they are propelled in the manner of a capstan, by men pressing against bars, or moved by horses walking in a circular path.

Those wheels in which men walk are considered dangerous; this species of crane has been much improved by Albert, in France, and Whyte, in England. In their cranes, the men are placed upon a platform, at the height of the horizontal diameter of the wheel, and step upon leaves placed on its circumference.

This species of crane is called the *Footmill;* it has been lately introduced into the Penitentiary of New-York. It is an advantageous application of human strength; for, in the crane of Albert, the weight raised in a given time was to the whole estimated force of the men employed, as 8 to 10. No more, therefore, than $\frac{1}{5}$ of the power was lost, in consequence of the friction, &c. See HACHETTE, p. 240.

To enumerate the great variety of cranes employed in different cases, would far exceed our limits; the curious may consult HACHETTE, FERGUSON, and GREGORY.

Of the Pile Engine.

405. In the pile engine a heavy weight or ram is raised to a great height, and discharged at once upon the head of the pile that is to be driven. This machine is in some cases composed of a simple pulley, elevated upon posts that serve as guides to the ram; the cord of the pulley is divided into a number of

smaller ropes at the extremity, in order to permit a number of men to act at the same time. In another form of the machine, the cord of the pulley is wound around an axle, whose wheel is cut into teeth, and driven by a pinion; to this pinion, manual strength is applied by means of a winch. Such is the engine used in the harbour of New-York.

Where the pile engine is used in great works, it is propelled by horses; of this, we have an instance in the famous engine contrived by Vauloue, to drive the piles used in building Westminster Bridge. See FERGUSON.

Of the Venetian Machine.

406. The Venetian machine is a contrivance for removing the deposit made by water in the bottoms of harbours. It is named from the place in which it was first used; and from its great simplicity and excellence, has superseded, in most parts of Europe, all other contrivances for the purpose.

In the use of this machine, a large instrument, resembling a spade, and like it composed of a wooden handle and iron blade, is driven into the muddy deposit; this is effected by attaching it to the extremity of a horizontal beam like the working beam of a steam engine, whose opposite extremity is elevated by the force of a screw. A motion of the screw in the contrary direction withdraws the spade after the operation is completed. When the spade is driven to a proper depth, a scoop that moves around a pivot, traversing the handle of the spade just where it is inserted into the blade, is forced forward to meet the spade, by the joint action of a lever and a pulley, whose cord passes around a capstan.

This machine is worked by 5 men, and raises 60 cubic feet of mud every 5 minutes from a depth of 15 feet. HACHETTE, p. 250.

A very imperfect and inefficient machine is used for a similar purpose in the harbour of New-York.

Of Wheel Carriages.

407. Where the strength of men or of animals is applied to *draught, the* effect that has been stated is the weight that species of exertion is capable of raising perpendicularly upwards, as in the case of its being suspended by a cord passing over a pulley. The same application of strength may be rendered efficacious in removing much heavier weights along a horizontal plane. In this case, the resistance consists, not in the weight of the body to be moved, but in its *Friction.*

The principle upon which wheels and rollers act, in the removal of friction, has been already explained. See § 91.

408. Where wheels are considered, without reference to the animal force by which they are propelled, it would appear that the larger they are, the more advantage is to be derived from their use, whether we consider them as employed to diminish friction, to surmount obstacles, or to depress them. The only case where small wheels are more advantageous than large, is where the obstacle is to be removed.

409. The height of wheels is however limited by the direction of the draught. The power of draught is exerted most directly, and consequently most powerfully, when the line of draught is horizontal. Besides the loss arising from obliquity of action, where the draught is not horizontal, a part of the power of the horse is employed, when the axle is lower than his breast, in raising the weight from the ground, and, when it is higher, in increasing the friction.

A horse, in drawing, leans forward, bearing a portion of his weight upon the collar; hence, in order that the draught be horizontal, a wheel with an axle a little lower than his breast must be made use of.

An axle lower than the horse's breast, is also advantageous on hilly roads, where, in rising, a portion of the power must be exerted to overcome the gravity of the load, and where, in descending, its spontaneous velocity is to be checked. On level roads, however, it is said that the axle may be made a little higher than the breast of the horse with advantage.

410. When a horse leans forward to the draught, he acts not only by his muscular strength, but also by his weight. For this reason, in moving heavy loads, horses of a large breed are found

more valuable than smaller ones, even of equal or superior muscular force ; oxen, also, are found of great value wherever the required velocity is small.

In carriages with two wheels, a portion of the weight rests upon the back of the animal, and, where this is not too great, adds to his effective force.

A load laid upon the back of a horse will often enable him to overcome a resistance his strength was before unequal to. In this way we may explain the curious fact, observed in carriages driven by postilions, that the mounted horse is always in the best order ; this observation, which holds good in the artillery services of Europe, and in posting on that continent, is not found true in England ; but in England the roads are so good, that the strength of the horses is employed in obtaining velocity, rather than in overcoming friction ; and in this case, an additional weight is an encumbrance.

The advantage of carriages with two wheels, ceases when they are made so large as to require more than a single horse for their draught. And for similar reasons, it is disadvantageous to draw even a four wheeled carriage by horses harnessed *tandem*. This rule, however, does not apply to a level road, or where velocity is the object in view.

411. In four wheeled carriages, the fore wheels are made lower than the hind wheels, for the convenience of turning. In every other respect, this plan is disadvantageous.

Some ignorant persons imagine that high wheels help to push forward the lower ones, that precede them. And many drivers consider that it is best to load the fore wheels with more than a due proportion of the load. Both of these impressions are erroneous. Where heavy loads are to be carried a considerable distance, four wheeled carriages, drawn by a number of horses, are most advantageous ; for, although the same number of horses would draw at least $\frac{1}{3}$ more in carts, yet each cart would require a driver ; in short distances, where the drivers are advantageously employed in loading and unloading, carts are preferable. In Scotland, the horses are so trained, that 5 or 6 carts may be driven by a single person, and their agriculturists boast much of the great superiority of this method, and of the good construction of their carts. The cart used in the city of New-York, and its vicinity, is however still better than that of Scotland; its usual load is 14 cwt. over unequal ground.

412. When horses are harnessed abreast, it seldom happens that their strength is equal; it is very easy however, by apportioning the length of the lever on which each draws to his power, to place them upon an equality in this respect.

413. Wheels are usually made dishing; an advantage is said to be thus gained, for where the carriage inclines, the lower spoke becomes vertical, and thus has the greatest strength at the moment when the strain is greatest. The axles are also usually inclined downwards; but this practice appears to be vicious, for it increases the resistance.

414. In carriages that are loaded with heavy weights, the wheels should be made broad. This, although not attended with any advantage on hard or very smooth roads, is of great value in those which are soft or unequal. It acts also to preserve and even improve the roads.

Hence in England, all heavy wagons are required by law to have broad wheels. This law, which met with great opposition from the carriers, has now been found to be as much for their private interest as for the good of the roads. On many of the turnpikes of the United States, broad wheeled wagons pass toll free; and it is to be hoped that this valuable improvement will rapidly come into general use.

The friction rollers of Mr. Garnett, mentioned in § 92, have also been applied to the wheels of carriages.

415. Springs, that were originally introduced for the ease of the person carried, have been found a mechanical advantage. Upon rough and uneven roads, they act on the same principle with the *Fly*.

See, for a more full dissertation on this subject, FERGUSON'S *Lectures, with Additions,* by BREWSTER.

416. In wheel carriages, the equability of motion is even of greater importance than in other machines; sudden checks to motion having a more pernicious effect on an animate than it can have on an inanimate body. PLAYFAIR, § 192.

Hence we may see the very great importance of good roads, and the necessity that they should be constructed upon scientific principles. In low countries, every exertion should be used to make the roads as nearly level as possible; in hilly countries, the ascent from the lowest to the highest points, over which the road must unavoidably pass, should be, if possible, made a regularly inclined plane of the least practicable angle.

In almost every case, it is better to avoid and go round a hill, than to pass over it; the effective length of a road is measured, not upon a level line, but by the number of turns made by the wheels that move upon it.

In some countries Rail-ways have been introduced, when the weight of the articles to be transported in one direction was very great; these are composed of two parallel inclined planes of cast iron, on which the wheels of the carriages move; they answer the double purpose of diminishing the friction and equalizing the resistance, hence a horse is capable of drawing 5 times the weight upon them that he could over a road of similar inclination. See REES, *Railways*.

Of Animals considered as Machines.

417. Animals may themselves be considered as machines, planned by the Creator with consummate wisdom, and admirably adapted to the several states and circumstances in which they are destined to exist.

418. The motion of animals is produced by the action of their muscles upon the bones. Each several motion is effected by the joint operation of a pair of antagonist muscles called the flexor and extensor; by the union of two or more pair of these, and by the simultaneous action of those of different bones, every variety of position and attitude of which an animal is capable *is produced.*

In man and quadrupeds, the flexor muscles are a little shorter than the extensors; hence the position of the joints, when the muscles are not in action, is a little bent. On this account a standing position, when the joints are straightened, cannot be assumed without effort; and when muscular action ceases, as in sleep, the upright position ceases also.

Man alone, of all the mammalia, is capable of maintaining an erect position. His standing posture is produced by the joint action of the extensor muscles of all his joints; the most powerful of these are situated in the calf of the leg, where they not only serve this important purpose, but add to his beauty and symmetry.

In birds, the erect position is also produced by the action of the extensor muscles; but as they are slender, their want of mass is compensated by their increased length. To this great difference in the length of the antagonist muscles of birds, it is owing that their feet collapse and close firmly when muscular action ceases, and thus birds cling to their perch during sleep, and even after death. When they stand upon a plane, their weight acts to prevent the contraction of the extensor muscles, and thus their feet remain spread out without any exertion on their part; birds therefore sleep in an erect position, and sometimes standing upon one foot. BORELLUS, *de Motu Animalium*, Cap. XVIII.

The antagonist muscles are applied to points of the bone, not far distant from the joint that serves as the fulcrum. Thus the frame of animals may be considered as a combination of levers of the third kind, where velocity is gained at the expense of intensity of force.

419. In the motion of terrestrial animals, the flexor muscles act to throw forward the line of direction of the centre of gravity; this, which of itself would destroy the standing posture, is met by a corresponding motion of the limbs. BORELLUS, Cap. XIX. and XX.

420. Birds are raised into the air by the rapid oscillation of their wings; the great feathers of which these are composed, strike the air directly when the wing moves downwards, but obliquely as it returns, a very great difference consequently results between the action of the wing upon the air while moving in the two opposite directions; this is sufficient to raise the bird

into the air, and support it there. The flight is directed either upwards horizontally, or downwards by the feathers of the tail. The muscles by which the wings are moved, are placed upon the breast, and compose by far the greatest part of the body of the bird. Man not being furnished with pectoral muscles, in vain attempts to fly by his own unassisted strength. Borellus, Cap. XXII.

421. Fish are nearly of the same specific gravity with the fluid in which they are immersed. They are furnished with an air bladder, by the inflation or contraction of which, they become either specifically lighter and rise, or specifically heavier and sink. When swimming in their usual position, the centre of gravity of a fish is above the centre of magnitude; hence the equilibrium is momentary. They are preserved in this position by the motion of the pectoral fins: and are propelled by the action of the tail. Borellus, Cap. XXIII.

MECHANICS.

APPENDIX II.—HYDRAULIC ENGINES.

OF MACHINES FOR RAISING WATER.

422. Machines for raising water may be divided into two classes : (1) those which are moved by the direct action of the water itself ; and (2) those which are moved through the intervention of some machine, of which any natural agent may be the prime mover.

Among the first class are the following machines, viz.

423. The *Fountain of Hero ;* this is a machine by which the motion of a column of water is transmitted to another column of the same liquid, through the intervention of a stratum of air that separates them. The principle on which it acts, may be generalized, and comprehend all machines in which the motion of one liquid is transmitted to another, through the intervention of a stratum of fluid, whether compressible or incompressible, of less specific gravity than either of the two liquids.

Hero's fountain in its original form consists of two vases ; the one placed perpendicularly over the other, and both air tight. A pipe, open at its upper extremity, passes through the upper vase without communicating with it, and enters the lower, reaching nearly to its bottom. From the upper part of the lower vase, a second pipe proceeds that reaches nearly to the top of the upper. A third pipe is inserted into the top of the upper vase, and reaches within a small distance of its bottom.

To use the machine, a portion of water is placed in the upper vase, and in the lower enough to immerse the lower extremities of the first and third pipes ; the several openings are then closed, and water is poured into the first pipe ; this column, of the whole height of the *instrument*, presses upon the air contained in the upper parts of both vases, and would condense it, did it not act upon the water in the upper vase, and force it out in a jet through the third pipe.

The action continues, if water is supplied, until the level of the water in the upper vase falls below the lower end of the third pipe.

Hero's fountain is but little used in its original shape, except for purposes of mere amusement. It has however, when modified, been applied to several valuable practical purposes. For instance, Girard, in France, has constructed a modification of Argand's lamp, where the oil is raised to the wick by a contrivance acting on the principle of Hero's fountain. See HACHETTE, p. 43.

At Schemnitz in Hungary, water is raised from the bottom of a deep mine by a machine of this same species. Its principal parts are two strong vessels; one of these (A) is placed on a level or horizontal shaft carried in from the open ground; the other (B) at the bottom of the mine. Above the *level*, the ground rises rapidly into a mountain, and at a considerable height on its side is a spring; the water of this spring is made to act on the air enclosed in the vessel A, this communicates with the air in the upper part of the vessel B, and forces the water from it through a rising pipe to the horizontal shaft, where it is discharged upon the plain. When the vessel A becomes filled with water, and B with air, stop-cocks are opened, by which the first is discharged along the same shaft, and the second allowed to escape; the machine is then ready to recommence its action. These cocks were, in the original machine, opened by hand, but an ingenious apparatus has been contrived by Boswell, in England, by the aid of which the machine is rendered self-working. HACHETTE, § 83. GREGORY, Vol. II.

424. Two buckets united by a rope passing over a pully are sometimes used as an hydraulic engine. One of these buckets (A,) weighs, when full of water, more than the other, (B;) but when both are empty, B is the heavier. If, then, they are both at the same level in consequence of the middle of the rope resting on the summit of the pully, and if water be poured into them, A will soon preponderate, and, in descending, raise B with its contents until it reach the pully. If at this moment, a pin or some *other contrivance* be applied, by which the buckets are overturned and emptied, (the water from A running to waste, and

that from B passing into a reservoir,) B will preponderate, and, in its turn descending, will raise A to the original level.

If the buckets be made of the figure of conic frustums, open at the larger end, and if the points of suspension be placed so that their axis passes between the centres of gravity of a solid and of a hollow frustum, the equilibrium of the buckets when full, will be momentary, but when empty stable; they may therefore be emptied by the slightest force.

425. The *Hydraulic Ram*, (*Belier*) of Mongolfier, acts upon the following principles :

When water is running with a rapid current through a pipe or close channel, if the end at which the water issues be suddenly stopped, the water by its acquired impetus, will act upon the sides or circumference of the pipe, and endeavour to escape with a force proportioned to its quantity and velocity; it may thus be made to issue with great violence from a small aperture opened at or near the end that is closed. If, then, an ascending pipe be joined to the aperture, a portion of the water will ascend. If the machine be provided with proper valves to close it at the end, and to prevent the return of the water through the ascending pipe, the operation may be repeated in continued succession.

The action of the principal valve of the hydraulic ram may be thus explained; when a body is acted upon by a force, it does not instantly acquire the whole velocity the force is capable of communicating. Thus, when a rope is tied to a shell placed in a mortar, it is broken when the mortar is fired; and thus a weight that if it passed slowly over a thin plank or weak ice would infallibly break them, produces no such effect when it moves rapidly.

The hydraulic ram is composed of an inclined pipe, down which the water flows; a valve that is of such a weight that the water can raise it when it has communicated to it its whole impetus, but which does not float on still water; a second valve that closes when the first opens, and vice versa; an air vessel that communicates with the descending pipe by means of the second valve; and an ascending pipe opening upwards from the air vessel.

The action of the fluid produces an alternate rise and fall of the first *valve;* the motion of the second valve reciprocating with that of the first, it admits water into the air vessel; here the air is condensed, and by its spring forces the water in a continued stream through the rising pipe. HACHETTE, § 94.

426. The *Syphon of Venturi* acts upon the principle discovered by him, of the lateral communication of motion in fluids. In it water is admitted into a vessel, and allowed to escape from an orifice near its bottom, by a cylindrical adjutage; a syphon is introduced into the adjutage at the vena contracta, and its longer branch descends into a reservoir; through this syphon the water rises, and is discharged along with that flowing from the upper vessel. HACHETTE, p. 35.

Among the second class of engines for raising water, are,

427. The *Tympanum*, a machine much used by the ancients; in it, a hollow cylinder whose axis is horizontal, is divided into a number of compartments, by partitions extending from the axis to the circumference. The lower part of the wheel dips into the water, and holes are pierced near the circumference, into which the fluid is received during the revolution of the wheel; as the wheel revolves, the fluid passes along the inner circumference until it reaches the next following partition, over which it is carried to the centre of the circle, where it is discharged.

In the use of this instrument, the centre of gravity of the fluid raised does not move in the shortest possible line. De la Faye has proposed to improve this machine, by causing the water to rise in a straight line; he effects this by making the channel in which the water moves in the form of the involute of a circle.

In both forms of this machine, water cannot be raised higher than half the diameter of the wheel.

428. The *Persian Wheel*; this has buckets hung upon its circumference by pins, in such a way, that they will be perpendicular when acted upon by no other force than their own weight,

and that of the water they may contain. The water received by the immersion of these buckets into a reservoir when at the lowest point of their revolution, is discharged at the moment they reach the summit of the wheel, by causing them to strike against a pin that overturns them.

The velocity of this wheel must not be very great, otherwise the whole of the water raised by each bucket will not be discharged into the channel placed to receive it.

This wheel is liable to a great resistance, in consequence of the difficulty of pressing the empty buckets into the lower reservoir ; but it may be avoided by placing a valve opening upwards in the bottom of each bucket.

429. *Vera's Pump*, called also the Hessian or Rope pump, is formed of an endless rope, that passes over two pulleys ; one of these is placed in the reservoir whence the water is to be raised, the other at the place of discharge ; a quantity of water adheres to the rope, when at the lowest point of its revolution, and it is compelled to part with it by a gentle pressure when it has reached its highest situation.

A machine of this sort, constructed by the inventor himself, drew water from a well ninety-five feet deep ; and a single man could raise six gallons per minute. REES, article *Water*.

430. The *Hydraulic Cane* is a long tube fitted with a valve opening upwards ; this valve is sometimes placed at the bottom and sometimes at the top of the tube ; the lower extremity of the tube is immersed in water, and a rapid alternating rectilineal motion is given to it ; when the valve is at the top of the tube, the water that has entered it during its depression, is supported, while it is drawn upwards, by the pressure of the atmosphere ; when it is again depressed, more enters, and the action is continued until the fluid issues from the top of the tube ; when the valve is at the lower part of the tube, it supports, during the motion upwards, the water that has entered during the time of descent.

431. *Vialon's Pump* is formed of two tubes, formed into screws, by wrapping them around the same cylinder, but in con-

trary directions ; the lower end of each tube is furnished with a valve opening upwards ; a rapid reciprocating motion around the axis of the cylinder is communicated by means of a winch ; at each alternation of movement, the water acts upon one of the valves, which opens and permits it to enter, at the same time the other valve closes and retains the water that has entered through it ; the two spirals are drawn together at top, and joined into one common tube, through which the water they have raised is discharged.

This machine, and the hydraulic cane, whose valve is at its lower extremity, are both constructed on the same principle. HACHETTE, § 129.

432. The *Centrifugal Pump* is composed of a number of tubes arranged upon the surface of a truncated cone, whose larger base is uppermost ; these tubes are bent outwards at their upper extremity, and a circular channel placed beneath their orifices ; the lower extremity of the tubes is placed in the water to be raised ; when the machine has a rapid motion given it around the axis of the cone, the centrifugal force in the lower circular base of the cone becomes sufficient to overcome the gravity of the water, it rises, and is discharged at the upper ends of the tubes into the circular channel. HACHETTE, § 130.

433. The *Screw of Archimedes* is composed of a solid cylindrical shaft, surrounded by a hollow cylinder having the same axis ; the threads of the screw are contained in the hollow space, between the outer and inner cylinder, and are generated by the motion of lines that are constantly perpendicular to the axis of the cylinders, and apply themselves to screws of the same thread, supposed to be wound around their surfaces ; from three to four such screws are usually wrapped around the solid cylinder. The upper part of the machine is furnished with a winch, the lower with a gudgeon, and these rest upon a frame of wood.

In order to apply the screw of Archimedes to raise water, the lower end is immersed in the fluid, and the axis inclined to the horizon ; the winch is then turned in a direction contrary to that of the motion of the point that is supposed to have generated the screws ; the water is by this motion carried along the surfaces of the screws, and falls at the upper part of the machine into a trough prepared to receive it.

The elevation of the axis of the machine has a limit that depends on the radius of the inner surface of the hollow cylinder, the angle of inclination of the screw, and the distance between its threads. See HACHETTE, § 134, et seq.

434. The *Chain Pump* is composed of a cylindrical or square barrel, and an endless chain furnished with several pistons of the same figure with the barrel, and placed at small distances from each other, on the links of the chain ; the chain is generally extended between two wheels, one below, the other at the top of the barrel, but the lower of these is sometimes omitted ; these wheels have forks fixed on their circumference at such distances as will permit a piston to fall into each of the intervals, while the fork takes hold of the links of the chain ; to set the machine in motion, the upper wheel is turned by means of a winch ; the pistons fit close to the barrel at the lower extremity, and for a few feet upwards, above this the barrel becomes larger, and the pistons move freely. When the lower end of the barrel is immersed in water, and the chain is made to circulate by turning the wheel, each piston will carry a portion of water into the barrel, until at last the barrel is filled and the water runs over the top ; the pistons succeeding each other in regular rotation, produce a constant stream as long as the pump is worked, and water supplied from beneath.

The chain pump, if tight, is attended with a great degree of friction. It is sometimes inclined, and is then found to raise a greater quantity of water to the same height. The chain pump placed vertically, is principally used on shipboard. The inclined chain pump is used in draining coffer dams, and in other places where hydraulic erections are going on beneath the level of the water.

435. The principle of the *Common Pump* has already been explained ; the difference that exists among machines of this *kind*, arises in part from the construction of their valves ; of *these*, there is a very great variety.

(1) The common valve; this is a frustum of a solid cone of wood that falls into a hollow frustum or socket, that it nearly fills up; to the top of the solid frustum is attached a piece of leather of a circular form, that is larger than its greater base, and thus covers the space between the hollow and solid cones; one side of the leather is firmly fastened to the hollow cone, and the valve plays around this side as upon a hinge.

(2) The butterfly valve is in two pieces, and the leather is fastened to a solid piece that crosses the socket in the direction of its diameter; the two parts of the valve moving on this as a hinge, have a motion that resembles the action of wings, hence the name.

(3) In the pyramidal valve, the socket is square; from its angles rise four pieces that meet at an apex like the edges of a pyramid; the valve is composed of four triangular parts that are leathered, and fastened down to the socket; it opens from the apex of the pyramid, and when open, the parts lie against the sides of the pump.

(4) The triangular valve; in using this, the barrel of the pump and the socket of the valve are made square; the valve is in two portions, of the figure of a right angled triangle, and the leather, which is square, is fastened down to a piece that crosses from angle to angle of the socket in the direction of its diagonal.

These four valves may also be constructed of metal as well as of wood and leather; in doing this, the parts are ground to fit each other, and for the flexibility of the leather, hinges of metal are employed. The remaining valves that we shall mention, are always constructed of metal.

(5) The conical valve; in this a solid frustum of metal is ground to fit a hollow one; as the first is raised, a passage is opened between them, and it is prevented from rising too high by a key that arrests its progress, and on its return, guides it back to its proper position.

(6) The spherical valve; this is a sphere of metal, hollow within; the socket is ground into the form of the zone of a sphere, and thus the valve adapts itself to its cavity in whatever direction it falls.

(7) The sphere is sometimes made very light, and to enable it to return, it is attached to a long rod of metal, at the lower extremity of which is a weight; this valve is less liable to be choked than any other.

The limit of the height to which a common pump will raise water, is about 34 feet, as has been already explained.

436. The ***Lifting Pump*** consists, like the common one, of a barrel and two valves; in it, however, the upper valve is fixed in the barrel, and it is the lower valve that moves in the piston. The piston is moved beneath the surface of the water by an iron rod of the form of a stirrup. When the piston is depressed the valve opens, a portion of water passes through it into the barrel; when the piston is drawn up it *lifts* the water upon it, and forces the air contained in the barrel through the upper valve; when the piston is again pressed down, the upper valve closes, prevents the admission of air, and more water is admitted through the lower valve; after a few strokes, the water will rise through the upper valve, be lifted at each elevation of the piston, and prevented from returning at each depression by the closing of the upper valve.

The power of this pump, or the height to which it will raise water, is limited only by the strength of the materials, and the intensity of the moving power. The common pump may be converted into an engine that shall combine the effect of the lifting pump with its own by making its piston work air tight in a collar of leather, and adapting an ascending pipe instead of a spout. See REES, article *Water*.

The lifting pump may be made with a solid plunger; in this form it has two pipes that open into the upper part of the barrel; one admits water from the well when the piston descends; through the other, the water is lifted as the piston rises; both pipes are furnished with valves opening upwards, that prevent the return of the water.

437. The ***Forcing Pump*** has a solid piston that works in the barrel; at the bottom of the barrel is a valve opening upwards that permits the water to enter when the piston is drawn up, but prevents its return; near the bottom of the barrel is introduced an ascending pipe with a valve opening upwards; at each descent of the piston the water in the barrel is forced up this pipe.

In the lifting, and in the common pump, the water is raised during the ascent of the piston, in the forcing pump during its depression. A pipe is frequently adapted to the lower valve of the forcing pump, not exceeding in length the limit of the common pump, and in this way the effect of the two may be combined.

In the pump of De la Hire, the lifting and forcing pumps are combined, and thus water may be raised by a single piston both in its ascent and descent. In this pump the moving power is resisted by but half the friction that would oppose it were two barrels used.

The forcing pump that is most commonly used in England, is the invention of Sir Samuel Morland; its barrel is not ground smooth and the piston fitted tightly to it, but the piston consists of a solid plunger of a cylindrical form nearly as large as the barrel of the pump, which it does not touch at any point; this plunger works air tight in a collar of leather, and in descending diminishes the capacity of the barrel, while in rising it increases it.

This kind of forcing pump is preferable to any other. There is a little nicety to be observed in making and fitting the collar of leather; for which, see REES, article *Water.*

438. To equalize the discharge of water from pumps, an air vessel is adapted to the ascending pipe. This, which may be a close chamber of any figure, communicates with the ascending pipe at its lower end; at its upper end, a pipe is inserted, reaching down through the chamber nearly to its bottom. The action of the pump will force water into the air vessel; the air being thus condensed, will by its elasticity endeavour to regain its former space, and equalize all sudden motions in the pipe.

439. The *Fire Engine* consists of two pumps, that force water into one common air vessel; they are placed in a wooden cistern, and the whole mounted on wheels.

440. When the height to which water is to be raised is small, a pump may be constructed in which the piston does not require to be fitted closely into the barrel, nor are any leathers necessary. The barrel of this pump must be as long as the whole height to which the water is to be raised, and as much more as the length of the stroke of the piston. The piston is a solid piece of wood, fitted to the barrel as closely as it can be without actually touching; it must be as long as the barrel itself

so that when it is let down, it will occupy the whole interior of the barrel, except the small space that is left to prevent their surfaces from coming in contact. The bottom of the barrel has a valve opening upwards, and a pipe rises from its lower part to convey the water raised by the pump to the reservoir. This pipe has a valve opening upwards, to prevent the return of the water from the reservoir, whose altitude must be less than the height of the barrel of the pump.

When this pump is fixed for work, the lower end must be immersed as far into the water as is equal to the length of the stroke of the piston, so that the lower end of the piston may never rise above the level of the water. When the piston is drawn up, the water enters the barrel to the height at which it stands in the well by its own pressure; when the piston descends, it forces the chief part of this into the ascending pipe, while another portion rises between the piston and the sides of the pump.

441. Pumps are sometimes constructed on the principle of the common pump, but with a diaphragm of leather, in which the upper valve plays. These are subject to but little friction, and have no other fault but the liability of the leather to crack.

OF THE WATER PRESS.

442. If there be a mutual communication between two columns of any fluid, whatever pressure or effort may be exerted on the one, will be transmitted to the other in a ratio proportional to the respective area of each column. In this sense, as we have already seen, every fluid is in itself a machine.

On the above principle, Bramah, an ingenious artist of Great-Britain, has constructed a water press engine that goes by his name. This is now applied to a very great extent, in various departments of the useful arts

It consists of a small forcing pump, whose plunger works through a collar of leathers like the pump of Morland; the pipe of discharge, which has as usual a valve opening upwards, is carried along horizontally, and enters a cylinder, whose diameter is much greater than that of the pump; to this cylinder is adapted a piston that works in it water tight. When the plunger of the forcing pump is pressed down, water is forced into the cylinder, and acts upon the piston, on which it presses with a force as much greater than that which presses on the plunger of the pump as the area of the section of the former is greater than the area of the section of the latter. If this ratio be multiplied by the ratio of the arms of the lever, applied as a pump rod, to each other, we shall have the proportion between the power applied to work the press, and the pressure it is capable of producing. See HACHETTE, § 173. REES, article *Press.*

OF WATER WHEELS.

443. A circular motion may be produced either by the impulse, by the weight, or by the re-action of water.

Undershot wheels are examples of the first of these methods, Breast and Overshot wheels of the second, and Barker's mill of the third.

In the undershot wheel, the water acts upon leaves or float boards, passing beneath the wheel in a current. The properties of this machine have been investigated analytically by means of the hypothesis we have already employed, and very accurate experiments have been made on its action by Smeaton. We shall give the theoretic deductions, and state how far they have been found to agree with the experiment.

444. If a stream of water impinge upon the float boards of an undershot wheel, and escape from it the instant after it has made its impact, the quantity of water which actually strikes the wheel will be to the whole quantity that passes by it in a given time, as the relative velocity of the wheel to the absolute velocity of the water.

Hence, the force exerted by the water to turn the wheel, is as the square *of the* relative velocity of the wheel. GREGORY, § 469.

445. The effect produced by an undershot wheel, is a maximum when its absolute velocity is one third of the velocity of the stream. GREGORY, § 470.

By experiment, this is found to vary between $\frac{1}{3}$ and $\frac{1}{2}$; the reason of this is, that the water does not escape immediately after impact, but is confined by the channel.

446. If the velocity of the stream be given, the effect is as the quantity of water expended.

This is fully confirmed by the experiments of Smeaton, as are also the following corollaries:

(1) Where the expense of the water is the same, the effect is as the square of the velocity.

(2) The expense of water being the same, the effect is as the height of the head of water, or rather of the effective head.

(3) The aperture being the same, the effect will be as the cube of the velocity. GREGORY, § 471. SMEATON, *Experimental Inquiry*, p. 15, et seq. PHILOSOPHICAL TRANSACTIONS, for 1759.

It was formerly believed that the floats on an undershot wheel should be so arranged, that when one was perpendicular and fully immersed in the stream, the preceding float should be just leaving, and the following one just entering the water. By Smeaton's experiments, it would appear that this is not true; he found that, in his apparatus, power was lost when the number of floats was diminished below 24, and that no power was gained by increasing that number. With 24 floats 3 are constantly immersed, and this appears to be the rule best adapted for practice. SMEATON, p. 24.

The effect produced by an undershot wheel is to the power as 3 to 10. SMEATON, p. 23.

447. In the overshot wheel, the water is received in buckets placed upon the circumference where it acts by its gravity; it is sometimes admitted at the top of the wheel without exerting any impulsive force, at other times it is permitted to act by its impetus. In the breast wheel, only half the buckets in a semi-circumference, or even fewer, are filled with water, it being admitted at the extremity of, or beneath the horizontal diameter.

448. The effective weight of water in an overshot wheel, may be computed by multiplying the quantity in each bucket by the sine of the angle which the diameter, passing through the centre of gravity, makes with the vertical diameter. Gregory, § 474.

If the number of buckets were 12, and all supposed equally full, the real weight of the water would be to its effective weight as 1 to .622008.

If the number of buckets were 24, the real weight would be to the effective as 1 to .632214.

If the number of buckets be so increased that the weight may be considered as diffused equally over every portion of the semi-circumference, the real is to the effective weight as 1 to .63662. Gregory, § 474.

449. Supposing the water to act partly by impulse, and partly by its weight, an overshot wheel will be, theoretically, in its greatest perfection when its height is two-thirds of the head of water.

In practice, many inconveniences are found to result from admitting the water to act by its impulse; and it is ascertained by experiment, to be best to make the height of wheel nearly equal to that of the fall, and to admit the water with a velocity but little greater than that of the circumference of the wheel. Smeaton, p. 81.

450. The efficacy of an overshot wheel is to that of an undershot wheel as 13 to 5, when the head of water, the aperture, and the diameter of the wheels, are equal. Gregory, § 476.

This theoretic deduction is not confirmed by experiment; Smeaton makes the proportion between the two effects, as 2 to 1. Smeaton, p. 30.

451. The more slow the motion of an overshot wheel, the greater will be its power; for the accumulation of water in wheels more than counterbalances the diminution of the velocity; but there is found a practical limit to this, for if
of the circumference become less than from 2
nd, the wheel is no longer steady, and is li
ities. Smeaton, p. 35.

452. Where a circular motion is produced by the re-action of water, the machine is called *Barker's Mill*, after the name of the inventor. In Barker's mill, the water is introduced through a cylinder moveable about a vertical axis into two horizontal boxes that communicate with it. The boxes have openings at the sides in opposite directions.

When the apertures are shut, and the whole filled with water, the machine is in equilibrio ; but if the apertures are opened, the columns having these apertures for bases, will cease to press on the sides, the equilibrium will be disturbed, and a rotary motion take place in a direction opposite to that in which the water issues. PLAYFAIR, § 306.

453. If A be the sum of the areas of the two orifices, and h the height of the vertical tube, the theoretic effect of Barker's mill will be A h, or equal to the whole momentum of the water expended. It would therefore appear to be the most advantageous application of a fall of water. In practice it does not answer the expectations that might be formed from the theory ; it is liable to great friction, and a centrifugal motion takes place in the vertical pipe, that diminishes the pressure of the head.

Mr. Rumsey, an American, has proposed to remove this last inconvenience, by introducing the water through a bent tube from beneath the boxes.

454. A variety of other ingenious contrivances have been proposed for the purpose of applying water as a moving power. None, however, have come into general use, wherefore it is unnecessary to mention them here.

455. If a rotary motion be given to a breast wheel by a windmill, a steam engine, or any other mechanic power, it may be applied to raise water from one level to another. It is then called the *Fen* or *Flash Wheel*

OF VESSELS.

The most important application of the theory of the motion of bodies in fluids, is to the subject of vessels intended for navigation. In the consideration of this question, we shall follow for the most part, EULER's *Complete Theory of the Construction and Properties of Vessels.*

456. Whatever difference there may be in the construction of vessels used in navigation, we always find in them this one general property, that each vessel is composed of two perfectly equal parts joined together in the middle, lengthwise.

The section that divides the vessel into these two equal parts is called the diametrical section.

As these parts are not only equal, but as care is taken to load them equally, the centre of gravity of the whole vessel must fall in the diametrical section.

When a vessel floats upon the water, if we suppose a plane to pass through the water line, it will separate the part immersed from the remainder of the vessel. The part below this plane is called the *Hollow* of the vessel, and its centre of gravity the *Centre of the Hollow.*

457. There are two conditions essential to the equilibrium of a vessel floating upon a fluid, viz.

(1) The immersed part must be equal in volume to a mass of the fluid whose weight is equal to that of the vessel.

(2) The centre of gravity and the centre of the hollow must be in the same vertical line. This line is called the *Vertical Axis* of the vessel.

The centre of the hollow is always below the water line, but the centre of gravity may be either above or below it. In large vessels, particularly in ships of war, it is above the water line, in canal boats, &c., it is below. EULER, Book I.

458. Were the parts of vessels firmly fastened together, and the materials inflexible, or if their figure were such that the pressure of the water upwards, and the action of the weight of the vessel downwards, were not only in general equilibrio, but also in equilibrio at every point on the bottom of the vessel, the above conditions would be all that need be considered. But vessels are neither perfectly well fastened, nor are their materials inflexible; their shape, too, is such, that the equilibrium between the two pressures does not take place at every point.

If we conceive the vessel divided into two parts, by a transverse section passing through the vertical axis, and if we find the centre of gravity of each of these parts, and the centres of gravity of the corresponding portions of the hollow, the pressure of the water and the weight of these parts are in equilibrio only when the centres are situated in the same vertical line. In all other positions of these points, one or other of the forces acts to bend or change the figure of the vessel.

In ships of the usual figure, the lines of direction of the centres of gravity of the two parts fall farther from the transverse section than the lines of direction of the centres of the corresponding portions of the hollow. The weight has therefore an action to depress the vessel at each extremity. This change of shape is called *hogging*. Such is the intensity of this force, that all vessels of any considerable burthen have their shapes changed in some degree at the moment of launching; and it acts continually while the vessel lasts. To lessen the effect of this cause, Seppings has proposed to substitute diagonal trusses in place of the ceiling plank of vessels; and his plan has been found effectual in several ships built under his direction for the British navy. See QUARTERLY REVIEW, for 1812.

459. When a vessel, by the action of the wind, or of any other cause, becomes inclined, one of the three following consequences must happen:

(1) The vessel may remain in this inclined state, in which case we say the equilibrium is insensible.

(2) The vessel re-establishes itself by its own efforts in its original *situation*, when the equilibrium will be permanent.

(3) The vessel, after being inclined, is entirely overturned; this equilibrium is called unstable or tottering.

Neither the first nor the last of these ought to take place in a vessel; the second is absolutely necessary to insure stability.

460. When a vessel inclines, the position of the centre of gravity remains unaltered, but the centre of the hollow changes its place; if it change its position so far that the line of direction of the centre of gravity falls farther from the vertical axis than the vertical line drawn through the centre of the hollow, the equilibrium will be permanent, and the vessel will restore itself to its original position; if the line of direction of the centre of gravity falls without the vertical drawn through the centre of the hollow, the equilibrium is destroyed.

Where the external force acts to depress either extremity of the vessel, and the vessel tends to restore itself to an upright position, the motion is called Pitching; where the inclination is to either side, the motion is called Rolling.

461. The figure which the section of a vessel ought to have, in order first to resist an effort to incline it, and then restore itself, has occupied the attention of analysts. It appears from their researches, that if the sections in planes parallel to the direction of the disturbing forces be all circular, and have their centres in a line passing through the centre of those forces, the stability will be the greatest possible.

This is also confirmed by experiment, for a quarter of the shell of a cocoa-nut, or of the skin of an orange, if thrown into the water, will float with its extremities elevated above the fluid, and resist the effort of the winds and waves to overturn it, or depress its extremities.

From a consideration of this circumstance, Mr. Greathead was led to the invention of his life boat: for an account of which, see *Philosophical Magazine*.

462. The stability of a vessel will also depend upon the proportion between its depth and breadth, upon the position of the centre of gravity, and of the centre of the disturbing forces.

When the vessel is propelled by sails, the last named point is called the *Centre Velique.*

In canal boats, vessels propelled by oars, &c., the centre of gravity is below the surface of the water; in many merchant ships, and in all vessels of war, the centre of gravity is above the water line. In the latter case, the breadth of the vessel can never be safely less than half her draught of water. This is of great practical importance, for many builders consider, that by increasing the depth of a vessel her stability is augmented, when the reverse is in fact the case.

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463. As long as a body immersed in a fluid continues at rest, all points of its surface sustain equal pressures; but as soon as it begins to move, it is acted upon by another force that is directly opposed to its motion; this is the resistance of the water.

As from theory it would appear, that the same resistance will take place whether a body move in a fluid with a determinate velocity, or the fluid move and strike it with the same, the investigation of the law which obtains in the one case, will hold good in the other. The following theorem is therefore applicable to the motion of ships, as well as to the action of their rudders and sails.

464. If a fluid in motion impinge against a plane, the force with which it strikes will be proportioned to the specific gravity of the fluid, to the area of the plane, to the square of the velocity of the fluid, and the square of the sine of the angle at which the plane is inclined to the direction of the course of the fluid.

Or, if f represent the force of the fluid, s its specific gravity, a^2 the area of the plane, v the velocity of the fluid, and o the angle of obliquity:

$$f = s\, a^2\, v^2\, \sin.^2 o$$

This theorem may be submitted to the test of experiment, by making use of a vessel of the form of a parallelopiped, and applying to it wedges of equal bases and unequal heights; and it will be found among them to be nearly if not exactly true.

When a vessel of the figure of a parallelopiped moves through water, the fluid, in consequence of its viscidity, does not close immediately behind it, and thus a *non-pressure* takes place. To obviate this, a wedge may be applied to the stern as well as to the bow, and the resistance will be found to diminish.

In consequence also of its viscidity, the fluid opposes a greater resistance to a wedge with plane surfaces, than to one whose surfaces are curved. And the wedge at the stern is better supported if it be curved also. The best form for the curves at the bow is found to be convex outwards, and that for the stern concave. If these be joined by a regular sweeping line instead of a straight one, thus removing all the angles, the resistance will be still further diminished. We thus arrive at the species of curve that the water lines of vessels should be composed of, and we find figures of the same kind adopted by nature in the shape of fishes, &c.

465. The action of the wind upon the sail of a vessel is such as would propel it in a direction perpendicular to the sail, provided the vessel were equally resisted in every direction, or were of a circular shape. As a vessel is less resisted in the direction of its length than in any other, the true course can be perpendicular to the plane of the sail only when the latter is perpendicular to the plane of the keel, and in no other case.

We consider the course as being in the plane of the keel, and apply a correction according to circumstances, that is called the *Lee-way.*

The lee-way will decrease with the increase of the length of the vessel, of her depth, and of the angle that the wind makes with the plane of the keel, and with the sails; but according to ratios differing in every different vessel.

466. Vessels are either rigged with square or with fore and aft sails; in the former, the yards may be braced until the angle they make with the plane of the keel is reduced to 40°, or even a little less; in the latter, the sails hang in the plane of the keel,

but cannot be used to advantage until the action of the wind has changed their position, and they make an angle of at least 20° with the keel. The primitive position of square sails is at right angles to the keel; fore and aft sails may be used in the same position.

By considering the action of the wind upon these sails, the possibility of plying to windward may be seen; thus, as a square rigged vessel will receive the wind in its sails, and be propelled nearly in the plane of its keel when the apparent direction of the wind makes an angle with the keel no greater than 45°; two courses, upon different tacks, will bring the vessel to a point directly to windward of that whence it first set off.

We say the *apparent* direction of the wind, because, when a vessel is in motion, the direction of the wind, as observed on board of her, is different from the absolute direction.

Vessels with fore and aft sails can lie still nearer the wind, and therefore can ply to windward with greater advantage.

Of all vessels, the flying prow of the Ladrone Islands lies nearest to the wind; the periauguas used in the harbour of New-York also lie very close to the wind; but of all fore and aft rigged vessels, the American pilot boat appears to combine the greatest number of advantages.

467. Ships and all other vessels are found to sail best when nearly in an upright position. As the action of the wind in oblique courses, tends to make them heel towards the lee side, it is a matter of no small importance to make the inclination as little as possible.

The force exerted by the wind upon the sail in propelling the vessel is proportioned to the surface of the sail; but that exerted to incline the vessel, acts upon the arm of a lever, and is therefore proportioned not only to the surface of the sail, but to the length of the mast. Hence it is a good rule, when the quantity of sail is to be increased, to do it rather by increasing the length of the yards than that of the masts.

468. Masts should in general neither lean towards the bow nor the stern of a vessel. In the former case, they tend to de-

press and bury the bow in the water; in the latter, a part of the power is lost in unnecessarily raising the bow from the water.

The natural action of masts, even when perpendicular, is to depress the bow; hence vessels are generally built so as to have, when at rest, a greater draught of water at the bow than at the stern.

Masts leaning aft are, however, sometimes advantageous, when the vessel is sharp built; that part of the force exerted to raise the bow from the water is not then lost, but tends to make the motion of the vessel more rapid, particularly in high seas.

469. The Rudder is applied to vessels for the purpose of guiding them and changing their course; were the vessel a plane surface, placed vertically in the water, the rudder would act with the greatest force when it makes with the keel an angle of 54° 44′. EULER, book II.

In consequence of the wedge-like form of the after part of a vessel, an angle of from 45° to 48° is found to be the best in practice. EULER, ib.

In oblique courses, the action of the rudder is much more powerfully exerted to turn a vessel from than towards the wind; it is for this reason that the manœuvre of veering must often be resorted to instead of that of tacking.

470. The time in which vessels whose breadth and depth are equal take to perform the manœuvres of tacking and veering, is proportioned to their length.

This gives the limit in practice to the length of a vessel, which could not be too great, were we merely to consider the facility of passing through the water, and the diminution of the lee-way. The usual proportion of the breadth to the length of vessels navigating the ocean, is between 1 : 4 and 1 : 3.

471. When ships of a great draught of water are to be transported over shoals, they are fitted to a machine that is called the *Camel*. This consists of two large hollow cases of timber, one side of each of which is of such a form as to apply itself closely to the convexity of a ship's figure. These cases are brought and sunk, by opening plugs, one on each side of the vessel to be

lightened; the plugs are then shut, and the water pumped out, the cases rise, embrace the ship, and lift it up with them.

These machines are principally used in Holland and in Russia.

OF THE STEAM ENGINE.

472. The Steam Engine is a machine where the expansive force of the elastic vapour of boiling water is applied, as a moving power to overcome any resistance whatsoever. It was originally a machine of little power, and was for a long time considered as applicable to but one purpose; but it has, since the time of Watt, become the most powerful of all those instruments by which natural agents are applied to the increase of human energy. We shall mention the several forms under which this engine appears to have existed, in a chronological order.

(1) Brancas, an Italian, appears to have been the first person who entertained the idea of using steam as a moving power; he proposed to apply the vapour issuing from the mouth of an Eolopyle to fans or sails placed on the circumference of a vertical wheel.

(2) The French claim the invention of the steam engine for their countryman Papin, known as the author of the digester. His project appears to have been similar to that of

(3) The Marquis of Worcester. In this engine, the steam was made to act upon the surface of water enclosed in a vessel by a valve, and to drive it up by a rising pipe, in the manner of a forcing pump.

(4) Captain Savary, who first executed a successful engine for the purpose of raising water, applied the steam on a principle composed of those of the common and forcing pump. In his engine, an ascending pipe reaches from the water of the reservoir to a chamber, the height of the upper part of which is less than 34 feet; if this be filled with steam, the water will rise into the vessel as soon as it is condensed, and will be retained there by the valve of the ascending pipe; if another pipe be adapted to the bottom of the chamber, furnished with a valve opening upwards, and if steam be admitted and made to act on the surface of the water, it will be drawn up through the second pipe to a height that will be determined by the elastic force of the steam.

In the machines of Savary, Papin, and Worcester, much heat is lost in consequence of the steam coming in contact with the cold water, and with the sides of the chamber, after it has been cooled down to the temperature of that fluid; they therefore use much fuel.

(5) In Newcomen and Cowley's engine, the steam was introduced beneath a piston fitted air tight into a cylinder; when the steam was condensed, the piston fell, being forced down by the pressure of the superincumbent air; when steam was again admitted beneath the piston, it rose, in consequence of its being more than counterbalanced by the weight of a pump rod attached to the opposite extremity of a beam vibrating upon a prop. Thus, a reciprocating motion was produced, and applied to the purpose of working a forcing pump.

In this machine, the atmosphere was evidently the moving power, and the steam merely applied to restore the equilibrium and produce a vacuum.

This engine is imperfect, in consequence of the great waste of steam; for this is introduced into a vessel exposed to alternations of heat and cold, in such a way that it can neither preserve the heat necessary to the full elasticity of the steam, nor become cold enough to produce its complete condensation.

For these reasons, the engine of Newcomen, in its best state, never acquired a power due to a pressure of more than 7⅓ pounds per square inch upon its piston; whereas the whole pressure of the atmosphere is double that quantity. The causes of the imperfect performance of these machines depend on the doctrine of latent heat; and until this was discovered by Dr. Black, no adequate remedy could be applied.

(6) The celebrated James Watt, the friend and pupil of Dr. Black, applied the principles of the doctrine of latent heat to the improvement of the steam engine. This he did by adding a separate vessel for the condensation of the steam. He called this vessel the condenser, and the effect it produced was such as to raise the power of the atmospheric engine from 7⅓ to 12 pounds per cubic foot.

With this addition, with more perfect workmanship, and more regular motion, (the latter of which is secured by the use of a fly,) the steam engine is known by the name of the *Single* or Atmospheric Engine of Watt.

(7) The Atmospheric Engine is incapable of being applied to produce any other than a reciprocating motion. In order to obtain a circular motion by means of the steam engine, Watt introduced steam both above and below the piston; in this way it is not only employed to form a vacuum, but becomes itself the moving power; it is not limited in its action to the limit of the pressure of the atmosphere, but may be extended to pressures of greater amount per square inch. These pressures will depend upon the heat of the steam employed.

(8) This part of Watt's plan was not extended by him farther than to steam of a temperature a little higher than that of boiling water. It being discovered that the elasticity of steam increased with the squares of the increments of temperature, steam is now employed at higher temperatures, and with much greater expansive powers. Engines where steam of this character is employed, are called high pressure engines. In a high pressure engine there is no condenser, but the steam is permitted to escape into the air. The steam is admitted alternately above and below the piston, and the action is due to the difference of the expansive energy of the high steam on the one side, and that which is issuing into the atmosphere and losing its temperature on the other.

Although the elasticity of steam increases with the square of the increase of temperature, this is not attended with the advantage it might appear at first sight to possess; for it seems, from late experiments, that the consumption of fuel is proportioned to the elasticity of the steam.

In the old atmospheric engine, the rectilineal reciprocating motion of the piston rod was adapted to the circular motion of the end of the beam, by means of chains; these applied themselves to successive portions of a circular arc whose radius was the distance of the extremity of the beam from its centre of motion. The same contrivance was adopted at the other end of the beam, to change the reciprocating circular motion communicated to it into the rectilineal motion, suited for working the pump rod.

In Watt's first atmospheric engines, the same plan was continued; *when however* he planned the double engine, where the rods require to be forced down as well as drawn up, he contrived an apparatus, by which a change might be effected, and the force of the ascending and descending piston made effective to produce reciprocating circular motions. This contrivance goes by the name of the Parallel Motion.

The principle in its simplest form is as follows, viz.

473. If two of the adjacent angles of a parallelogram be made to describe concentric circles, so that the side between them passes through their centre, and one of the remaining angles another circle having its convexity opposed to that of the two former; then the fourth angle of the parallelogram describes a line that does not differ sensibly from a straight line. Playfair, § 355. Prony, § 1478. Rees, article *Parallel Motion.*

474. To convert the reciprocating motions into continuous circular motions, Watt invented several ingenious contrivances acting on principles similar to those of a crank. The earliest of these was known by the name of the Sun-Planet wheel.

475. In the old atmospheric engines the valves were opened and shut by hand; after a time rack work was applied to this purpose and worked by the machine; this is still employed in some of the late engines. A more neat contrivance for this purpose is the eccentric motion; of this, there are two kinds, one by Murray, of Leeds, the other by Watt. See Rees, article *Steam Engine.*

476. Most of the valves that are applicable to pumps, (see § 435,) have, at different periods, been adapted to the steam engine. There is also another, that appears preferable to any of them; it is known by the name of the *Slide Valve.*

477. To regulate the admission of steam into the cylinder, a Governor is employed; the production of steam is itself regulated by a self-acting damper, that increases or diminishes the *draught of the* chimney of the furnace.

478. In converting a reciprocating rectilineal into a continuous circular motion, by means of the working beam and parallel motions, a small portion of the force is lost, in consequence of obliquity of action. More is lost in consequence of the irregularity of the motion of the piston ; the velocity of this is continually accelerated during the first half of its descent, continually retarded during the second, and is for an instant at rest ; it then returns in the opposite direction, with a motion that is for a time accelerated, reaches its maximum, and is again retarded.

For this, no adequate remedy was discovered by Watt, for the best workmanship and the heaviest fly will not entirely overcome the defect. It has however been met, and fully corrected by Francis Ogden, an American engineer, to whom Watt paid the compliment of erecting, for his own purposes, an engine on this construction.

479. In Mr. Ogden's engine, there are two cylinders that are so arranged, that the motion of one shall be at its maximum when that of the other is null. The velocity of the impelled point of the machine is therefore very nearly uniform.

480. Besides the original purpose of working pumps, and thus drawing water from a great depth, steam engines are now employed to propel machinery of every possible kind ; to impel boats, and to draw carriages upon rail-ways.

It is also said that they have applied them of late in England, to draw carriages upon common roads. An attempt of the same kind has likewise been made in Pennsylvania.

The earliest idea of the possibility of propelling boats by steam, seems to have originated with Savary ; he even mentions the method of paddle wheels as the one he would propose to use. No practical result however followed from his theory, nor could his engine have been used for such a purpose. Watt has intimated in conversation, that he had at one time entertained views of attempting to apply steam to navigation, but was prevented by the pressure of other business connected with his engine.

Steam-boats appear to have been actually constructed and set in motion by Miller, of Dalswinton, and Symington in Great-Britain It is fore necessary that the sail should be inclined to the vertical plane.

When a surface at rest is acted upon by a fluid in motion, the maximum of the force by which it would be carried in a direction perpendicular to the course of the fluid is attained when the plane makes with the direction of the current an angle of 54° 44′. When the surface has a motion around a fixed axis in the prolongation of one of its sides, this condition is found no longer to hold good, nor is there any constant angle at which every part of the sail ought to be inclined to the vertical plane. Those parts which are nearest to the axis are most inclined, and those most distant are less so; the greatest angle of inclination in practice is 25°, and it is gradually diminished to 0°.

In the vertical wind-mill, the pressure of the wind acts upon the sail during its whole course; all the four sails, therefore, concur in producing the effect.

Smeaton has laid down a number of maxims in relation to wind-mills; these may be found in the Philosophical Transactions, and in the separate editions of his works; they are also detailed at length by GREGORY, in his 2d volume, by HACHETTE, § 185, and in [illegible], article *Wind-mill.*

486. Horizontal wind-mills have been sometimes constructed, but they are found to be much less powerful than those which are vertical. Their effect appearing, by the experiments of Smeaton, to be no more than [illegible] of that produced by a vertical wind-mill, whose sails are of equal dimensions.

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OUTLINES

OF

NATURAL PHILOSOPHY:

BEING THE

Heads of a Course of Lectures

DELIVERED IN

COLUMBIA COLLEGE, NEW-YORK,

BY

JAMES RENWICK, A. M.

PROFESSOR OF NATURAL AND EXPERIMENTAL PHILOSOPHY, AND
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OUTLINES

OF

NATURAL PHILOSOPHY.

PHYSICS.

SECTION FIRST—HEAT.

487. Among the first circumstances that would attract the attention of the inquirer into Physical and Chemical subjects is, the powerful and active agency of Heat. We find it continually employed both in the processes of nature and of the arts; we perceive its effects upon our own organs; we find it destroying some substances, melting others, rendering others luminous, and converting others into vapour. When its action is less intense, it alters the dimensions of bodies, and changes their volume.

Most philosophers regard Heat (or, as the French school call it, Caloric) as a material substance, to which is attributed several of the properties found in ponderable bodies, such as elasticity, compressibility, and the power of entering into combination with other bodies. Others again refuse to admit the agency of a specific substance. For the present we shall avoid the discussion of the relative merits of the two theories, and shall treat of the effects produced when bodies are heated; making use of the term *Heat* to denote the hidden cause of these phenomena.

OF HEAT CONSIDERED AS EXPANDING BODIES.

488. It is a general law of the action of Heat, that when it is applied to bodies their dimensions are increased, and when they are again cooled their dimensions become less. This effect on the one hand is called expansion, on the other contraction; it is found to take place in all the different classes of bodies, whether solids, liquids, or elastic fluids.

(1) The expansion of solid bodies is less perceptible than that of fluids, being but trifling when the heat is distant from that at which they melt. It may however be rendered very evident by a variety of experiments:

When long lines of iron pipes are laid to conduct water or gas, and have flaunches connected together by bolts and screws, the difference of their lengths in summer and winter is so great, that it is absolutely necessary to place at intervals, joints where one pipe can slide freely into the other, otherwise the flaunches would be torn asunder.

We have already, in Vol. I. spoken of the effects produced on the pendulums of clocks by change of temperature.

A bar of iron that, when cold, will just pass between two parallel planes, and just enter into a hole of a figure similar to its own section, is so dilated by the application of heat that it will no longer do either.

(2) The expansion of liquids may be shown by enclosing a small quantity in a bolt head, or in a Florence flask to which a long and slender neck is attached; if the apparatus be immersed in boiling water the fluid will after a short time rise rapidly in the neck of the vessel. In this experiment it will frequently be observed that the liquid will sink in the neck when heat is first applied; this arises from the expansion of the glass vessel itself, which is at first greater than can be counteracted by that of the liquid.

(3) If a bladder only partially inflated with air, and tightly closed, be immersed in boiling water, or heated in any other manner, it will soon be distended, and finally burst. If a flask be heated, the air will expand and pass out; and if it be inverted, while warm, in a liquid, the liquid will rise into the flask as it cools.

489. We make use of this property of heat in constructing an instrument to measure its intensity. This instrument is called a *Thermometer;* its form and exterior appearance are so well known that it is not necessary to describe them.

The first Thermometer on record was constructed by Heron of Alexandria, about the year 250 of our era. It appears to have been re-invented by Sanctorio, an Italian physician, about the year 1590, and by Drebbel, of Alkmaer in Holland, about 1610.

In these three thermometers heat was measured by the expansion of air. A portion of this fluid was enclosed in a tube with a bulb, by inverting it in a vessel filled with a liquid. Sanctorio made use of water for this purpose, and Drebbel employed sulphuric acid.

Air dilates so rapidly on the application of heat, that the scale of these instruments was necessarily very limited; they were also affected not only by change of temperature, but by a change in the pressure of the atmosphere; and there was no means of rendering them comparable with each other.

Notwithstanding these defects, thermometers of this construction are still occasionally used, where very great sensibility is required.

The Florentine academy, about 1675, constructed thermometers resembling in form those used at present, and enclosing spirits of wine. These instruments were originally graduated with one fixed point, viz. that of the greatest heat of the sun at Florence. As all the thermometers subsequently constructed were graduated from the first, the philosophers of Europe were furnished with instruments that could be compared with each other. These thermometers were however liable to many objections, for the point whence the scale began was by no means a constant one, even in Florence, and could not be recovered in any other place; the fluid employed has itself the disadvantages of differing in strength, of boiling at a low temperature, and of expanding very unequally with equal increments of heat.

To obviate the first of these objections, various modes of obtaining fixed and constant points, whence the scale might begin, were proposed.

Mr. Boyle recommended the use of the freezing point of essential oil of aniseed; Dr. Hooke proposed to graduate thermometers in a deep pit; and Newton preferred the boiling and freezing points of water. These last have since been universally adopted.

The objection arising from the nature of the fluid led to the use of a variety of other liquids. Newton used linseed oil; this has the advantage of bearing great changes of temperature between its freezing and boiling points; but its viscidity was such as to cause it to adhere to the tube, and render it impossible to ascertain with accuracy the place at which its surface stood.

Mercury has finally superseded all other fluids, except where great cold is to be observed, in which case spirit of wine is to be preferred. Mercury was first used in thermometers by Fahrenheit; his mode of graduation was as follows, viz. His zero was fixed by him at a cold he had himself observed in Iceland, and which he afterwards produced by means of a freezing mixture. When the thermometer, after having this point marked upon it, was immersed in melting snow, or water beginning to freeze, the mercury rose: he marked the point at which it stood, and divided the intervening space by continual bisection into 32 equal parts. When a scale of parts equal to these was carried up the instrument, he found that it rose 64 of them from the freezing point of water before it reached the temperature of the human body, and 180 before water in which it was immersed boiled. These points are therefore marked on his thermometer at 96° and 212°.

The thermometers now called by the name of Fahrenheit are graduated by immersing them first in water containing a portion of ice still undissolved, and then in water boiling, when the mercury of the barometer stands at 30 inches. The stem of the instrument, between these two positions of the mercury, is divided into 180 equal parts; 32 of these are set off below the freezing point to the zero, and as many are carried up above the boiling point as the length of the tube will admit. Fahrenheit's scale is used by the English, the Dutch, and in this country.

In France a thermometer that goes by the name of Reaumur was principally employed before the revolution. In this the freezing point is the zero; the space between it and the boiling point is divided into 80 degrees. The invention of this was justly due to De Luc, but, in consequence of its containing the same number of degrees on its scale as a spirit thermometer constructed by Reaumur, it obtained its appellation from the latter.

The thermometer of De Luc has been superseded in France by the Centigrade; this is so called because its scale comprises 100 degrees between the boiling and freezing points of water. A scale of this kind was first used in Sweden by Celsius.

In Russia a thermometer contrived by De Lisle is used; the zero is at the boiling point, and the scale comprises 150 degrees between that and the freezing point.

490. Except when very low temperatures are to be observed, mercury is the best fluid that can be used in thermometers; for, its dilatations at mean and low temperatures are very nearly proportioned to the increments of heat; it is of all liquids the most easily freed from air, any other impurity is also readily separated; it is speedily affected by the alternations of heat and cold; its scale is long, reaching from $-40°$ to $+575°$ of Fahrenheit; finally, it is homogeneous, and of the same quality, when properly purified, at all times and in all places.

The manner of filling and graduating thermometers may be found in HENRY's *Chemistry*, Vol. I.

491. Thermometers may be constructed in such a way as to register their own greatest and least heights; the best that have been used for this purpose are those of Six and Rutherford.

492. We have, in directing the distance between the boiling and freezing points to be divided into a number of equal parts, supposed that the tubes were truly cylindrical, and that no other cause of error existed. When it is wished to construct a standard thermometer, a large tube may be taken with a bulb of proportionate dimensions, and the mercury introduced by means of a small measure in portions exactly equal to each other; in this way we shall have a means of dividing the distance between the two fixed points into parts that shall correspond with equal expansions of the mass of mercury. BIOT.

493. The thermometer is one of the most useful of all philosophical instruments. Its applications in every department of natural science are almost innumerable; its variations form the basis of every theory of heat; it is a check upon all chemical operations; it is consulted by the astronomer at every instant of his observations; to it we owe our knowledge of animal heat, as kept up and supported by the processes of respiration, &c.; we observe by it the mean heat of the earth, and of different climates, and the decrease of this mean between the equator and the poles; and, in short, we have instances of its value at every step. See Biot, *Precis*, Vol. I.

494. The ratio of expansion is not the same in other fluids as in mercury; when, therefore, we use any other, alcohol for instance, in a thermometer, its scale will not correspond with that of a mercurial thermometer, unless its divisions be unequal. Neither is the expansion of mercury absolutely uniform. A measure of equal increments of heat may be found, from a knowledge of the fact, that if two portions of the same liquid be mixed at unequal temperatures, the bulk of the mixture is exactly equal to that of the two parts; hence it is inferred that the resulting temperature is the true arithmetic mean of the two that previously existed.

It has been proposed to make use of this principle in the construction of thermometers, but it has been thought best not to deviate from that we have already pointed out; for it is easy to apply a correction, in such investigations as are of so nice a character as to demand an allowance for this cause of error.

495. The expansion of solid bodies may be measured by means of instruments called Pyrometers. In this way it is found that the absolute dilation of different substances is very different, but that the relative expansion of each particular substance follows nearly the same law as mercury.

For tables of the expansion of solid bodies, see Biot, *Precis*, Vol. I. *p.* 238. and Brande, in *M'Neven's* edition, p. 20.

496. The number that represents the cubic expansion of solid bodies, when not very great, is exactly three times that which represents the linear expansion, at equal temperatures.

For a demonstration of this remarkable law, see Biot, *Traite Complet.*

A knowledge of the relative expansibilities of the metals, has many valuable practical applications; particularly in the construction of clocks and watches. See Vol. I. § 174. and 394.

497. All elastic fluids whatsoever, exposed to equal increments of temperature, under equal pressures, are equally expanded, and the law of their expansion is the same as that of mercury.

This is the result of the experiments of Gay Lussac. See Biot, *Precis*, vol. ii.

498. The uniform relation that exists when the individual expansions of solid bodies, and the general rate of dilatation in gases, are compared with that of mercury, does not hold in the case of liquids. The variation of this class of bodies from the law, is most remarkable when they approach the temperatures at which they boil, or at which they become solid.

(1) Fluids expand with a rapidity that is much increased as they approach the point of ebullition.

(2) When water is cooled to a temperature approaching its freezing point, it ceases to contract; and at 40° of Fahrenheit's scale it again begins to expand. At this temperature of 40° of Fahrenheit, the density of water is of course the greatest possible. Water, if kept from the contact of the air, and at rest, may be cooled below the temperature of 32°; but if it be agitated in the slightest degree, or have a piece of ice placed in it, it congeals at once. At the moment of congelation, the bulk of the water is again changed, the nascent ice suddenly expands, and a difference in its specific gravity of nearly $\frac{1}{9}$ is produced.

There are other substances that expand in the same manner as water, when they pass from the liquid to the solid state; among these are cast iron, bismuth, antimony, and sulphur.

Some substances, on the contrary, suddenly contract, as is the case with mercury.

499. The expansion of bodies by heat is attended with a proportionate change in their specific gravities. For this reason, when heat is partially applied to masses of fluids, those portions which are heated rise to the surface, and their place is supplied by the adjacent portions. This may be made manifest in liquids, by placing powders in the vessel that contains them; when heat is applied to the lower part of the vessel, the motion of the powder will show the directions of the ascending and descending currents.

We apply this to several practical purposes; viz. to the construction of fireplaces and chimneys, the erection of air furnaces, and the ventilation of apartments.

OF LATENT HEAT.

500. Heat, in expanding bodies, appears to act in opposition to their attraction of aggregation. Its agency in this respect is more remarkable, when it is of such intensity as to change their state of existence. Thus, at certain temperatures, solids become liquids, and liquids gases. All the different substances in nature are probably capable of assuming these three several forms. DAVY, *Elements of Chemical Philosophy.*

Instances of the conversion of solids into fluids by heat, are too familiar to require any particular example.

When water becomes steam in the act of boiling, it is converted into an elastic fluid. Ether furnishes a simple experiment by which this may be illustrated. Mercury, if held over a lamp in a platina spoon, is converted into vapour; and if it be cooled to the temperature of — *40°* it becomes solid.

When the heat of furnaces or lamps is insufficient to liquify or evaporate the substance submitted to it, it may notwithstanding be acted upon by the Compound Blowpipe, or by a powerful Galvanic Deflagrator.

Different bodies change their states at different temperatures; thus, water becomes solid at 32° of Fahrenheit, and boils at 212°; mercury becomes solid at — 40°, and boils at 575°; while ether boils at 98°.

501. The conversion of solids into liquids is attended with a loss of thermometric or sensible heat. Thus, if ice be placed in a vessel over a lamp, its temperature speedily becomes 32° of Fahrenheit, and it begins to melt: but although the heat is continually acting upon it, the temperature cannot be raised above this point until the whole of the ice be dissolved. And thus, if equal quantities, by weight, of ice at 32° and water at 172° be mixed together, the resulting temperature is no more than 32°, but the whole of the ice is dissolved. From this it would appear that 140° of thermometric heat had disappeared.

In all other cases of liquefaction, heat is in like manner absorbed. Thus, when snow and salt are mixed together, they rapidly become fluid, and absorb heat from the neighbouring bodies. Mixtures of this sort are called freezing mixtures; the most powerful of these in its effects is a mixture of snow and muriate of lime; by means of it a cold sufficient to freeze mercury may be readily produced.

502. On the other hand, when liquids are converted into solids, heat is disengaged.

503. When fluids are converted into gases or vapour, there is a similar loss of thermometric heat. Thus, when water is heated, it soon reaches the temperature of 212°, at which it boils; steam rises rapidly from it, and the whole finally disappears, without any greater elevation of temperature, either in the liquid or the steam.

The quantity of heat absorbed when steam is produced may be taken at about 900° of Fahrenheit. This may be shown in two different ways:

(1) A vessel of sheet iron may be filled with cold water and placed on a red-hot plate; if the time necessary to raise it to the boiling point be remarked, and compared with that which is sufficient to make the whole disappear, a simple arithmetical calculation will give the quantity of heat lost. Dr. Black, who planned this experiment, made it 810°.

(2) A pipe may be led from a vessel in which water is boiled, and immersed to the bottom of a vessel containing a known weight of water of a given temperature. The steam will pass through this pipe into the vessel, and be there condensed; its heat will be communicated to the water, and from a comparison between the weight of steam condensed and the increased temperature of the water, the heat that is latent in the steam may be estimated. In this way the quantity of heat that disappears is found to be from 900° to 1000°. See HENRY, *Chemistry*, Vol. I.

If one part of steam at 212° be mixed with six parts by weight of water at 62°, the whole of the steam will be condensed, and the temperature of the fluid will be 212°.

504. The heat which is thus absorbed is said to be *latent*, in contradistinction to that which is sensible, and capable of showing its presence by affecting the thermometer. Sensible heat, as measured by the thermometer, constitutes what we have had occasion more than once to speak of by the name of *Temperature*.

The phenomena of latent heat were first discovered by Dr. Black, who laid down as a general rule, "that whenever a body changes its form, its relations to temperature are likewise changed." DAVY.

505. The discovery of this law, or of the doctrine of *latent heat*, has been productive of some important results, and is applicable to a great variety of useful purposes.

(1) The knowledge of it led Mr. Watt to his great improvement in the steam-engine.

(2) We apply the heat produced by the condensation of steam, to cooking, in the processes of brewing and distillation, to warm large rooms and extensive buildings.

(3) The cold produced by spontaneous evaporation is applied to the cooling of liquors in warm climates, where ice cannot be procured.

(4) The evaporation of ether under the receiver of an air pump, will produce a cold sufficient to freeze water.

(5) Water may be frozen by its own evaporation under the receiver of an air pump; this experiment was contrived by Professor Leslie; in it, sulphuric acid is placed in a vessel on the plate of the air pump, and the water in another vessel raised above it; when the receiver is exhausted, the sulphuric acid rapidly absorbs the vapour rising from the water, fresh vapour is then formed, and in a short space of time crystals of ice are seen to shoot over the surface of the water.

(6) The Cryophorus acts upon similar principles; it is composed of two balls of glass, united by a tube bent twice at right angles; water is placed in one of the balls, and the air driven out by boiling the water. If all the liquid be in one of the balls, the other will contain a small portion of aqueous vapour, which, if it be placed in a freezing mixture, will be condensed; fresh vapour will now rise, until the cold its formation causes is sufficient to freeze the water.

506. The absorption and disengagement of latent heat, are also of great value in the economy of nature. When aqueous vapour is condensed in the atmosphere, heat is produced; and the formation of rain, hail, and snow, tends to mitigate the severity of the winter. In summer, the surface of the earth is kept cool by constant evaporation. The melting of the polar ice moderates the heat that would otherwise be felt in the polar regions, in consequence of the long continuance of the sun above the horizon during the summer. And the evolution of heat during the congelation of water, renders the transitions of temperature more slow and gradual.

507. We have seen that different bodies change their states at different temperatures; the change that takes place from

liquidity to solidity, and vice versa, is performed in the same substance at a constant temperature, under all circumstances whatsoever ; the change from liquidity to a gaseous state is performed at no constant temperature, but is affected by pressure, whether of the atmosphere, or produced artificially.

Thus, ether will boil at the temperature of freezing water, under the exhausted receiver of an air pump ; water will boil in a similar place at 90°. On the other hand, water enclosed in a strong vessel may be heated to a temperature of at least 300°. Davy.

508. We have, under the head of Mechanics, made a distinction between permanently elastic fluids, and those which are condensible by pressure or cold. This distinction, which is of importance in that department of natural science, and very marked between atmospheric air and aqueous vapour, cannot be maintained in Physics or Chemistry, where the one class runs insensibly into the other. Ether, whose vapour is classed among condensible fluids, becomes a gas at the temperature of 98°, which is not above the heat at which the air is sometimes found in tropical countries ; while ammonia, although permanently elastic under all common circumstances, is condensible by intense cold aided by pressure. Davy.

509. All bodies that boil at a moderate temperature, seem to evaporate, so as to produce a certain quantity of elastic matter at the common temperatures of the atmosphere ; and this in quantity greater in proportion as the temperature is high.

The elastic force of vapour increases in geometrical progression, while the temperature increases in geometrical ; but the rate differs in different fluids. This law was discovered by Dalton.

In the first volume we have treated of the several phenomena produced by the evaporation and subsequent condensation of aqueous vapour. See § 336, et seq. It is barely necessary here to remark, in addition, that, although we have there assumed the hypothesis that moisture is held in chemical solution by the air, this is by no means of absolute necessity ; for by the experiments of Dalton it would appear, that the quantity of moisture taken up in a given space, and at a constant temperature, is constant, whether air be present or not. The facts might have been represented on this hypothesis, but not as concisely.

Those who wish to pursue this subject, may consult DALTON's papers in the *Manchester Transactions*, and BIOT, *Traite Complet*, Vol. II.

OF SPECIFIC HEAT.

510. When equal volumes of two different bodies, of different temperatures, are suffered to remain in contact till they both attain the same temperature, it is found that this temperature is not the mean of the other two. Thus, if a pint of quicksilver at 100° be mixed with a pint of water at 50°, the resulting temperature is not 75°, but no more than 70°. DAVY.

In the language of Chemistry, this is said to depend on the different capacity of the bodies for heat; and in the above instance, mercury appears to have a much less capacity for heat than water.

511. In consequence of the different capacity of bodies for heat, equal increments of heat will not produce equal increments of temperature in different bodies; nor will the cooling of different bodies an equal number of degrees of temperature, communicate equal quantities of heat to the surrounding bodies.

In this case, as well as in the change of the forms of bodies, sensible heat disappears, and is again developed on cooling. The heat that disappears is in this case said to be *Specific.*

512. To measure the capacity of bodies for specific heat, an instrument has been contrived by Lavoisier. The action of this consists in comparing the quantity of ice a body is capable of melting in cooling a certain number of degrees, with the quantity melted by an equal portion of water in cooling from the same

high to the same lower temperature. See Lavoisier, *Chemie.* Biot, *Precis*, Vol. II.

In this way the following results were obtained:

Specific heat of Water,	1.00000	
Sheet Iron,	0.11051	
Crown Glass,	0.19290	
Mercury,	0.02900	
Lead,	0.02819	
Tin,	0.04754	
Sulphur,	0.20850	
Olive Oil,	0.30961	Biot.

From this table it may be seen that mercury has but a very weak capacity for specific heat; to raise this liquid a certain number of degrees of temperature will not require more than $\frac{29}{1000}$ of the heat that would be necessary to produce an equal effect upon an equal mass of water. It is therefore evident that it will be speedily affected when placed near substances of unknown temperatures; and it is for this reason, in addition to its other properties, the very best substance with which thermometers can be filled. Biot.

Professor Leslie has proposed to ascertain the specific heat of bodies, by observing the rates at which they cool.

513. When the forms of bodies are changed by mechanical means, or when mechanical forces are made to act upon them, there is usually a change of temperature; for in this way the capacity of bodies for specific heat is changed, being increased when they expand, and diminished when their dimensions are reduced. Of this there are many instances.

(1) A bit of caoutchouc, extended by mechanical means, and then suffered to contract, becomes warm.

(2) Metallic bars are heated by a few blows of a hammer; and a nail may thus be actually made red hot. The friction of solids produces an increase of temperature sufficient to cause ignition, and by strong pressure fluids may be made luminous.

(3) When an elastic fluid is compressed by mechanical means, its tem-

perature is increased; and when the compressing forces are great and suddenly applied, the effect is such as to cause the ignition of bodies. A machine for setting fire to tinder, by the compression of air, has been for some years in use.

(4) Where air expands in consequence of the removal of the forces by which it is compressed, its capacity for heat is increased, and a diminution of temperature is occasioned. Thus, the mercury in the thermometer sinks when the air is exhausted from the receiver of an air pump in which it is placed.

A curious phenomenon, arising from this cause, is observed during the action of the hydraulic machine of Schemnitz, in Hungary. (See § 423.) When one of the stopcocks in the lower vessel is opened, and the air suffered to escape, its sudden rarefaction produces a degree of cold that not only precipitates aqueous matter, but causes it to assume the form of hail.

The production of snow on the tops of mountains, has been explained upon the same principle. Even under the equator, mountains are to be found covered with perpetual snow; these are of such a height as to reach into regions where the air is very rare, and its capacity for heat very much increased; its temperature is therefore lessened. DAVY, *Elements*, Part I. § 5.

OF THE PROPAGATION OF HEAT.

514. Heat may be propagated by radiation, or may be conveyed from one body to another through the intervention of a third.

515. When a body is heated to a temperature greater than that of the surrounding bodies, the heat tends to escape until an equilibrium of temperature takes place. The escape of the heat appears to take place on every side of the heated body, in right-lined directions; it is hence called *Radiation.*

The rate at which a body is cooled by radiation, abstracting from the action of the medium in which it is placed, is uniformly retarded; the diminution of temperature for any small space of time, being proportioned to the excess of the temperature of the body above that of the surrounding air. BIOT.

516. If, while heat is radiating from a heated body, a mirror of polished metal be presented to it, it will be reflected.

The phenomena of the reflection of heat are best investigated by means of an apparatus contrived by Pictet, of Geneva. This apparatus is composed of two concave mirrors of metal, placed opposite to each other, and having a common axis. A heated body, that is not luminous, is placed in the focus of one of the mirrors; the rays that proceed from it, and fall upon the nearest mirror, are reflected in a direction parallel to the axis of the mirrors; they then meet the second mirror, and are concentrated by it at its principal focus. If a sensible thermometer be placed in this last named point, it will be affected by the heat thus concentrated, and that in proportion to the heat of the body that radiates.

This experiment may be rendered more striking, by placing the mirrors at such a distance from each other, that the heat that radiates directly from a body placed in the focus of the one will produce no effect on a thermometer in the focus of the other. This distance may be found by removing the mirrors, and making the experiment without them; or the same thing may be shown by covering the mirrors.

517. This experiment is best performed by means either of the Differential Thermometer of Leslie, or of the Thermoscope of Rumford. The first of these instruments consists of two bulbs of glass united by a tube bent twice at right angles; the two bulbs contain portions of air separated from each other by a liquid placed in the tube. If one of the bulbs be heated while the other is not, the expansion of the air it contains will mark the excess of temperature. The thermoscope of Rumford differs from the differential thermometer, only in having its scale drawn on the horizontal branch. It is more convenient than the differential thermometer, because the length of its horizon-

tal stem places the bulb that is not to be heated beyond the influence of the calorific rays.

If, instead of employing a body that is not heated to such a degree as to be luminous, an ignited substance be placed in the focus of one of the mirrors, the heat condensed in the other will be sufficient to set fire to combustible bodies. Thus, if a small piece of red hot charcoal be placed in the focus of one mirror, gunpowder may be made to explode in the other.

When the apparatus is in action, the heat may be intercepted by the interposition of a screen, and permitted again to act on withdrawing it. In this way it may be shown that heat is propagated with very great rapidity.

The heat of ordinary flame is intercepted by the interposition of glass screens, that permit the light to penetrate them; but the heat is not separated by this means from the solar ray, nor from the flame of carburetted hydrogen. From this it might be inferred, that heat is, like light, susceptible of polarization.

518. The phenomena of the radiation of heat have been supposed to arise from undulations in the air, or in some other fluid medium. Sir Humphrey Davy, by enclosing mirrors in the receiver of an air pump, has shown that this is not true; but that heat radiates nearly three times as fast, when the body is enclosed in the receiver of an air pump, and the air exhausted, as in the open air.

519. From experiments of Professor Leslie, it appears that the nature of the surface has a great effect on the radiation of heat. The following table of the radiating powers of different substances has been deduced from his observations:

Lampblack,	100
Water,	100
Writing Paper,	98
Crown Glass,	90
India Ink,	88
Ice,	85
Mercury,	20

Lead, (brilliant,)	19	
Polished Iron,	15	
Gold, Silver, Copper, Tinplate,	12	Biot.

From this it would appear that colour has little or no effect on the radiation of heat. See Leslie, *Relations of the Air to Heat and Moisture.*

520. The rate at which bodies receive heat is affected not only by the nature but by the colour of their surfaces. Bodies that absorb most light receive most heat, and are consequently most rapidly heated; black bodies are more heated than red, red more than green, green more than yellow, and yellow more than white. Metals are less heated than earthy or stony bodies, or than animal and vegetable substances. Polished surfaces are less heated than those which are rough.

There are various applications of these principles to the useful arts and the processes of domestic life. Thus, vessels intended to retain their heat should be metallic, and highly polished; steam or air pipes for warming houses should be polished in those parts where the heat is not intended to communicate, and covered in others with some radiating substance; culinary implements should be blackened and not polished on those parts intended to receive heat.

The heated surfaces of fire-places or stoves should not be metallic, but of some substance that radiates well. See Davy.

521. When a body colder than those which surround it is placed in the focus of one of the mirrors of the apparatus we have described, a thermometer placed in the other will descend; hence it might be inferred that cold radiates as well as heat; but when we consider that there is no limit to the temperature at which bodies may radiate heat, it is evident that there is no necessity for such an inference, and that the radiation of heat from the thermometer itself will be sufficient to furnish the explanation of this phenomenon.

The apparent radiation of cold was first observed by the Academy del *Cimento* at Florence.

With this apparent radiation of cold are connected several natural phenomena.

If a body be exposed during a serene night to the free aspect of the heavens, all the heat that it radiates will be lost; if that which it receives from the contact of the air, and of neighbouring bodies, is not equal to the quantity thus radiated, the temperature of the body will be lowered. In this way we may account for the rapid cooling of the earth's surface when the sun has set, and the consequent formation of dew and hoar frost. (See § 344.) If bodies be supported at a small distance from the earth by bad conductors, they may be cooled by the radiation of their heat towards the sky, to a degree sufficient to cause the congelation of water, even when the temperature of the air is not below 50°. It is in this way that we can account for the manner in which ice is prepared in Bengal. Biot, *Precis*, Vol. II. Young, § lvii.

The sky must be clear, or these effects will not be produced, for clouds will interrupt the radiation of heat from a body whose temperature is not very high. Currents of wind will also prevent the formation of dew.

Dew will be formed in greatest abundance on such surfaces as radiate well.

The diminution of temperature necessary for the deposit of dew on the surface of a body will depend on the quantity of moisture contained in the air. It is now well understood, from the researches of Dalton, what quantity of moisture is necessary to saturate a given space at a given temperature. Hence, if it were possible to observe the exact point at which dew begins to be deposited, we should have a measure of the moisture contained in a given portion of the atmosphere. Mr. Daniell has invented an instrument by which this is readily effected; for a description of which, see *Quarterly Journal of Science*, Vol. X.

He has still more recently prepared tables for adapting his hygrometer to be used with the mountain barometer; thus obviating the imperfection mentioned in § 262. See *Quarterly Journal*, Vol. XII.

522. When a metallic bar has one of its ends immersed in a medium hotter than the air that surrounds it; for instance, if it

be placed in the fire of a forge, or in a bath of melted metal, the heat will be transmitted to the other end, but not instantaneously, for a time will elapse that depends upon the nature of the substance of which the bar is composed, and on its dimensions.

The bar is said to *conduct* heat.

523. If slender cylinders of silver, glass, and charcoal, be held in the central part of the flame of a candle, the silver becomes heated throughout in a very short space of time, and cannot be retained in the hand; the heat is more slowly propagated through the glass; but the charcoal will become red hot at one extremity long before any heat is felt at the other. BIOT.

This difference is said to arise from the difference in the powers of these bodies for conducting heat; and we say that silver is a better conductor than glass, and glass than charcoal.

524. It may be laid down as a general rule, that those bodies that are densest, and have the least capacity for heat, are the best conductors; thus, the metals conduct better than any other solids; solids better than liquids; and liquids than gases. To this rule, however, there are some exceptions; one of the most remarkable of these is platina, which, although the densest of all known substances, is the worst conductor among the metals. DAVY.

525. Among solids, animal and vegetable substances are the worst conductors: thus, the hair and wool of quadrupeds, and the feathers of birds, are well fitted to retain the animal heat. They also enclose and confine air, which is a still worse conductor than themselves.

526. When a heated solid is plunged in a liquid, it is rapidly cooled, and hence it might be inferred that the liquid was a good conductor. This is by no means the case; the cooling of the solid is caused by the motion which the heat, abstracted from it, causes among the particles of the fluid. Those which are first

brought into contact with the solid, expand and rise to the surface; others take their place, and the motion continues until the solid and liquid have reached a common temperature. That liquids are bad conductors of heat, may be shown by a variety of experiments, when the heat is so applied as to produce no motion.

(1) If water be frozen in the bottom of a long and slender tube, another portion of water may be poured over it, and the ice will remain at bottom attached to the tube; the water may now be boiled by the application of the heat of a lamp to the upper portion of the tube, and, although it has its temperature so much raised, the ice will remain below unmelted. If ice be placed on the surface of water, contained in a similar tube, and heat applied from beneath, it will be almost instantly melted, and very long before any appearance of ebullition is manifested. HENRY, *Chemistry*, Vol. I.

(2) If a thin stratum of ether be poured on the surface of water contained in a glass vase, and set on fire, it will produce little or no effect on an air thermometer, whose bulb is sunk just beneath the surface of the water. If a series of air thermometers be inserted at different depths in the water, the last of which is in contact with the bottom of the vase, this last thermometer will be the first to show signs of an increase of temperature; this is owing to the heat being conducted to it through the glass. From this it was at one time inferred, that water would not conduct heat; Murray has, however, shown, by making an experiment in a vessel of solid ice, that liquids do conduct heat in a small degree.

527. Gases also cool bodies placed in them, by means of the currents that are thus produced; but they are themselves the very worst conductors.

For this reason, confined air is frequently made use of to prevent the escape of heat.

528. The effect that bodies produce on our organs of sensation, is proportioned to their power of conducting heat, as well as to their temperatures. This is evident, both when they are at a temperature higher and when at one lower than that of our

own bodies: Thus, the heat of metals at the temperature of 120° is insupportable; water scalds at 150°; while air may be heated to 240° without being painful, and may even be respired with safety. Mercury, when frozen, will take the skin off a hand that is applied to it, while, in high northern latitudes, a cold air has been experienced without injury, in which mercury froze. DAVY.

There is an application of this principle to the discovery of artificial gems. After metallic substances, the diamond is the best conductor of heat, and all the precious stones are better conductors than glass or paste.

When, therefore, a real gem is applied to the tongue, it will produce the sensation of cold; while a counterfeit, at an equal temperature, will produce no such effect.

PHYSICS.

SECTION SECOND—ELECTRICITY.

OF ELECTRICAL ATTRACTION AND REPULSION.

529. When certain natural bodies are subjected to friction, they acquire the property of attracting light substances. This phenomenon was first observed in the case of amber, and from its Greek name the general class of similar appearances are called *Electrical*.

In order to study the nature of these phenomena, tubes of glass, sticks of sealing wax, or sulphur of 3-4ths of an inch in diameter and a foot in length, should be rubbed with a dry woollen cloth, or the fur of a catskin. When this is done, light bodies will be drawn towards the body subjected to friction; some of them will remain sticking to it, while others will be repelled after having touched it. If the tube be brought near the face, a sensation, similar to that produced by the contact of a spider's web, will be felt. If the finger or a metallic knob be presented to the tube, a slight crackling sound is heard, which, if the experiment be performed in the dark, is attended with a luminous spark.

A body, when the power of producing electrical appearances is developed, is said to be excited.

We are entirely ignorant of the nature of the principle that is the ultimate cause of these phenomena; but, to avoid circumlocution, it is distinguished by the name of *Electricity*.

530. All vitreous and all resinous substances are capable of producing these phenomena when rubbed; so also is silk

There are, however, others, in which no such appearances are manifested. For this reason, bodies were long distinguished as either *Electrics* or *Non-electrics*. This distinction is not well founded, for if a metallic body be supported on glass feet, or suspended by silken threads, it may be excited; but its state of excitation will last no longer than while it is exempt from any communication; for if it be touched, it parts instantly with the whole of its electricity. Glass or resin, on the other hand, retains electricity after being touched, except in those parts that are nearest to the point of contact. BIOT.

From this it would appear, that the difference between bodies arises from the greater or less facility with which they transmit electricity. They are, in consequence, divided into the two classes of *Conductors* and *Non-conductors*.

Atmospheric air is evidently a non-conductor; for if it were not, no body could long retain electricity; but we find that an excited substance will retain its electrical virtue for a considerable time, although immersed in the air.

Water, on the other hand, is a conductor, and so is its vapour and steam; the aqueous vapour that is floating in the atmosphere will, therefore, alter the non-conducting properties of that fluid, and thus we find that electrical experiments succeed best in dry and cold weather, when the air contains but little moisture.

We know of no constant relation between the state of bodies and their conducting power. Among solid bodies, metals are good conductors, while gums and resins do not conduct at all. Liquids are almost all good conductors, with the exception of the oils, which are imperfect conductors. Wax and tallow, when cold, are not conductors; but, when melted, they conduct well. The power of conducting electricity, is found in bodies in the most opposite states; for instance, in the flame of alcohol and in ice. Temperature appears to have no effect, for sparks drawn from ice are not cold, and those from red hot iron do not burn. BIOT.

531. Atmospheric air and dry gases appear to possess, in addition to their insulating power, the faculty of retaining electricity on the surface of bodies by their pressure; for if a conducting body be electrified, insulated by placing it on non-conducting supports, and enclosed under the receiver of an air pump, when the air is exhausted it will lose all its electricity, which passes with a bluish light to such conducting bodies as have a communication with the ground. BIOT.

532. If a small ball of pith be suspended by a fine silken thread, it will be insulated; and will, from its lightness and conducting quality, be an excellent apparatus for investigating the phenomena of electrical attraction and repulsion. If this ball be touched by an excited electric, it will be found, after separation, to have acquired electrical properties; it will attract very light bodies; and is itself attracted by a finger presented to it. In a word, it is electrified by communication.

The ball will retain the communicated electricity for a considerable space of time, provided it be not touched, and the air is dry. If it be touched, it returns to its natural state, having lost all the electricity it had acquired. BIOT.

The manner in which this is effected may be seen by the following experiment: Instead of touching the pith ball with the finger, let it be brought in contact with another ball, suspended in like manner by a silken thread, but whose bulk is 80 or 100 times as great as the first: after the contact it will be found that the small ball has lost its electricity nearly as completely as if it had been touched by the finger. A given quantity of electricity is therefore diminished in intensity, by being distributed over a larger surface. In like manner the same small ball seems to part with its electricity when touched, because it is distributed between the ball and the whole mass of the earth, with which it is in communication by the intervention of the body of the experimenter.

It is for this reason that we consider the earth as the common reservoir of electricity. BIOT.

533. When the excited tube is first brought near the pith *ball, it flies towards* the tube, and attaches itself to its surface;

after a short space of time, sufficient to give it a proportion of the electricity of the tube, it flies off, and is repelled by the tube as long as it preserves its electricity.

This repulsion takes place, whatever be the nature of the electric, and is exerted on the ball by other excited tubes of the same substance. But if, after having communicated electricity to the pith ball by means of a glass tube excited by friction with a woollen cloth, an excited tube of resin, sulphur, or sealing-wax, be presented to the ball, it will be attracted instead of being repelled; and this attraction will be evidently more powerful than if it had not been electrified. The same will happen if the ball be first electrified by sealing-wax, and then presented to an excited tube of glass.

We are led by these experiments to distinguish two different kinds of electricity; one of these, being manifested by glass when rubbed with woollen, is called *vitreous*. The other, being produced by resin rubbed with the same substance, is called *resinous*.

534. If two pith balls, each suspended by a silken string, be electrified from the same excited substance, they will repel each other; but if one be electrified with vitreous, the other with resinous electricity, they will attract each other. These phenomena, taken in connexion with those already mentioned, may be expressed by this simple law, viz.—Bodies, when charged with electricity of the same nature, mutually repel; and when charged with electricities of different natures, attract each other. BIOT.

Although this furnishes a simple rule for expressing the facts, we are by no means to consider it as absolutely true. It is possible for similar appearances to be produced where there is neither attraction nor repulsion; and in this case the particles of the electrified bodies do not act upon each other, but the action takes place between the vitreous and resinous electricities that envelope them. When we speak, then, of attraction and repulsion, we are to be understood as employing these words merely as the most easy method of expressing the circumstances of the motions that are observed, and not as meaning by them to indicate the ultimate causes of the phenomena. *BIOT.*

535. Electrical attraction and repulsion are not only perceptible when air intervenes, but they are also felt through the other bodies that do not conduct electricity.

Thus, if an excited stick of sealing-wax be suspended in a glass matrass, it will attract bodies situated without the matrass, precisely as if the glass did not intervene. Biot.

536. The nature of the electricity acquired by any substance when rubbed in a certain manner, is ascertained by presenting it to a pith ball, charged with an electricity whose nature is known.

For instance, if the pith ball be touched by a glass tube rubbed with woollen, it acquires vitreous electricity. The substance whose electricity is to be examined, is then excited, and brought near the pith ball; if it be repelled, the friction of the substance has produced vitreous electricity, but if it be attracted, the electricity is resinous.

The proof may be inverted, giving the pith ball resinous electricity by means of sealing-wax. Biot.

537. When the electricity developed by a great number of substances has been thus examined, it will be found that its nature depends as much upon the body employed as a rubber as upon the body rubbed.

For instance, polished glass rubbed with woollen cloth acquires vitreous electricity, but when rubbed with a catskin the electricity is resinous. Silk rubbed with resins acquires resinous, but when rubbed with polished glass, vitreous electricity. Biot.

The following substances acquire vitreous electricity when rubbed by those which succeed them on the list, and resinous electricity when rubbed by those which precede them:

Catskin,
Polished Glass,
Woollen Stuffs,
Feathers,
Wood,
Paper,
Silk,
Gum Lac,
Ground Glass,

Biot.

538. From the above list, it is evident that there is no apparent natural relation between the nature and constitution of substances and the electricity they develope, when rubbed one by the other. Experiment gives but one general law applicable to the phenomena, which is, that the two bodies that are rubbed against each other always acquire electricities of different natures, the one being always vitreous when the other is resinous. BIOT.

539. Electricity is developed not only by the mutual friction of solid bodies, but by the friction of liquids and gases upon solid bodies; nor is friction the only mode in which it is produced; it is observed when bodies are compressed and when they expand; it is shown when certain bodies are melted by heat; several crystallized minerals become electric when merely heated, and in this case one of the extremities of the substance takes the vitreous, while the opposite takes resinous electricity; finally, electricity is developed in several chemical combinations, and by the mere contact of heterogeneous substances. BIOT.

540. The law, according to which the forces of electrical attraction and repulsion decrease with the increase of the distance, has been determined by means of a modification of the Balance of Torsion of Coulomb. Numerous experiments with this instrument give for their result, that the force of electrical attraction or repulsion is inversely as the square of the distance. BIOT.

541. The attractive or repulsive force varies at each distance, in the same ratio as the quantities of electricity contained in the two bodies that act upon each other. BIOT.

When the same electricity is distributed among a number of bodies of similar figures and equal magnitudes, the force of repulsion or attraction that each acquires is the same, and has no relation either to the nature or density of the body.

Hence it may be at once inferred, that bodies do not receive electricity *in consequence* of any chemical action that depends either on their nature or the arrangement of their particles, but are merely vessels in which it is received according to mechanical laws.

OF THE TWO ELECTRICITIES.

542. If a cylindrical conductor be taken, whose ends are rounded into the form of hemispheres, and which is supported by means of insulating substances in a horizontal position; then, if a number of linen threads be tied to it at short distances from each other, and small pith balls attached to their extremities, the following phenomena will be observed when an excited electric is brought near it, but not within such a distance as to permit a spark to pass:

(1) The threads at the two extremities of the cylinder will diverge, and thus show that it is electrified.

(2) The divergence becomes less and less as the threads approach the middle of the cylinder, near which a point is found at which no divergence is manifested.

(3) The position of this point varies according to the distance of the excited electric.

(4) If a pith ball that is not electrified be passed along the cylinder, it is attracted at every point except at that where no divergence is manifested.

(5) But if the pith ball be electrified, it is attracted by one end of the conductor and repelled by the other; thus showing that they are charged with electricities of different natures.

(6) If a small insulated conductor be applied in succession to the two extremities of the cylinder, and the nature of the electricity communicated to it at each contact examined, it will be found that the end nearest the excited electric is charged with electricity of a nature contrary to that the electric possesses, while that at the opposite end is of the same nature.

(7) The signs of electricity cease gradually as the conductor and excited electric are removed from each, or when the electricity of the *latter* is drawn off by the contact of another conductor.

(8) The excited electric transmits no part of its electricity to the cylinder, for it will be found to lose no more than is due to the slow dissipation through the air.

(9) If the excited electric be removed, and replaced again several times, without losing any part of its electricity, the same phenomena will cease, and recur each time, without any change whatsoever. Biot.

543. The following deductions have been made from this experiment:

(1) Since the cylinder takes no electricity from the excited electric, it must contain within itself the principles of the two electricities, and they are developed by the influence of the electric.

(2) Since these electricities disappear the moment this influence ceases, they must exist in such proportions as mutually to neutralize each other.

(3) This neutralization must be effected without any destruction of either principle, for they reappear with their primitive force each time that the cylinder is submitted to the influence of the electrified body. Biot.

544. In the natural state of bodies, they contain the principles of the two electricities in a state of combination that renders them neutral; friction, which, at the first glance, would seem to call these principles into existence, does no more than separate them. It is for this reason that we find the body rubbed, and the rubber, always manifesting electricities of contrary natures. And because the mere influence of an electrified body, presented at a distance, is sufficient to separate the two principles, and make them distribute themselves in such a manner that those of opposite natures shall be nearest to each other, we must, in order to account for this circumstance, admit, that the electric principles of the same name are mutually repulsive, and those of different names mutually attractive of each other. Biot.

This law once admitted, all the phenomena of electrical attraction and expulsion become simple, necessary, and evident consequences of it.

The phenomena of the two electricities have been explained by means of two different hypotheses, either of which might, at the first view, be thought sufficient to explain the phenomena.

545. Franklin supposes they are produced by the action of a single fluid, of which a certain quantity is distributed throughout all bodies, and forms their natural state. An excess of this fluid in some bodies, gives rise to that state which we have called vitreous, but which he called positive or plus electricity. A quantity less than that which constitutes the natural state, gives rise to resinous, or, in conformity with this hypothesis, negative or minus electricity. To account for there being no action when bodies are in their natural state, it was supposed that this fluid was attracted by their particles, but that its own were mutually repulsive of each other.

546. The other hypothesis conceives, that each of the two electrical principles is a perfect, incompressible fluid, whose particles mutually repel each other, and attract those of the other principle with forces that are inversely as the squares of the distances.

The question of the comparative merits of these two hypotheses, has been long contested with much warmth; some of the most respectable names have been numbered on each side. It appears, however, to have been lately decided in favour of the latter hypothesis, that of the two fluids; for when the aid of analytic investigation is called in, and the consequences that would follow mathematically from this hypothesis are deduced, they are found to agree in every respect with experiment; on the other hand, those deduced from the Franklinian hypothesis, have no analogy with the phenomena. These investigations are due to Poisson, who has, in them, exhausted all the means of modern analysis. His process is detailed at length by Biot, in his Traite Complet de Physique. Vol. II. p. 308. et seq.

547. When electricity is developed in a body or system of bodies by the influence of an excited electric or charged conductor, and where no spark passes, it is said to be caused by *Induction.*

OF ELECTRICAL APPARATUS.

The Electrical Machine.

548. The Electrical Machine is essentially composed of three parts :—the rubber, the electric, and the conductor.

The rubber must be of such a nature as to apply itself exactly to a considerable portion of the surface of the electric ; for this reason it is usually made in the form of a cushion, covered with pliable leather, stuffed with hair, and pressed against the electric by means of a spring. When leather alone is rubbed upon glass it developes but little electricity ; but its effect is much increased when it is covered by an amalgam of mercury, tin, and zinc.

If the electric be glass, it acquires by the friction vitreous electricity, and the cushions acquire resinous.

When the machine is in use for ordinary purposes, the resinous electricity of the rubber cannot be examined, for it is necessary that this part of the machine should communicate with the ground ; if it do not, the quantity of electricity developed is very small.

This is a necessary consequence of the manner in which the two electricities are developed by friction. Biot.

549. The electric, which is usually of glass, may be made in the form of a plate or of a cylinder ; very powerful machines have been constructed in both ways. It appears, however, that while the cylinder machine is better for general use, that formed of a plate is capable of producing the most powerful effects.

550. The prime conductor is usually in the form of a long cylinder of metal, with two hemispheric ends; attached to it are branches furnished with pointed wires that present themselves to the surface of the glass after excitation, and receive the electricity that is called into action on its surface.

Besides the prime conductor, others that communicate with it are frequently hung from the ceiling of the room.

551. There are several precautions that should be attended to in the construction of an electrical machine.

(1) Those parts of the glass which are rubbed should reach the wires that are attached to the prime conductor, before they lose the electricity they have acquired. For this reason, a piece of silk is attached to the rubber, which extends over the surface of the glass in the direction in which it is turned. As soon as the glass is electrified this silk adheres to its surface, and preserves it from the contact of the air.

(2) The prime conductor should have as many branches as there are rubbers, so that no portion of the glass shall return to the rubber before its electricity is discharged. In the plate machine there are usually two rubbers; in the cylinder machine, but one.

(3) The electric, the prime, and secondary conductors, should be insulated as perfectly as possible; for this purpose they are mounted on slender supports of glass, and these are frequently coated with gum lac; this substance is a more perfect insulator than glass, and is less liable to be affected by moisture. The rubber or the conductor may then be made to communicate at pleasure with the ground by means of a chain.

552. When the rubber communicates with the ground, and the conductor is insulated, the machine gives vitreous electricity. But both the plate and cylinder machine may be made to afford resinous electricity for the purposes of experiment. This is done, in the plate machine, by making the conductor moveable around an axis, in such a manner that the rubbers may be brought in contact with its branches, and the communication

between the rubbers and the ground is cut off. In the cylinder machine, a conductor of the same shape and size with the prime conductor is attached to the rubber; the prime conductor is made to communicate with the ground, and the communication between the rubber and the ground interrupted.

553. When an electrical machine is in good order, and the room in which it is placed is darkened, its use is attended with brilliant phenomena of light. Flashes appear to pass from the rubber over the surface of the electric to the prime conductor; brilliant stars are seen upon the points by which the conductor draws the electricity from the electric; if a point be placed upon the prime conductor, when the rubber communicates with the ground, it will exhibit a luminous appearance of a brush-like figure, that is called a pencil of rays; if the rubber be insulated, and the conductor communicate with the ground, a point placed on the rubber will exhibit a bright star; if the point be presented to the prime conductor when insulated, it will show the star; if it be presented to the rubber it will be attended with the pencil of rays. These two different phenomena are constantly attendant on the two electricities.

Points placed in this manner on the conductors of a machine, or presented to them when the machine is in action, have the property of drawing off the electricity silently, so that it is impossible to charge a conductor when a sharp point is either affixed or presented to it at a short distance.

Electroscopes.

554. Electroscopes, or, as they are more commonly called, Electrometers, are instruments intended to discover the presence of even the smallest quantity of electricity. The general principle of their construction is the repulsion that exists between bodies charged with electricity of a similar nature. The *balance of torsion* that has already been spoken of in § 540, is one

of the best of these. They are more usually formed of two long pieces of straw, two narrow strips of gold leaf, or two pith balls, suspended from a common support by conducting substances. A knob of metal is usually attached to the support; to prevent disturbance from the air or from external violence, the parallel pieces of straw, the slips of metallic leaf, or pith balls, are enclosed in a small glass phial, the support is firmly fixed to it, and the knob projects from the neck.

555. When the electroscope is used, it may be charged with electricity of either kind by a tube or rod, of a substance capable of producing by friction electricity of the nature that is required. Upon the principle of the developement of electricity by induction, a single electric may charge the electroscope with either positive or negative electricity at will.

If, for example, a stick of sealing-wax is used, rubbed with a bit of catskin, and presented at a distance to the knob of an electroscope, the knob will acquire vitreous, and the leaves of the instrument will diverge with resinous electricity; on removing the sealing-wax, the equilibrium is restored, and the leaves collapse. But if the knob of the electroscope be touched with the finger at the moment the sealing-wax is presented, the separation of the two electricities takes place as before, but the resinous passes into the ground through the body of the experimenter, the vitreous electricity is retained and attracted to the knob of the instrument, while the leaves being void of electricity do not diverge; if now the finger be first withdrawn, and then the sealing-wax, the leaves will diverge with vitreous electricity, for it cannot escape after the finger is withdrawn. BIOT.

The Condenser.

556. When a conducting body, insulated, and in the natural state, is brought in contact with an electrified conductor, it acquires a determinate charge of electricity; this may, however, be much increased by bringing near it at the same moment ano-

ther conducting body that has a free communication with the ground.

Upon this principle is constructed the condenser of electricity. It is usually composed of two metallic plates, separated from each other by an insulator in such a way as will prevent them from ever coming into contact. This is sometimes effected by a plate of glass; at others, by a piece of silk; at others, by a stratum of air; and at others, by a coat of varnish applied to each plate. If the charge is a large one, the condenser, whose plates are separated by a plate of glass, must be used; but if these are not well varnished, the greatest part of the electricity attaches itself to the glass, and thus the upper plate will no longer possess it when it is lifted up.

The condenser, when modified, assumes the shape of the Doubler of Bennett, and the Multiplier of Cavallo.

557. This instrument is used to great advantage in rendering small quantities of electricity evident. For this purpose it is frequently connected with an electroscope.

The Electrophorus.

558. When a body is charged with electricity and insulated, and another body communicating with the ground is presented to it, the latter will acquire a contrary electricity, and on being suddenly insulated, it will be found charged with that species of electricity.

This may be shown by charging the conductor of a machine with a certain quantity of electricity, and bringing near it a metallic disk, supported by a glass handle. If the disk be withdrawn without being touched, it will be found in the natural state; but, if it be touched for an instant while near the conductor, and then immediately withdrawn, it will be found charged with electricity of a nature contrary to that of the conductor.

If a metallic disk be taken, supported on an insulating stand, and charged by means of a spark, it may be used in the above experiment instead of the conductor of a machine, and it will retain the power of communicating electricity to the disk that is presented to it, until its whole charge has been dispersed by the air.

The separation of the two electricities in the body which is presented to that charged with electricity, becomes more complete the more near the bodies are brought to actual contact. This distance cannot, however, be diminished beyond certain limits, or else a spark will pass between the bodies. For this reason, a thin plate of a non-conducting substance must be interposed. BIOT.

559. This principle is applied to the construction of the electrophorus; thus, a metallic disk, similar to the lower plate of a condenser, may be insulated, covered with a plate of glass, and electrified by a spark. If another plate, with an insulating handle, be placed on the glass, touched, and withdrawn immediately, it will be found to be charged with the opposite electricity; and this experiment may be continually repeated for a considerable space of time. Electrophori may be constructed when the thickness of the insulating plate is insensible. To effect this, it is only necessary to form the lower plate of glass or resin, and electrify it by friction. These substances have so strong an attraction for electricity, that a metallic disk may be set upon their upper surface, without depriving them of any appreciable quantity of it; on the other hand, their influence in decomposing the two electricities in the metallic plate that is presented to them, is powerfully exerted. BIOT.

The electrophori that are most generally used are composed of a cake of resin, cast in a metallic ring; they are electrified by striking this plate with a dry catskin; they are then charged with resinous electricity, and communicate the vitreous to the upper plate.

This apparatus is frequently used in the laboratory instead of more expensive machines, and if set in a dry place it will retain its electricity for whole months. The electricity appears to be finally dissipated in consequence of the attraction between the two fluids, which overcomes the resistance that the resin opposes to the separation of its electricity.

The union of the two fluids, and consequent cessation of electrical appearances, may be more rapidly brought about, for if the edge of the metallic disk be applied to the plate of resin, instead of its flat surface, the electricity of each portion of the latter, with which it comes into contact, will be destroyed. After a few contacts, so arranged as to include the whole surface of the resin, it will be entirely discharged.

If the metallic disk be applied in a similar manner to a cake of resin in the natural state, it will charge it with vitreous electricity; and if a metallic plate be set on its surface, we shall have a second electrophorus, giving electricity of a nature contrary to that of the first.

The surface of the cake of resin may be only partially electrified, by affixing a knob to the metallic plate, and drawing it over the surface in those places it is wished to electrify. The path of the knob may be rendered visible by spreading over the surface of the cake of resin a light powder formed of a non-conducting substance; its particles will remain attached to the parts that are electrified, and arrange themselves in a variety of figures. See Biot, *Precis*, Vol. II.

The Leyden Phial.

560. When the plates of a condenser are separated by a sheet of glass, not varnished, the electricity will leave the plates and attach itself to the glass; in this way the opposite surfaces of the glass become charged with electricities of opposite natures, and the metallic plates have in fact no other office than to keep up a free communication between the different points of each surface of the glass. If a communication be made between the metallic plates, the two opposite electricities will instantly unite and neutralize each other, leaving the condenser in its natural state. In like manner, if a vessel, partly filled with water, be grasped in the hand, and a metallic conductor communicating with the prime conductor of an electric machine, be plunged into the fluid; and if, after a few turns of the handle of the machine, the other hand be brought in contact with the prime conductor, a sensation will be perceived that is caused by the pas-

sage of the electricities through the body; this sensation is called the electric shock, and will be violent in proportion to the size of the vessel, the power of the machine, and the time it has been in motion.

These two experiments are analogous; the latter was first in order of time, and was made at Leyden, by Cunæus and Muschenbroek. In pursuing their researches, they fell upon the construction of the instrument that we call, after the place of its discovery, the Leyden Phial.

561. The most convenient form of the Leyden Phial is a bottle or jar of common glass, on whose outer surface a thin coat of metallic foil is glued, and whose interior is covered in the same manner, or merely filled with strips of a similar foil.

562. The theory of this instrument is the same as that of the condenser. The electricity introduced into the inner part of the phial acts by induction on the natural electricities of the outer surface, and decomposes them. If the electricity introduced be vitreous, the outside of the jar is charged with resinous electricity; when the two sides are made to communicate, the two electricities that are there accumulated unite with great velocity, and rapidly traversing the human frame, produce the shock.

In order that the instrument shall act, the outer coating must communicate with the ground when the charge is introduced into its inside, and vice versa.

563. When electricity is communicated to different Leyden phials, the charge they receive is found to increase with the increase of the coated surface. For this reason, when we wish to accumulate great quantities of electricity, we connect together a number of large jars, making the inner coatings of the several jars communicate with each other by means of metallic wires, and placing their outer coatings in contact.

This apparatus is called an electrical battery; it is usually placed upon an insulating support, and has a communication with the ground by means of a conductor that may be removed at pleasure.

When a battery is composed of a number of bottles, it will take a great space of time to charge it in the usual manner. But any number of bottles whatever may be charged in the time it would take to charge one; for if one Leyden jar be insulated, and its inner surface connected with the conductor of a machine giving vitreous electricity, another jar may be charged from its outer coating; and if this second jar be insulated a third may be charged from the outer coating of the second, and so on for any number of jars, provided only that the outer coating of the last jar is connected with the ground.

564. In charging jars, and batteries composed of combinations of jars, it is important to know the state of their electricity at every instant of the process; for there is a point of accumulation when the intensity of the two opposite electricities will be so great as to overcome the resistance of the air, and cause a spontaneous discharge; and in the case of the battery this is frequently attended with the fracture of several of the jars. To enable us to stop the action of the machine before this explosion is likely to take place, we make use of the quadrant electrometer of Henley. This instrument consists of a wire moving around one of its extremities as a centre, and having a pith ball at the other end; it is placed in communication with one side of the jar or battery to be charged, and the state of its electricity is judged of by the repulsion of the pith ball from the body of the instrument. The divergence is measured in degrees marked upon a graduated circle.

This instrument does not give any absolute measure of electricity; but if it has once been used, and the point at which a spontaneous discharge is to be feared observed, it will enable us thereafter to avoid that danger. Biot.

565. In discharging phials and batteries, instruments called *dischargers* are employed. These have insulating handles of *glass, to preserve* the experimenter from the shock. Indeed, every

precaution must be taken when large batteries are used, to avoid being exposed to their explosion, the effects being sufficiently violent in many cases to cause loss of sensation, or even death.

566. Electrical batteries may also be constructed of plates of glass, interposing a leaf of metallic foil between each of them, and making communications between the alternate leaves. In this construction a very powerful combination may be enclosed in a small compass. It is mentioned by Franklin, (see the 3d vol. of his works,) but has since been constructed by Professor Dana of Harvard University. See SILLIMAN'S *Journal*, Vol. I.

Effects produced by the Electric Shock.

567. In the same manner that the electricity with which a Leyden jar or battery is charged, becomes sufficiently powerful to force its way through the interposing air, it may, when accumulated, acquire sufficient power to discharge itself through other substances more dense than air; in doing so, it separates their particles from each other, and will produce many remarkable effects.

(1) The discharge of a powerful battery will tear in pieces cylinders of wood that are interposed in the circuit; will kill living animals, whose flesh will putrefy as rapidly as those which are struck by lightning; and will break and pass through plates of polished glass.

(2) If it be transmitted through wires of iron, silver, or copper, it melts them into small globules; and if the charge be still more powerful, it burns and converts them into vapour, and will do so even to gold.

(3) Where these last experiments are made in close vessels, containing gaseous substances, but where oxygen is not present, the metals are merely melted.

(4) The electric discharge of even a small phial, will inflame a mixture of oxygen and hydrogen gas; if the proportions be one of the

former by bulk, to two of the latter, the whole mass will be converted into water.

(5) A similar discharge will also inflame phosphorus, ether, alcohol, gunpowder, &c.

(6) The electric discharge will not only form water by combining its elements, but it will separate water into its component parts. The best apparatus for this purpose is one contrived by Wollaston. By means of the same apparatus, solutions of some of the metallic salts may be decomposed, and the metal separated from the acid; the latter will be found arranged in a metallic state around the wire proceeding from the resinous side of the jar.

OF ATMOSPHERIC ELECTRICITY.

568. There are many phenomena that occur in the atmosphere, in the production of which electricity performs an important part; among these, lightning is the most remarkable.

Soon after the discovery of the Leyden Phial and Electric Batteries, the analogy between their effects and those of lightning was remarked. Franklin was the first who pointed out a method by which their identity might be ascertained. Being aware of the effect of points in drawing off electricity from a distance, he proposed to make use of a metallic rod, elevated against a lofty spire, and terminating in a sharp point; were lightning and electricity identical, he inferred that electrical appearances must manifest themselves at the foot of this during the occurrence of thunder gusts. Having no fit place within his reach for the performance of this experiment, he called the attention of the learned of Europe to the subject; and, in consequence of his representations, an apparatus was erected in France, by Dalibard, who, during a storm of thunder, received sparks from it, was enabled to charge a phial, and perform the other experiments of which an electrical machine is capable.

The method was afterwards improved by Canton, who contrived an apparatus to give warning of the appearance of electrical phenomena; he found some clouds charged with vitreous, others with resinous electricity, discovered that snow and rain, unaccompanied by lightning, electrified the apparatus, and that the phenomena occurred as well in winter as in summer.

Before the news of Dalibard's success reached Franklin, he had himself succeeded in drawing electricity from the clouds, by means of a kite. See FRANKLIN'S *Works*, Vol. III.

569. In consequence of the full proof thus afforded of the identity of lightning and electricity, Franklin conceived that the effects of the former might be prevented by furnishing buildings with pointed conductors; the points acting to draw off the electricity without explosion, and the conductor itself to convey it to the ground. This apparatus is called a lightning rod; in some countries of Europe, it goes by the name of its contriver.

A space of upwards of half a century has fully proved the great value of this instrument.

There are several conditions to be fulfilled, in order to render it efficacious.

(1) The point must be of a substance that will retain its sharpness, otherwise the discharge will not be a silent one. For this reason, it was formerly made of steel and gilt; but the best substance for the purpose is platinum.

(2) The communication with the ground, and between the several parts of the rod, must be perfect, for without this precaution, explosions would occur, and the lightning rod would be no longer a protection.

(3) The rods must not be less than an inch in diameter; if less than this, they might run the risk of being melted, like the metallic wires submitted to the experiments in § 567.

Cords, formed of metallic wires, are sometimes employed instead of rods; these have the advantage of readily adapting themselves to the figure of the building. To make the communication with the ground complete, the rod must be buried until it meet with moisture.

When bars of metal are numerous in the construction of a building, they should be united into a system of conductors; if this be done, they tend to add to the security. It is usual, when an edifice is large, to place lightning rods at intervals of 60 feet.

570. There is occasionally a lateral shock, when the conductor, through which an electrical battery is discharged, is not large enough to convey the electricity; the same will occasionally happen in the case of lightning rods; although this phenomenon is but of rare occurrence, it is for this reason unsafe to carry a lightning rod through a building that contains inflammable substances.

571. It sometimes happens, that animals are killed during a thunder storm, although the lightning is at the moment remote. This may be explained as one of the phenomena of electricity by induction; for if we imagine an electrified cloud to be hanging over the earth, with two portions near its extremities lower than the intermediate parts, this cloud will electrify the ground beneath by induction, which will assume electricity of a contrary nature to that of the cloud. When the spark passes in the form of lightning, the equilibrium will be restored, and this will be done so rapidly at the point beneath the other extremity of the cloud, as to cause a violent shock, sufficient in some cases to destroy animal life.

572. When very sensible electroscopes are exposed to the action of the atmosphere, it will rarely happen that they do not manifest signs of the presence of electricity. For this purpose, they are furnished with a long pointed rod, inserted into the knob of the instrument.

The intensity of the electricity of the atmosphere, is found to increase as we recede from the earth. This fact was first pointed out by Saussure; and his inference has been fully proved by Gay Lussac and Biot.

OF THE ELECTRICAL LIGHT.

573. The light which is produced by electric explosions, was long considered as a modification of the electric fluid, that was thought to have the property of becoming luminous when accumulated to a certain degree. Biot is, however, of opinion, that it is caused by the mechanical compression of the air, that takes place when the explosion occurs, in the same way that light is produced when air is compressed in a syringe.

It might be objected to this hypothesis, that the electric light may be shown in the vacuum both of the air pump and the barometer. But even these contain the vapour either of mercury or of water, whose compression will be sufficient to account for the light that is observed.

The appearance of the electrical light may be varied in a number of different ways, and thus give rise to many experiments of great beauty. As none of these have any immediate scientific application, we shall not detail them. The curious may consult Adams, Cavallo, and Singer, and the several articles on *Electricity* in Rees' *Cyclopedia*, and the *Encyclopedia Britannica.*

OF GALVANIC ELECTRICITY.

574. The phenomena of electricity are produced, not only by the means already explained, but may be developed in several other ways. Thus, the friction of liquid mercury on glass produces electricity, as does also a current of air blown from a bellows on a plate of glass; all changes in the state of bodies appears to be attended with like phenomena, as is manifested in the conversion of water into vapour, the heating, cooling, and congelation of resin and sulphur; finally, electricity is developed by the mere contact of conducting substances.

575. The development of electricity by the mere contact of different substances, was first observed by Galvani, a physician of Bologna, in Italy, in 1789. In making experiments on the

excitation produced in the muscles of animals by electricity, he perceived, that when the nerves and muscles were made to communicate by a metallic arc, they were convulsed and set in motion; he also found that this effect was very much increased when the arc was formed of two different metals.

Galvani supposed that these phenomena were produced in consequence of the nerves and muscles having electricity of the different kinds developed in them, and that the arc of metal acted merely as a discharger does when a communication is made between the opposite sides of a Leyden phial.

Volta, who repeated the experiments of Galvani, finding that the presence and contact of two metals was almost essential to the production of the phenomena, conceived that they were caused by this very contact. Pursuing the investigation of this subject, he ascertained that the contact of zinc with silver or copper, was capable of exciting more violent convulsions than the contact of any two other metals.

Volta having reached this point, was enabled to explain, by means of his theory, several phenomena of sensation, that had, until his time, been considered as anomalous. See Biot, *Precis*, Vol. II.

576. The truth of Volta's theory may be proved by taking two plates, the one of zinc, the other of copper, of the diameter of two inches, perfectly plane, unvarnished, and provided with insulating handles. If these plates be held by their insulating handles, brought into contact, and then separated, they will exhibit electrical phenomena. It may so happen, that these are not sufficiently powerful to affect the electrometer; in this case, the electrometer must be joined to a condenser, and the electricity of several contacts accumulated. If after seven or eight contacts, the upper plate of the condenser be withdrawn, the leaves or pith balls of the electrometer will diverge. Biot.

This experiment requires certain precautions; for which, see Biot, *Precis*, Vol. II. p. 619.

577. The result of similar experiments upon the other metals, and on many different substances, tends to show that the contact

of all heterogeneous bodies whatsoever, is attended with the developement of electricity. This is not, however, of equal intensity, but varies with the nature and properties of the two substances.

578. If two circular plates, the one of silver, the other of copper, be pressed firmly together, and if the other face of the zinc be covered with a moist conductor, that does not itself act to produce electricity in the zinc ; for instance, a circular piece of pasteboard, or cloth moistened in water ; every conducting body that is placed upon this system will receive vitreous electricity from the zinc surface. If then a second system of plates be laid upon the first, the copper surface being next to the moistened cloth ; this second system will in the first place, as a conducting body, partake of part of the vitreous electricity of the zinc plate ; and the second plate of zinc will acquire in addition a fresh quantity of vitreous electricity from its contact with the copper. If several similar systems be laid upon each other, they will constitute an apparatus in which the electric energy of the successive parts will increase with the number of pairs of plates.

An instrument, constructed upon this principle, is known by the name of the Galvanic or Voltaic Pile. To it natural philosophy and chemistry are indebted for many important discoveries.

Volta also contrived another instrument, called the *Couronne des Tasses*. In this a number of cups are filled with a liquid, and in each is placed a plate of zinc and a plate of copper ; these plates are connected together by wires, so that the zinc plate of the first cup may communicate with the copper of the second, and so on.

In the original apparatus of Volta, a number of circular disks of silver and zinc were arranged in pairs, with paper or cloth moistened in water interposed between each pair. A few alternations were found capable of affecting the human body, and of causing the leaves of an electrometer to diverge. When the electricity produced in this manner is examined, the electrometer in contact with the most oxidable metal is found to be charged with positive, the other with negative electricity.

579. The power of the Galvanic apparatus, to cause the leaves of the electrometer to diverge, increases with the number of alternations, and appears to be independent of the chemical action of the fluid with which they are moistened. Indeed, the several pairs of plates may be separated by dry paper, and still affect the electrometer, although not very powerfully.

This is manifested in De Luc's *Electrical-Column*, which consists of a number of alternations of silver leaf, zinc leaf, and thin paper. Piles of this description will retain their electrical action for a great length of time.

580. The power of the Galvanic pile to produce shocks on the human body, appears also to increase with the number of alternations.

581. The Galvanic apparatus was discovered by Carlisle and Nicholson to possess strong powers of chemical decomposition. It has since been very extensively applied to this object. The chemical powers of the apparatus are found to depend not only on the number of alternations, but upon the chemical action that the interposed liquid has upon the plates, and, in some small degree, on the size of the plates.

In the earlier experiments of Carlisle and Nicholson, the imperfect conductors, interposed between each pair of plates, were moistened with water; to this succeeded solutions of neutral salts, and finally dilute acids.

582. The application of these liquids to the pile, is by no means convenient; for this reason, various modifications of the original apparatus have been introduced.

One of these consists in soldering the plates of zinc and copper together, and cementing them into troughs of baked wood, so as to form cells to be filled by the fluid. In arranging this apparatus, each surface of zinc must be opposed to one of copper.

In another modification, plates of copper and zinc are connected together in pairs by a sheet of copper, and introduced into a trough formed of a non-conducting substance, that is divided into a number of cells; troughs of wood, with plates of glass cemented into them, were first used, but troughs of porcelain, formed into cells by partitions of the same material, have superseded them. Dr. Hare proposes to use cells of glass. Dr. Wollaston has discovered that the action of a Galvanic apparatus is much increased by extending the copper plate around the zinc, so that each surface of the latter may be opposed to one of the former.

583. Among the chemical effects produced by the Galvanic apparatus, the following are the most remarkable :

(1) If two platina wires be adapted to the two poles of a Galvanic apparatus, and both inserted into the same vessel of water, a continual current of oxygen will be found to rise from the wire that communicates with the vitreous pole, and one of hydrogen from the resinous.

(2) By different modifications of the apparatus, the two gases may be collected and measured; when this is done, the quantity of hydrogen is found to be double that of oxygen. This is exactly the proportion in which they unite to form water; they may be again united by exposing them to the spark of an electrical machine, or the discharge of a Leyden jar.

Water may also be decomposed by common electricity, in an apparatus invented for the purpose by Wollaston.

(3) The rapidity with which a certain number of plates decompose water is found to be much affected by the state of the water that is acted upon. Thus, if it be nearly pure, but little gas will be disengaged; if common pump water be used, the production of gas will be greater; if solutions of neutral salts be employed, the discharge is still farther increased; and mixtures of acid with the water are found to give out the gases in the greatest abundance.

(4) The quantity of gas disengaged in a given time is also affected by the greater or less strength of the dilute acid, with which the cells are filled. In the Battery of the Royal Institution, it was found that 120 plates acted upon by a mixture of one part of nitric acid to three of water, was as powerful as 480 acted upon by acid diluted by fifteen parts of water.

(5) When a more powerful Galvanic circuit is employed, the substances held in solution or combined with the water, as mentioned in (3) are themselves decomposed; thus, if a solution of acetate of lead be employed, the negative pole will be found covered with a number of needles of metallic lead. This was first observed by Cruikshanks, while repeating the preceding experiments. He obtained similar results with solutions of sulphate of copper and nitrate of silver.

(6) In pursuing this subject, Cruikshanks found, that when neutral salts were contained in the water, and the electric circuit transmitted through infusions of vegetable blue, that the separation of the acid from the alkaline base, and their collection at the two poles, was marked by a change of the colour.

(7) This effect was found to be produced even when distilled water was acted upon, and hence it was supposed that the Galvanic action actually formed the acid at one pole and the alkali at the other. By making use of the greatest precautions, Sir Humphrey Davy showed, that this was not the fact, but that the alkali was furnished by the decomposition of the glass vessel, while the acid is formed by a union of the oxygen that is let loose with the azot of the atmosphere.

(8) When separate vessels are used containing a solution of a compound substance, and the communication between them is affected by animal fibre or moistened amranthus, the substance will be separated into its component parts, and each of them will be found in a cup by itself. In this transfer, acids will be carried towards the vitreous, alkalies towards the resinous pole. This discovery was made by Hesenger and Berzelius.

(9) When the caustic alkalies are submitted to the action of a powerful Galvanic battery, they are decomposed; a metallic substance being found at the resinous pole, and oxygen given out at the vitreous. This very remarkable fact was first observed by Sir H. Davy. BIOT, *Precis*, Vol. II.

See also BOSTOCK on *Galvanism*; DAVY, *Bakerian Lecture*; DAVY, *Elements of Chemical Philosophy.*

584. In consequence of the union of electrical and chemical action in the Galvanic pile, and the fact that electrical effects are exhibited by the same bodies when acting in masses, which produce chemical phenomena when acting by their particles, Davy was led to infer, that the primary cause of both might be the same; and that the same powers which place bodies in the relations of positive and negative, or vitreous and resinous electricity, may render their particles attractive of each other, and enable them to combine when they have full freedom of motion.

585. When the communication between the two poles of a Galvanic apparatus is made by metallic wires, if the circuit be weak, and they be brought gently together, they will be found to adhere strongly; if the wires be of iron a spark is perceived, and this will be sufficient to set fire to inflammable substances. If the energy of the apparatus be increased, the wires are themselves inflamed.

It is found that this species of action follows a very different law from that which produces electrical, or that which causes the chemical phenomena. The power of deflagrating appears to depend rather upon the size of the plates than upon their number; still, however, a certain number of plates is absolutely necessary to produce any very striking effects, as is evident from the experiments of Professor Hare, of Philadelphia.

The first powerful apparatus for deflagration, was constructed by Mr. Children; each of his plates had a surface of sixteen square feet.

Apparatus of still greater power has been since constructed by Professor Hare; he calls his instrument the Deflagrator; the experiments performed with it are of the most striking and splendid kind.

The best form of this apparatus is described in the 11th number of Silliman's Journal.

586. Polar electrical arrangements, similar to those formed of zinc, copper, and an imperfect conductor, may be made by various alternations of conductors and imperfect conductors;

the series must consist of three substances or more, and one at least must be a conductor.

> Thus, silver or copper, when brought into contact with a solution of sulphuret of potassa on one side, and with water or a solution of nitric acid on the other, some saline solution being interposed between the two liquid substances, forms an element of a powerful Galvanic combination. DAVY, p. 82.

587. There are certain animals that are furnished by nature with organs capable of exciting electrical phenomena. These organs appear to be formed of arrangements of exciting bodies, similar to those employed in Galvanic electricity. The shock of the torpedo and gymnotus electricus, resembles the Voltaic shock; and the power resides in organs which consist of a number of similar alternations of different substances. It has also been imagined, that other phenomena of vital action may be connected with the operation of electrical powers. Some ingenious hints on this subject have been suggested by Wollaston and Home, and experiments relating to the subject instituted by Mr. Brande. DAVY.

> Such inquiries must not be confounded with certain vague hypotheses that have been advanced by some authors, on the general dependance of nervous and muscular action on electricity. DAVY.

PHYSICS.

SECTION THIRD—MAGNETISM.

GENERAL PROPERTIES OF THE MAGNET.

588. There is a class of the ores of iron, where the metal exists in the state of protoxide, that possesses the property of attracting pieces of the purified metal. This force of attraction is sometimes so powerful as to raise considerable weights, and the ore is then called a *Magnet.*

The most simple method of exhibiting this property is, to roll the magnet in iron filings; when it is withdrawn, a portion of them will remain attached to its surface. Upon examination, it will be found that the iron filings are not equally distributed over the whole surface, but are most accumulated at two parts that are on opposite sides of the magnet; at these places, the filings stick together, and are arranged in a number of parallel threads. Biot, *Precis,* Vol. II.

The two opposite sides of the magnet, when this appearance occurs, are called its *poles.*

Each pole, when presented to filings of iron, attracts them from a distance, precisely as excited electrics attract light bodies.

589. That this attraction is not electric, is proved by its acting with equal force through substances that are conductors, and those that do not conduct electricity; neither water, glass, flame, nor paper, interrupt its action; insulation is not necessary, nor does the magnet lose its virtue when touched. Biot.

590. We are ignorant of the ultimate cause of these phenomena, but in order to avoid circumlocution, we call it by the

name of ***Magnetism***, and employ it hypothetically, in the same way that we have done the terms ***Heat*** and ***Electricity***.

591. If one of the poles of a magnet be brought near to the two poles of another magnet in succession, it will be found to attract one of them and repel the other. Its opposite pole will, on being presented to the poles of the other magnet, attract that which the first pole has repelled, and repel that which it attracted.

Hence it may be seen that the forces exerted by the two poles are not the same, for one attracts what the other repels. This is analogous to the phenomena of the two electricities, and may be explained by supposing the existence of two magnetic principles, precisely as we assume the existence of the two principles of vitreous and resinous electricity.

592. A piece of iron placed in contact with a magnet becomes itself magnetic, exactly as an insulated body becomes electric when placed near an excited electric.

This may easily be shown by using bars of soft iron; these readily acquire magnetic properties by mere contact; they however part with them almost as speedily as they receive them.

Bars of steel may be also magnetized by contact, but with greater difficulty; when they have acquired magnetism they retain it much longer than soft iron. Magnetism may be communicated to a bar of steel sooner and more powerfully by means of two magnets than by one; its extremities are to be placed at the same time in contact with those poles of the two magnets that attract each other. In this way the same two magnets will communicate magnetism to a number of bars in succession, without losing any portion of their own power. BIOT.

593. If a steel bar or needle, in which magnetism has been communicated, be suspended horizontally by a thread of unspun silk, or set to float upon water on a small piece of wood, it will remain at rest in one determinate direction only; this, in *the United* States, deviates but a few degrees from the true meri-

dian. In Europe it is at this moment a direction that differs but little from N. E. and S. W.; and it differs in different places of the earth, being in some exactly in the meridian, inclining in others towards the West, and in others again towards the East.

This deviation from the meridian is called the *Magnetic Declination;* the vertical plane passing through the poles of the magnet, the *Magnetic Meridian.*

594. The declination of the magnet is not constant at the same place, but is found to have both a diurnal and periodic variation.

595. When a magnetic needle is suspended freely by its centre of gravity, it will be found not only to place itself in the magnetic meridian, but will assume a position inclined to the horizon. In northern latitudes, the northern pole of the magnet will incline beneath the horizontal plane, and the other will be raised above it. This angle of inclination varies but little at the same place, and is called the dip of the needle. There is a zone not far from the equator, where the needle hangs in a horizontal position; to the south of this, the northern pole will be elevated and the southern will dip.

596. If a bar of soft iron be placed in the direction that would be assumed by the dipping needle, it immediately acquires magnetic properties; these are transient, remaining no longer than the bar retains its position, and its poles may be changed in an instant, by reversing the position of its extremities. When a bar has been long fixed in the magnetic meridian, like those crosses which sometimes surmount the spires of churches, the magnetism acquired will be permanent.

In these instances, the magnetism is received from the earth, which, in fact, acts upon ferrugineous bodies precisely as if it were a great magnet. It has, like a magnet, poles, but these do not coincide with the poles that form the extremities of the earth's axis of rotation.

The north magnetic pole of the earth has magnetism of the same quality as that end of the needle that points to the south, and vice versa.

597. Iron and its compounds were long thought to be the only bodies capable of exhibiting magnetic phenomena; but of late years nickel and cobalt have been found to possess the same property.

598. A red heat is found to destroy the magnetism of a bar or needle, and it does not entirely recover it on cooling; if cooled in a direction perpendicular to the magnetic meridian, it will show no magnetic properties whatever; but in any other position it will recover some of its virtue, and most when it is cooled in the direction of the magnetic meridian. BIOT.

599. The analogy of the phenomena of magnetism to some of those of electricity has been already remarked; this analogy is most striking when the properties of the magnet are compared with those of the tourmaline and the electric pile. BIOT.

OF THE COMPASS.

600. The compass is an instrument wherein a magnetised needle is employed to ascertain the direction of objects from the point in which it is placed. It is more especially useful in pointing out the course of a vessel when at sea, and out of sight of land; without this valuable invention, it would have been next to impossible to have traversed the great oceans, that form so large a portion of the surface of the globe.

The mariner's compass has a magnetic needle, supported in a horizontal position on a pivot; the needle is for this purpose fitted with a counterpoise that destroys the action of the dip. It is affixed to a circular card in the direction of its diameter; the card is graduated on its outer circumference into 360 degrees, and upon an inner circle is divided into 32 equal parts, called *points of the compass;* by means of these divisions the bearing is ascertained. The card is enclosed in a circular box that is suspended by means of two concentric rings from two axes at right angles to each other. By this contrivance, the needle is maintained in its horizontal position, even when exposed to the violent motions of the sea when agitated by storms.

Magnetic needles are also used by land surveyors, attached to various instruments, such as the Circumferentor, the Graphomentor, and the Theodolite.

MODES OF DETERMINING THE DIRECTION AND INTENSITY OF THE MAGNETIC FORCE.

601. The several circumstances which it is necessary to know in relation to the force of magnetism at any point on the surface of the earth are :

(1) The declination or angle which the magnetic makes with the true meridian.

(2) The dip or inclination of the needle to the horizon.

(3) The intensity of the magnetic force.

(4) The variations in the declination and dip, if any.

602. The declination is observed by means of an instrument called an Azimuth Compass. This enables us to ascertain the angle of the horizon intercepted between the magnetic meridian and a vertical circle passing through the sun or some other heavenly body. If the angle intercepted between the true meridian and the same vertical circle be calculated, the difference will be the magnetic declination.

The best construction of the azimuth compass, appears to be that by Captain Kater; in this, an observer sees at the same moment a fine thread that is made to coincide with the position of the body whose place is observed, and the reflected image of a point on a graduated circle that marks the direction of the visual ray in relation to the magnetic meridian.

Another azimuth compass is described by Biot, as constructed by a French artist, of the name of Gambey.

603. The dip of the needle is observed by means of the *Dipping Needle*. In the old construction of this instrument, the needle was suspended by a horizontal axis passing through its centre of gravity, and free to move only in a vertical plane. Around the point of suspension, was described in this vertical plane a graduated circle. The dipping needle has been lately much improved by Captain Sabine, who suspends the needle from a point above its centre of gravity; it is thus acted upon by the combined forces of gravity and magnetism. How much is due to each of them, is ascertained by means of a formula of Professor Mayer, originally published in the Gottingen Transactions.

To use this instrument, it must first be placed truly in the magnetic meridian, and accurately levelled. The arcs of the graduated circle, corresponding to the two extremities of the needle, are then to be noted, and the mean taken for the proximate dip. To diminish the chances of error, the instrument should then be turned in azimuth, until the face which faced the east in the former observation, faces the west; the same observation is then to be repeated, and the mean of the two taken as the true result. In Captain Sabine's improvement, the result thus obtained is to be corrected by the formula. See PHILOSOPHICAL TRANSACTIONS, for 1822.

604. The intensity of the force of magnetism, is deduced, from the number of seconds in which a needle performs a certain number of vibrations in a horizontal plane, and the dip at the same place.

If R be the intensity of magnetism at a place where the number of vibrations in a given time is N, and the complement of the dip Z; r, the intensity at a place where the vibrations in an equal space of time is n, and the complement of the dip z, may be found by the formula:

$$r = R \frac{N^2 \sin. Z}{n^2 \sin. z}$$

BIOT.

To avoid error, the needle must be suspended in such a way that the torsion of the string shall have no effect upon the number of vibrations.

605. If there be any diurnal variation in the dip of the needle, no instrument has yet been contrived by means of which it may be observed. To ascertain the diurnal variation of the declination, a very delicate and beautiful instrument has been contrived by Captain Sabine; the needle is suspended by a thread of unspun silk, and the divisions of the instrument are read to great minuteness by means of microscopes, similar to those used by Troughton, in his Astronomic Circles.

606. The direction of the needle, both as regards its dip and declination, is very liable to be affected by local causes; thus, the presence of iron, even in small quantities, will affect the result of nice and delicate experiments. This is very remarkable on shipboard, when the quantity of iron present has such an effect that the absolute direction of the needle is changed at each change of the course of the vessel.

This last circumstance was first observed by Captain Flinders. See Biot, *Precis Elementaire*, Vol. II.

OF THE ACTION OF THE MAGNET ON ALL MATERIAL SUBSTANCES.

607. It has been said, in § 597, that iron, nickel, and cobalt are the only magnetic substances with which we are at present acquainted. They are, in fact, the only ones to which a permanent state of magnetism can be communicated.. It has, however, been lately discovered by Coulomb, that if very fine needles of any substance whatsoever be taken and suspended between the opposite poles of two very powerful magnets, and if they be made to oscillate, their oscillations will be more frequent than if the magnets were not present.

This remarkable fact is as yet without any satisfactory explanation. That the increased frequency of the oscillations is caused by the presence of the magnets is evident, but whether their action is in this case merely magnetic, or whether some other cause of motion is called into action, is not yet determined. BIOT.

OF THE LAWS OF TERRESTRIAL MAGNETISM.

On this interesting subject many new and important facts that will materially affect former theories and inferences, may be hourly expected from the researches of Captain Sabine. Until these shall be made public, we must content ourselves with following the track pointed out by Biot.

608. To observe the circumstances of terrestrial magnetism, the instruments that have been described should be carried to different places on the earth's surface. Previous to doing this, great care should be taken, if the same instruments cannot be used in succession at the several places, to compare those intended for any place with the instruments that are to be employed in making similar observations at other places. A want of this precaution may render the observations worse than useless.

Experiments of this sort have been performed in many parts of the globe; their results may be stated as follows, viz.

609. There is a line (called the magnetic equator) on the surface of the earth, where the needle has no dip; it was formerly, from partial observations, supposed to be a great circle of the terrestrial sphere, cutting the equator at angles of 12° in the longitudes of 117° east, and 63° west from Greenwich. More recent discoveries have shown, that the magnetic equator is not a great circle of the sphere. It is found, by actual observation, to cut the equator first in longitude 18° east; thence, proceeding towards the west, it passes to the south of the equator until it reach the longitude of 26° west, where it is distant from it 14° of south latitude; from this point it continues for some time parallel to the equator; it afterwards inclines again towards the equator, and in the longitude of 96° west, at the dis-

tance of 300 miles west from the Gallipagos islands, becomes nearly parallel to it, and afterwards inclining towards the equator, touches it in the longitude of 118° west; after this the direction of the magnetic equator again bends towards the south until it attain a second limit of 3° 15′ southern latitude, in the longitude of 16° west; thence, it inclines to the north, and cuts the terrestrial equator in the longitude of 184° west, or 176° east, and passes to the north, attaining the limit of 9° of north latitude on the meridian of the Philippine islands; it now inclines again to the south to the longitude of 108° east, where its latitude is no more than 7° 44′; after this it again bends to the north, and attains its maximum north latitude, (11° 47′ north,) in the longitude of 64° east; hence it passes to the south until it reaches the point at whence we began to trace it. BIOT, *Precis*, Vol. II.

Observations of the dip show that it increases in proportion as the place is more removed from this magnetic equator. If we confine our views to that portion of the globe that includes Europe, the Atlantic ocean, and the eastern shores of America, the magnetic equator is not far from being a portion of a great circle. The poles of this circle would be in latitude 76° north, longitude 23° west, and in latitude 76° north, longitude 207° west. These points have, consequently, been considered as the magnetic poles of the earth, and here the dipping needle should assume a vertical position. But this is not confirmed by the observations of Captain Parry.

610. Were the magnetic equator a great circle, the tangent of the dip would be double the tangent of the magnetic latitude. BIOT.

Accurate observations on the dip show, that it does not remain constant at the same place. By the observations of Gilpin, in 1775, it was 72° 30′ at London; in 1805, by the observations of Cavendish, it was 70° 21′; and in 1822, by the observations of Sabine, it was 70° 02′ 91″.

611. There are lines on the surface of the globe in which

the needle has no declination. These lines are not in the same direction as the meridians, but are oblique to them, and by no means regular. There is, at present, a line of no declination in the Atlantic ocean; it cuts the meridian of Paris in latitude 65° south; thence extends in a north-west direction to longitude 35° west from Greenwich, after which its direction becomes north and south to the latitude of Cayenne, when it suddenly turns to the north-west, and is directed to the continent of North America, which it crosses in the same direction.

The position of this line is not fixed. In 1657 it passed through London, in 1664 through Paris, and has since been regularly passing towards the west. Whether this be a change that is continuous, or whether the motion of this line be reciprocating, is not yet ascertained.

There are three other lines of no declination on the globe. One of them is very nearly opposite to that already mentioned; it is first remarked in the southern ocean, cuts the western point of New Holland, passes through the Indian ocean, and enters on the continent of Asia at Cape Cormorin, thence it extends through Persia and western Siberia to Lapland. Another is a branch of this, leaving it in the Indian Archipelago, and passing nearly due north through China, and the eastern portion of Siberia. No change has been hitherto observed in the position of these lines. The last line of no declination was remarked by Cook in the southern ocean. BIOT.

612. The intensity of the force of magnetism, measured by a comparison between the dip and the oscillations of a horizontal needle, has been found to be least at the magnetic equator, and to increase on each side of that line. The same needle that made 245 vibrations at Paris, was found by Humboldt to make no more than 211 in Peru. The action of magnetism has been found, in ærostatic ascensions, to exert its influence in the air, as well as upon or beneath the surface. If this action be extended to the heavenly bodies, their magnetic properties would be shown by a change both diurnal and annual, in the direction of the needle. Both these effects have been actually found to exist,

and more lately a periodic change has been remarked in the intensity of magnetism. BIOT.

The needle is not only subject to periodic changes, but has been observed to be affected by sudden variations; these have been found to coincide with the appearance of the Aurora Borealis. BIOT.

OF THE RELATIONS OF MAGNETISM AND ELECTRICITY.

613. When an electrical current is transmitted through any metallic body whatsoever, it acquires magnetic properties, and becomes capable of attracting pieces of soft iron that are not magnetised. If a magnetised needle be presented to the metal through which the current is passing, one of its poles will be attracted, and the other repelled.

Metals, when thus electrified, act only upon iron and steel needles; those of other substances are neither attracted nor repelled. These phenomena were originally discovered by Professor Oersted, of Copenhagen; they were found by him to be temporary, and to cease with the electrical action. Sir H. Davy, by placing a needle at right angles to the magnetic meridian, succeeded in giving it permanent electricity, by means of a powerful Galvanic apparatus.

Since the discovery of Oersted, many philosophers have turned their attention to this interesting subject, but, with the exception of Oersted and Davy, they appear to have done but little more than contrive apparatus by which the experiments can be performed with greater ease. See BIOT, *Precis*, Vol. II., edition of 1821; SILLIMAN'S JOURNAL; JOURNAL OF SCIENCE, Vols. X, XI, XII, and XIII.

614. An intense electricity has also the power of depriving the needle of its magnetism. This is frequently observed at sea, where lightning is known to destroy the polarity, and even

reverse the poles of the mariner's compass. The shock of a powerful electrical battery, passed through a magnetised needle, will produce the same effects.

PHYSICS.

SECTION FOURTH—OPTICS.

OF LIGHT.

615. When the sun is above the horizon, a mode of communication is evident, that, in spite of his great distance, acquaints us with his existence. This mode of communication is called *Light*. Biot, *Traite de Physique*, Vol. III.

> The bodies which excite this sensation, and thus inform us of their existence, are called *Luminous*. Generally speaking, all bodies whatsoever become luminous when their temperature is sufficiently elevated, and lose this property upon cooling.
>
> Bodies, when not luminous, are said to be opaque, and even these may, when enlightened by a luminous body, acquire the power of transmitting light in the same way as if they were themselves luminous. They are then said to be visible by *Reflection*. Biot.

616. Light is transmitted from the luminous body in right lined directions.

> The straight lines drawn from a luminous body to the eye, are called *Rays of Light*, and we conceive that vision is effected by means of these rays.

617. The transmission of light from the luminous body, is not instantaneous, but requires a certain portion of time. It is, however, performed with great rapidity; so much so, as to render the interval of its passage between bodies, placed upon the surface of the earth, absolutely insensible.

> That light is not transmitted instantaneously, is shown by means of

astronomical observations; and particularly by the eclipses of Jupiter's satellites, and the aberration of the fixed stars. It is found, that when a satellite of Jupiter is eclipsed, or enters into the shadow of the planet, we still continue to see it for a short space of time; so that when it seems to us to disappear, it has actually been in the shadow of the planet for some time. Thus, the communication that results from the presence of a luminous body in any place, continues to be transmitted even after it has quitted it.

618. There are two different methods of accounting for these phenomena. Some suppose that light is transmitted by means of pulsations excited in an elastic fluid, in the same manner that sound is transmitted through the air; others are of opinion, that luminous bodies throw off emanations of particles of matter with great velocity.

In either case the sensation is transmitted through the mass of certain bodies that are called *Transparent.* It is, therefore, necessary that the pulsations of the elastic medium should be propagated through the pores of these substances, or that the luminous particles should be able to penetrate, and even pass through them. Biot.

The opinion which conceives that the sensation of light is produced by an actual emanation of material particles from the luminous body, appears to be most consistent with the phenomena. Of this we shall meet with continual proofs, and shall, therefore, adopt the language of this hypothesis, which is that of Newton.

619. When we look at an object whose magnitude is sensible, the rays that proceed from its opposite extremities reach the eye in different directions, and thus cross one another, forming an angle at the point of their incidence on the front of the pupil.

This angle is called the *visual angle,* or the apparent diameter of the object.

620. The rays which constitute a pencil of light are sometimes *parallel,* sometimes divergent, and sometimes convergent. The

point where the directions of two or more rays of light intersect each other, is called their focus, and this is either actual or virtual, accordingly as they either meet in it, or only tend to or from it. Young, § xxxv.

621. Where light is proceeding from a luminous object, it reaches us, after passing through different media, such as air, water, or other transparent bodies. The rays of light, upon entering these bodies, continue sometimes to proceed in the straight line in which they originally set out, but more frequently they are bent from this direction.

When this takes place, we call the phenomenon *Refraction.*

622. When the rays of light fall upon polished surfaces, they are thrown off, or *reflected;* and we may thus see objects indirectly, if we are placed in the course that is pursued by the reflected rays.

The laws that govern the phenomena of reflection are more simple than those of refraction. We shall therefore consider them in the first instance.

They constitute a division of the subject that is known by the name of *Catoptrics.*

OF CATOPTRICS.

623. In order that a body shall reflect light regularly, it is necessary that its surface be polished.

624. When a ray of solar light, admitted through the window-shutter of a darkened room, falls obliquely upon the surface of a piece of polished glass, placed horizontally, the following phenomena occur.

(1) The ray is not entirely transmitted; a part is reflected upwards, in a direction determined by the obliquity of the incidence. If the

eye be placed in the ray thus reflected, it will see a bright image of the sun, that will appear to be on the lower side of the plate of glass.

(2) The point where the ray falls upon the polished plate, is visible not only in the direction of the reflected ray, but from every part of the room. It however appears incomparably less brilliant when thus viewed; the direction, too, of the reflected ray is the only one that gives a regular image of the sun, or of any body that may be viewed by reflection from the plate of glass.

(3) A part of the incident light is not reflected from the upper surface, but penetrates into the interior; when it reaches the lower surface a portion is a second time reflected, while the remainder passes out into the air beneath the glass.

Hence we perceive that three distinct operations are performed; a part of the incident light is reflected in a determinate direction; another part is reflected in every possible direction, precisely as if the body were not polished; and the residue passes into the medium without being reflected.

Biot distinguishes these two species of reflection, by calling the first *Specular*, the second *Radiating* reflection.

If a metallic body with a polished surface be substituted for the plate of glass, the same phenomena of reflection still occur, but no light will be transmitted. Polished metal therefore reflects in a specular manner one portion of the incident light, disperses by radiation another portion, and absorbs the remainder. Biot.

625. When these experiments are repeated with different reflecting bodies, and with different degrees of polish, it is found that the irregular reflection is owing to the imperfection of the polish; for the less perfect the polish the greater is the quantity of light dispersed by radiation, and the less the specular reflection. Biot.

The ratio between the angles made by the incident ray with a normal to the point of incidence, and that made by the portion of the ray that undergoes specular reflection, may be examined by means of an instrument contrived for the purpose; and which is described by *Biot*.

626. The incident ray, and that caused by specular reflection, to which the name ***Reflected Ray*** is restricted, are both contained in the same plane perpendicular to the reflecting surface; and the angle of reflection is equal to the angle of incidence.

Of the Plane Mirror.

627. When we view the image of an object by reflection from a plane mirror, it appears to us as if it were in the perpendicular, let fall from the object upon the surface of the mirror produced, and as far behind the mirror as the object is distant from its surface; but it will appear with its right and left sides reversed.

If then the mirror were removed, and an object similar to the original one, but with its sides reversed, were placed in the position just pointed out, it would appear precisely like the image, subtending the same angle, having the same magnitude and the same degree of clearness. Biot, *Precis*, Vol. II.

228. Plane mirrors may be constructed either of polished metallic plates, or of glass covered with a metallic coating or silvering. In the former case, the reflection takes place from the front of the mirror; in the latter, from the metallic coating.

A reflection also takes place from the upper surface of a glass mirror, and the ray reflected from the silvering is again partially reflected within the substance of the glass; when it reaches the silvered surface it is reflected a second time towards the front of the mirror, a portion passes out, and another portion is a second time reflected through the substance of the glass, and so on.

It thus happens, that if a bright object be viewed by a very oblique reflection in a glass mirror, the observer will see not one image but several. The second of these will be the brightest, being formed by reflection from the metallic coating.

629. If two plane mirrors be placed parallel to each other, with an object between them, the rays of light that issue from the object and fall upon each mirror will be reflected backwards

and forwards from one to the other an infinite number of times. Thus, there will appear in each mirror a great number of images, situated one behind the other, in a straight line drawn through the object and perpendicular to the surfaces of the mirrors. HELSHAM, § xxiii.

This property of plane mirrors may be applied to the purpose of embellishing apartments and making them appear of boundless extent, and to a variety of amusing contrivances.

630. If two plane mirrors be inclined to each other in such a manner as to meet in an acute angle, and if an object be placed between them, an eye looking upon the two mirrors will see a number of images of the object arranged in the circumference of a circle, whose radius is the distance of the object from the intersection of the mirrors. HELSHAM, § xxiii.

This principle is also applied to the purposes of embellishment and amusement; among the contrivances for the latter purpose, the most remarkable is the Kaleidoscope of Brewster.

Of Mirrors with Curved Surfaces.

631. In order to investigate the apparent place, the form, and the magnitude of images formed by reflection from curved surfaces, of any figure whatsoever, we have only to conceive the reflection of each ray as effected by a plane that touches the curve at the point of incidence.

It is unnecessary to solve this proposition in a general manner, for in practical optics no curved mirrors are employed with surfaces other than spherical. Spherical mirrors may be either concave or convex. They are used in consequence of their being the only ones that can be brought to a true figure, and accurately polished.

Spherical mirrors do not form a distinct image, except when the rays fall nearly perpendicularly on their surface. In the investigation of their properties it is, therefore, usual to pre-suppose two conditions; (1) that the surface of the mirror contains but a small number of degrees of the sphere of which it is a portion; and (2) that the incident ray makes but a small angle with the central normal, or axis of the mirror.

632. In a concave spherical mirror, if d represent the distance of the focus of incident rays, and f that of the focus of reflected rays from the vertex of the mirror, and if r represent the radius of the mirror,

$$f = \frac{d\,r}{2\,d - r}$$

Hence, if the object be placed at an infinite distance, the image will be formed in front of the mirror at the distance of half its radius; and thus the rays of the sun, if reflected from a concave spherical mirror, will converge to this point, and form an image there that is so intensely heated as to inflame combustible bodies. This point, to which parallel rays converge, is called the *Principal Focus.*

As the object approaches the mirror, the image recedes, and when the object reaches the distance of the radius of the mirror, the image and object coincide in place. When the object is brought still nearer to the mirror, the image continues to recede until the object come to the principal focus, when the reflected rays proceed in parallel directions, and the distance of the image is infinite.

The images that are formed while the object passes from an infinite distance to that of half the radius of the mirror, are all inverted.

When the image is nearer to the mirror than the distance of its principal focus, the reflected rays have no longer an actual focus in front of the mirror, but a virtual one behind it. The image appears behind the mirror, is magnified and erect.

633. In a convex spherical mirror, using the same notation as in the case of the convex mirror:

$$f = \frac{d\,r}{2\,d + r}$$

Convex mirrors have no actual focus. The image constantly appears behind the mirror, is erect and less than the object. **Helsham**, § xxiii.

634. In spherical mirrors, whether convex or concave, the superficial magnitudes of the object and image are to each other as the squares of their distances from the mirror. **Helsham**, § xxiii.

635. The above deductions being made from an hypothesis that is not absolutely, but only nearly true, are themselves liable to a small error. In fact, spherical mirrors have no real focus, whether actual or virtual. The conic sections, on the other hand, have true foci; the parabola for parallel, the ellipse for diverging, and the hyperbola for converging rays. Were it possible to introduce them practically, they would be, for this reason, more perfect instruments than spherical mirrors.

No mechanical contrivance has yet been discovered, by which mirrors can be readily and accurately ground into figures, generated by the revolution of conic sections. Parabolic mirrors are, however, sometimes constructed with great pains and tolerable success, to be used in reflecting telescopes.

The inconvenience arising from the multiplication of images in glass mirrors, is but slight when they are plane, but becomes of importance when they are curved; for this reason, the mirrors employed in philosophical and astronomical instruments, are generally made of metallic substances.

636. Mirrors are sometimes constructed for purposes of amusement, of the form of cones and cylinders, both convex and concave. Their properties are intermediate between those of plane and spherical mirrors, but are not of sufficient importance in philosophy to merit mathematical investigation.

Of the Cause of Reflection.

637. When light is considered as an emanation of material particles from the luminous body, the phenomena of reflection would, at the first glance, appear to be the immediate results of an elasticity inherent in the particles of light, and it would seem that these were reflected, making the angle of incidence equal to the angle of reflection, precisely as a ball of ivory is reflected from a plane of marble. Biot.

This idea, however, cannot stand the test of a close examination; for, although we do not know the dimensions of the particles of light, it is evident that they must be extremely small; and as the highest polish that we can give to any reflecting surface, consists merely in rendering its asperities less by the action of fine powders, the most highly polished surfaces are in fact nothing but a collection of cavities, each sufficiently large to receive a multitude of the particles of light.

If then the particles of light should come actually in contact with the reflecting surface, they would not be reflected in the regular manner in which the specular reflection is found to operate.

638. The cause of reflection then appears to be in a repulsive force, which the particles of certain bodies are capable of exerting upon the particles of light. This force acts at a distance from the surface, that is so small as to be insensible, and its intensity diminishes with great rapidity as the distance increases. If then the surface be polished, so that the prominent points, which are thus brought into one plane, shall be within the sphere of each other's repulsion, the reflection would be performed by them alone, and it may be easily shown, that this would be in the same vertical plane, and make with the reflecting surface, equal angles with the incident ray. This high state of polish cannot, however, be given; some of the particles pass between the prominencies and come within the sphere of action of the repulsive force of the less prominent parts, by whose influence the irregular reflection is caused.

Hence it is clear, that the higher the polish, the more intense will be the specular, and the less so the irregular reflection; and, for similar reasons, bodies that have no specular reflection, when the incident ray is nearly perpendicular to their surface, become capable of producing it when the inclination of the ray is increased.

639. We have seen, that in transparent bodies, many of the particles of a ray of light are not reflected from the surface, but enter into it and pass through. This seems to arise from the circumstance, that all of them are not equally capable of reflection, but that they exist in two different physical states; these states either render them capable of being attracted by, or repelled from, the reflecting body, and are found, by late experiments, to be periodical.

This curious property of the particles of light is called polarity, from its resemblance to the phenomena of magnetism. It will be more fully illustrated hereafter. This subject is treated of at length in Biot's *Traite Complet de Physique*, Vol. IV.

OF DIOPTRICS.

That portion of the ray of light that is not reflected from the surface of a body, enters into it, and our next task is to follow it, and ascertain the course that it pursues. The branch of optics which considers this class of phenomena, is called *Dioptrics*.

640. Where a ray of light falls obliquely upon the surface of a transparent body, and enters into it, it is found not to proceed in the same direction as before, but is bent from its course. This phenomenon arises from the difference in the action of the different media upon the particles of light, and it may be fully announced as follows:

(1) When a ray of light passes from one medium to another, if the media are of the same nature, and equal densities, its course is not changed.

(2) When the media are different in nature or density, the course is not changed when the incident ray is a normal to the common surface of the two media.

(3) When the incidence is oblique, and the nature of the media the same, but their densities different, the ray is bent from its course. In passing from the rarer medium into the denser, it is brought nearer to the normal line passing through the point of incidence, and vice versa.

(4) When the incidence is oblique, and the densities and natures of the media both different, the ray is also bent from its original direction, and both these causes concur in the change.

These phenomena may be illustrated by a few simple experiments.

(1) By placing a piece of money at the bottom of an empty vessel, and stepping back until it is just hid by the sides of the vessel; on pouring water into the vessel, the money becomes visible.

(2) When a stick is plunged into clear water, whose surface is smooth, it appears broken at the place where it meets the surface of the liquid, and the immersed part seems shorter than it really is.

(3) If a ray of light be admitted into a dark room, and permitted to fall on the surface of water, or of any other liquid in a vase, the portion of the ray that enters the liquid is bent into a direction more nearly approaching that of the normal to the surface of the liquid.

641. The angle contained by the incident ray and the normal to the surface at the point of contact is called the angle of *Incidence;* that contained by the refracted ray and the same line produced the angle of *Refraction.*

These two angles have a relation to each that is determined by the following law.

642. The incident and refracted rays are always comprised within the same plane, perpendicular to the common surface of the media. And the sines of the two angles have a constant ra-

tio to each other in similar radii, whatever be the magnitude of the angle of incidence. BIOT.

This law is the foundation of the Theory of Dioptrics, and was discovered by Des Cartes. Before proceeding, it will be proper to mention two phenomena, one of which is constantly attendant upon refraction, while the other takes place only in certain bodies.

(1) The refracted ray does not continue of the same magnitude as before, but spreads itself in the plane of refraction over an angular space whose vertex is at the point of incidence; and this angular space is filled with rays of different colours. This phenomenon is called the *Dispersion* of the rays of light.

(2) There are certain substances in which the ray is not refracted in one direction only, but in two pencils that are separate and distinct, and each of these is affected by the dispersive power.

Only one of these pencils follows the general law, the other is found to be governed by one that is much more complex. The change in the direction of the first is said to be produced by *Ordinary*, that in the direction of the other by *Extraordinary Refraction.*

Mode of Determining the Ratio of Refraction.

643. The Ratio of the angles of refraction in solid bodies may be determined by forming a triangular prism out of the substance under experiment, and placing it in such a position that a distant object may be seen both by the direct and refracted rays from one and the same point. If the angle that the direct and refracted rays make with each other be measured by the repeating circle, or some other proper instrument, and the refracting angle of the prism by the reflecting goniometer, the ratio between the sines of incidence and refraction may be calculated by plane trigonometry.

The instrument used for determining the ratio between the angles of incidence and reflection, may also be employed to measure that between the angles of incidence and refraction. BIOT, *Precis Elementaire*, Vol. II.

644. Liquids may be enclosed in a hollow prism of glass, and the ratio between the angles of incidence and reflection calculated from similar observations. It is necessary that the surfaces of the plates of glass which compose the prism should be exactly parallel.

A neat apparatus for this purpose has been constructed by Biot and Cauchoix. See Biot, *Precis Elementaire*, Vol. II. p. 176.

645. The refractive power of gases may be observed on the same principle as that of liquids ; but their peculiar natures render several modifications in the apparatus, and great nicety in the observations necessary.

As the density of gases is much less than that of either solids or liquids, the refracting angle of the prism must be large. That employed by Biot had an angle of 143°. Biot, *Precis*, Vol. II.

The refractive power of gases is effected by their density, and by changes of temperature. In order then to compare experiments made at different times, and under different circumstances, together, these two elements must be taken into account.

The apparatus employed in these experiments by Biot and Avrago is described in the *Precis Elementaire* of Biot, Vol. II. p. 177.

Of Lenses.

646. A lens is a detached portion of a transparent substance, of which the opposite sides are regular polished surfaces, of forms described by lines revolving around a common axis. These lines may be portions of circles, of parabolas, ellipses, &c. or they may be straight. Young, § xxxv.

Although in theory it would be better that the curved surface should be formed by the revolution of a conic section ; yet, as it is difficult to grind lenses into any form but spherical, one side at least of a lens is always a portion of a sphere ; and the other, either a portion of a sphere or of a plane surface.

There are six combinations of these surfaces, by means of which are formed all the lenses that are used by opticians; they are as follows:

(1) The Double Convex Lens.

(2) The Plano Convex Lens.

(3) The Double Concave Lens.

(4) The Plano Convex Lens.

(5) The Meniscas; this has one surface concave and the other convex, but the convex surface is a portion of a less sphere than the concave.

(6) The Concavo Convex; in this the concave surface is a portion of the least sphere.

These may be arranged into two classes, for the 1st, 2d, and 5th, have properties similar to each other; so also have the 3d, 4th, and 6th. The first are called in general terms converging or convex; the second, diverging or concave.

The first class cause parallel rays to converge, diverging rays to diverge less, and converging rays to converge more rapidly. The second class cause parallel rays to diverge, diverging rays to diverge more, and converging rays to converge less.

The first class have actual, the second virtual foci, for parallel rays.

647. If F be the focal distance of a lens, when the object is at an infinite distance, r and r' the radii of the two surfaces, n the ratio of refraction:

$$F = \frac{r\,r'}{(n-1)\,.\,(r \pm r')}$$

Biot, *Precis*, Vol. II.

The difference of the radii is taken in the second term of the denominator of the fraction, when the curves are turned towards the same direction; the sum in the other case.

The ratio of refraction in crown glass is taken by many writers upon optics, as $\frac{3}{2}$, from which it does not differ materially, and substituting this value for n, we obtain the following deductions.

(1) In the double convex lens, whose surfaces are similar, the principal focus is situated at the centre of convexity of the face that is turned towards the incident rays.

(2) The distance of the principal focus of a plano convex lens, from its vertex, is equal to the diameter of the convex surface.

(3) The distance of the principal virtual focus of a double convex lens of similar surfaces is equal to the radius of the lens.

(4) In a plano convex lens, the distance of the principal virtual focus is equal to the diameter of the lens.

In investigating the above properties, the lenses are considered as very thin, and as formed of very small portions of spherical surfaces. As this hypothesis is not true, spherical lenses in consequence are liable to an error; those rays which pass farthest from the axis of the lens converge soonest. If the intersections of the several adjacent rays be conceived infinitely near to each other, they will form a curve that is called a *caustic*. The same error also affects spherical mirrors; in the case of mirrors, the curve is said to be a caustic by *reflection*, in that of lenses, by *refraction*.

This cause of error may be in a great degree obviated when lenses are used in the construction of optical instruments, by introducing diaphragms that intercept the extreme rays, and permit the passage of those only that are nearest to the axis. Biot, *Precis*, Vol. II.

648. When the rays that proceed from an object have been refracted by a lens of the second class, their course becomes exactly the same as if they had proceeded from the point that is rendered their virtual focus, by the refractive power of the lens. And thus we shall not see the object in the place where it really is, nor of its true magnitude, but an image that is nearer to us, and subtends a less angle. Biot.

A remarkable optical deception attends this experiment, for as we are apt to judge of the distance of objects by the angle which they subtend, when compared with their known magnitude, it will frequently happen that we fancy the image to be more distant than the object itself.

649. If an object is presented to a lens of the first class, it forms an inverted image on the opposite side of the lens, which may be received on white paper, ground glass, or any similar substance, and thus viewed. If the eye be placed at a proper distance, it will see this image, but will, by an optical deception, refer its position to the opposite side of the lens.

This image is seen to the greatest advantage when the distance of the lens from the eye is equal to the sum of the focal distance of the lens and the distance at which the vision of the eye is most distinct. If the lens be withdrawn further from the eye, the image becomes less, and its detail less clear, as happens to distant objects; if the lens be brought nearer, the image becomes confused until the eye reach the focus of the lens, when it disappears. If the lens be brought still nearer to the eye, the image reappears, but is confused; it is now erect, and when the lens is brought almost into contact with the eye, becomes distinct. Biot.

650. Where a number of planes are ground upon the convex surface of a plano convex lens, the multiplying glass is formed; it takes its name from its property of transmitting to the eye as many images of the object as there are planes ground upon the lens.

Theory of Refraction.

651. The refraction of the rays of light is caused by the attraction which the particles of refracting bodies possess for the particles of light. This is analogous to capillary attraction, and, like it, is only sensible at very small distances.

This conclusion, which is a necessary consequence of the phenomena, appears, on the first view, to contradict the deduction obtained from the phenomena of reflection. But we have already observed, that it is probable that the particles which are reflected are not in the same physical state with those which are refracted. BIOT.

652. The theory of refraction depends upon the investigation of the circumstances that attend the motion of an infinitely small particle of matter, that after being projected in vacuo in a determinate direction, with a determinate velocity, passes through a zone contained between two parallel planes, and is, during its passage, acted upon by an accelerating force.

When this problem is resolved, in relation to the first zone, we shall know the direction and velocity that have been acquired. With these data, the calculation may be repeated for a second zone; from this we may proceed to a third, and so on.

This problem is precisely the same as that which relates to the motion of projectiles in vacuo. Its solution informs us, that in the same substance the ratio between the sines of incidence and refraction is constant under every possible inclination. This is the same with the law that has already been given, as deduced from experiment. BIOT, *Precis*, Vol. II.

A full investigation, by the analytic method, of this problem, enables us to deduce from experiment, a quantity which, if it do not mark the absolute value of the attractive force, yet is proportional to its intensity. This quantity is equal to the square of the ratio of refraction, diminished by unity and divided by the density of the refracting substance. BIOT, *Precis*, Vol. II.

Newton gives to this quantity the name of the *Refracting Power* of the substance. NEWTON, *Optice.*

653. When a ray of light is about to quit a medium in which it has been refracted, the attraction of this medium will act upon the particles of the ray, to diminish their velocity and change their direction.

When the surfaces of the refracting medium are planes parallel to each other, the forces that act in this case will be equal to those which caused the refraction, but in a contrary direction. The ray ought, therefore, to pass out in a direction parallel to that which it had before refraction.

When the surface of emergence is not parallel to that of incidence, the ray will not pass out in a direction parallel to that it had at first. But the sine of its angle of emergence will have a constant ratio to the sine of the interior angle of incidence. This ratio will be the same with that of the sines of incidence and refraction at the surface of incidence. BIOT.

654. If the same investigation be extended to the case of the passage of a ray of light from one medium to another, it would appear, that whichever medium acted most powerfully on the particles, the ratio of the sine of incidence to that of refraction must be constant, and reciprocally proportioned to the ratio of the velocities of light in the two media.

We have already seen this same law deduced from experiment. See § 642.

655. It may sometimes happen, that a ray of light, on reaching the second surface of a body by which it has been refracted, will not pass out into the air, but will be retained by the attraction of the body. This takes place when the obliquity of the incidence becomes great, and, in this case, the ray will be reflected.

At the instant when the possibility of emergence is about to cease, a part of the ray will sometimes pass out, while the remainder is refracted; that portion of the ray which continues for the longest time to emerge, is of a red colour.

When a drop of a transparent substance is placed upon the second surface of the refracting body, the angle at which emergence becomes impossible, and the reflection of the whole ray takes place, is lessened. If the body thus applied becomes gradually opaque, the angle at which the emergence ceases becomes less and less. As this angle diminishes with the increase of the refractive power of the *second* substance, we have thus a mode of determining the refractive power of opaque bodies, when they can be placed in contact with transparent bodies whose refractive power is greater than their own. This method was first proposed by Dr. Wollaston.

656. The refractive power of a variety of substances was determined by Newton, and the table constructed by him has been shown to be correct by succeeding experimenters. In examining his table, it appears that substances of very different densities have sometimes equal powers of refraction, and that some rare substances have higher powers than others which are more dense. His inference was, that this power depended both on the density and on the chemical constitution of the bodies. Remarking that the general class of substances of the highest refractive powers were the oils and resins, and that pure water was but little inferior to them in the same property, he inferred a similarity in their chemical composition.

Modern discoveries in chemistry have determined hydrogen to be a common constituent portion of the oils and resins and of water. This gas appears, by the experiments of Biot and Avrago, to have the highest refracting power of any known substance, and it is to its presence that the high refractive power of the above-mentioned substances is due. BIOT.

In addition, Dr. Brewster has shown, that the refractive power of bodies is proportioned to their inflammabilities.

These experiments were extended to the different gases, and to mixtures of them in a gaseous form, but in proportions similar to those in which they existed chemically combined in the solid and fluid substances. In this way it has been discovered, that the passage from a gaseous to a solid state always produces a certain increase in the attractive power of the body for the particles of light. BIOT, *Precis*, Vol. II. p. 228.

This increase is not great, and thus it may be stated, that the refractive power of water in the state of vapour differs but little from that it possesses when liquid. The excess of its force over that of the air is very nearly compensated by its decrease of density, and thus the whole refraction produced by masses of moist and of dry air does not sensibly differ when the atmospheric pressure and temperature are equal. Biot.

Newton also observed the high refractive power of the diamond, and hence inferred that it was of the class of inflammable substances. It has not, however, any hydrogen in its composition, and is rather an instance of the increase of refractive power by a dense and crystalline structure.

657. When the refracting medium is not of equal density throughout, the phenomena that it produces may be investigated by supposing it to be divided into a number of very thin layers, each of which is of uniform density.

658. When the medium is of such a nature that the refractive power is constant to a certain altitude, and then decreases by insensible gradations, until it again become constant, it will cause a double image of the object to be seen, the uppermost of which is inverted, the lower erect. If the refractive power increase, the upper image will be erect, the lower inverted.

These phenomena may be shown experimentally, as follows, viz.

(1) By introducing concentrated sulphuric acid by means of a funnel into the bottom of a vessel containing pure water, the attraction of the two liquids for each other will be counteracted by the difference of their specific gravities, and thus they will mix but slowly, forming a medium gradually increasing in density within the limits where the chemical combination is going on. If a slip of paper be pasted on the vessel opposite the pure sulphuric acid, it will be seen by the rays transmitted through that liquid, in an erect position; it will also be seen at the same time inverted, and more elevated, by means of rays passing through the variable medium.

(2) If an object be viewed over the surface of a red hot iron bar, or of a rod of blackened wood, heated in summer in the sun, the stratum of air in immediate contact with the heated substance will be much rarefied, the adjacent strata will be also rarefied, but less so, until the influence of the heated body ceases. An object seen in this way presents two images, the upper erect, the lower inverted. *Biot, Precis*, Vol. II. Young, § xxxvii.

Upon this principle we may account for a number of natural phenomena, some of which frequently occur, while others are only to be met with under peculiar circumstances, and in particular situations; among these may be mentioned *Looming*, the *Mirage*, and the *Fata Morgana*. For the above experiments, and the explanations of the phenomena which they afford, we are indebted to Dr. Wollaston. See BIOT, and YOUNG, ubi supra. PHILOSOPHICAL TRANSACTIONS ABRIDGED, Vol. XVIII.

Of Double Refraction.

659. The rays of light in passing through crystalized bodies are frequently divided into two pencils. One of these follows the law of ordinary refraction, the other is differently affected, and its refraction is said to be extraordinary.

One of the most remarkable instances of this is to be met with in a species of carbonate of lime, called Iceland Spar.

There are generally in double refracting substances, two directions in which the ordinary and extraordinary pencils coincide. These are called the axes of the crystals.

It sometimes happens that a crystal may have but one axis.

660. The phenomena of Extraordinary Refraction appear to be caused by a force existing in the crystal, that separates a portion of the particles of light from the ordinary refracted ray, and repels them from one of the axes of the crystal.

With the nature of this force we are unacquainted; but we can ascertain the law by which it acts.

This was deduced by Hyggens, in the case of Iceland spar, from his hypothesis of undulations; and has been considered a proof of its truth. The same law has, however, been demonstrated on the principle of emanations by Laplace, and has been extended by Biot to every possible case. It is as follows:

661. If v represent the velocity of the particles of light acted

upon by the ordinary refraction, v' the velocity of the extraordinary ray, U the angle formed by the extraordinary ray with the axis, (where there is but one,) and k a constant coefficient for each species of crystal,

$$v'^2 = v^2 + k \sin^2 U.$$

662. If the crystal have two axes, let U represent the angle contained by the extraordinary ray, and the first of them, U′ the angle it makes with the second, then

$$v'^2 = v^2 + k \sin U . \sin U'.$$

ANALYSIS OF LIGHT.

Dispersion of Light produced by Refraction.

663. When an object is viewed through a prism, it does not appear in its true place, but we see an image that is made to deviate towards the refracting angle of the prism. This image is lengthened in a direction perpendicular to the edges of the prism, and is fringed with colours resembling those of the rainbow.

The merit of the complete investigation of this phenomenon is due to Newton, and it is one of the most valuable discoveries of that illustrious philosopher. His methods are detailed at length in his *Optice.*

664. When a ray of solar light is admitted through a circular aperture into a dark chamber, and then intercepted by a white screen, it will form an image of the sun of great brilliancy.

There is an instrument called the Heliostat, by means of which a ray may be introduced to great advantage in one constant direction, and thus the image will be immovable.

665. If this image be attentively examined, it will be found not to be of equal brilliancy throughout, but brightest nearest *its centre.*

This is owing to the sun's subtending a sensible angle; in consequence, the outer part of the image does not receive rays from every part of his disk.

666. If the ray of solar light be intercepted by a triangular prism of glass placed in a vertical position, and whose refracting angle is not less than 60 degrees, the following consequences will be observed:

(1) The ray will be bent from its course in a horizontal direction; and if the prism be made to revolve slowly around its axis, there will be found a certain position in which the image is stationary; it then begins to return backwards towards the original direction of the ray, although the prism still continues to move forwards. This position is, for many reasons, the best for the experiment.

(2) If the ray be received after refraction upon a white screen, the image is not circular, but the ray forms upon the screen an oblong coloured spectrum, bounded on the sides by two straight lines, and at each extremity by a semicircular arc. The extremity that is least bent from its original course is of a brilliant red, the other extremity of a violet hue; between these there are an infinite variety of shades of different colours. NEWTON, *Optice*, Lib. I. Pars. I.

This elongation of the image is not owing to any imperfection in the prism, for all transparent prisms whatsoever produce similar effects.

Newton varied this experiment, by looking through the prism at the image of the sun as transmitted through the hole, and thus observed there the same appearance and the same order of colours. NEWTON, *Optice*, Lib. I. Pars. I.

From his experiments he deduced the following important law:

667. The light of the sun is composed of rays of different degrees of refrangibility; and this is not an accidental property, but inherent in the nature of each ray; for if the rays be separated from each other, by permitting any one to pass through a screen, it will not be farther dilated by a second or even a third *refraction*. NEWTON, Lib. I. Pars. I. Prop. 2.

This unequal refraction of the rays of light must affect the action of spherical lenses. Each ray will have a different focal distance, the most refrangible rays crossing one another nearer to the lens than those which are less so. This is a greater difficulty in the construction of dioptric instruments than that which arises from the spherical aberration. It was first pointed out by Newton, and may be shown to exist by actual experiment. Newton, Lib. I. Pars. I. Prop. 7.

668. If the rays of light fall upon the prism in such a way as to pass through its substance until they meet its base, and if their incidence is very oblique, they will, instead of passing out, be reflected. The angle at which this internal reflection begins to take place is different in each ray; those which are most refrangible being also most easily reflected. Newton, Lib. I. Pars. I. Prop. 3.

The ratio of the sines of incidence and refraction is constant for each particular ray. Newton, Lib. I. Pars. I. Prop. 6.

669. It would be impossible to give separate denominations to each particular coloured ray. But it is sufficient to establish a number of divisions that shall include all the different shades, and to fix their place in the spectrum with as much accuracy as our senses will permit.

The colours chosen for this purpose by Newton, and adopted by most of his successors, are seven in number, viz.—red, orange, yellow, green, blue, indigo, violet.

The proportion of the spectrum, which each of these colours occupies, was supposed by Newton to be the same, whatever the nature of the refracting substance. In this, however, it appears that he was mistaken.

If the whole length of the spectrum be divided in 360 parts, the number of these occupied by each colour, according to Newton's determination, will be as follows, viz.

45	27	48	60	60	40	80
red,	orange,	yellow,	green,	blue,	indigo,	violet.

Biot, *Precis*, Vol. II. Newton, Lib. I. Pars. II.

This determination appears to have been influenced by a wish to show an analogy between sound and light, and is, notwithstanding the great authority of Newton, altogether imaginary.

670. Dr. Wollaston has attempted to determine the division of the spectrum, by looking through the prism at a narrow line of light. The spectrum formed in this way, consists of four colours only, red, green, blue, and violet; these occupy spaces in the proportions of 16, 23, 36, and 25. Young, § xxxvii.

671. Newton was of opinion, that the dispersive powers of bodies followed the same law as their refractive powers, or that the dispersive power was greatest in substances endowed with the highest refractive powers.

672. If the rays of light, after having been dispersed by a prism, fall upon a lens, placed at a distance from the prism equal to twice its focal length, they will again converge and be reunited into a common focus; beyond this, they diverge, and if they be intercepted on a screen, will form a spectrum similar to the first, but inverted. At the focus, the image will be circular and white; at all other points, a coloured spectrum.

Hence it appears, that a white light is produced by the mixture of all the colours of the spectrum, in the proportions in which they there exist. It might be inferred, also, that all the shades of gray, intermediate between white and black, might be composed of the same colours, taken in different proportions; and this Newton found, by experiment, to be true. Newton, Lib. I. Pars. II. Prop. 5.

All other colours whatsoever that we meet with in nature, are either the primary colours of the spectrum, or formed of a mixture of two or more of them. Newton, in the 6th proposition of the second part of his first book, lays down a rule by which the colour may be determined when the quantity and quality of its components are known.

In pursuing this subject, it will be found that compound colours may be formed of the same tone and tint with the primary ones. Thus, the orange and greenish yellow of the spectrum, when mixed, form pure yellow; the yellow and blue form green; greenish blue and indigo, blue; and what is still more remarkable, the blue mixed with red forms violet. All these mixtures differ from the primary colours in being separated by a prism into their component parts. Similar results may be obtained by mixing coloured powders together, but the colours will be less brilliant. Newton, Lib. I. Pars. II. Prop. 5.

673. Extraordinary refraction is accompanied with similar phenomena, the ray being separated by it into the same number of coloured parts as in ordinary refraction.

674. The dispersion of the rays of light produced by refraction, was long a cause of imperfection in optical instruments. This is now obviated by an application of the following principle:

If two prisms be taken of the same substance, and applied to each other, their refractive and dispersive powers would exactly counteract each other, when their angles were equal, and turned in contrary directions. In this case, the two prisms would, together, form a plate with parallel faces, and the ray would pass out in the same direction in which it had entered the first prism. If, now, it were possible to procure substances with equal dispersive, but unequal refractive powers, a system of prisms might be formed with them, by means of which the direction of the ray would be changed, without any dispersion of colours being produced.

Such substances do actually exist in nature, and it is evident, that they may be applied to the construction of lenses, that shall bring the rays that fall on their surface to a focus without affecting them with colours.

675. We have seen, that Newton was of opinion that this

was not practicable. His error was pointed out by Dollond, who made numerous experiments on this subject; his inferences and his application of them to practice, shall be mentioned hereafter.

OF VISION.

676. The organ of vision is among those parts of the human frame whose constitution and use is best understood. The most consummate wisdom is evidently shown in its construction, its position, and the manner of its use.

All optical instruments act with most advantage upon rays that fall nearly parallel to their axis; the eye is, for this reason, furnished with muscles that give it motion, so that it may be directed to any object at will.

It is situated in a bony cavity to protect it from danger, and is enveloped in masses of fat that are compressible, and permit it to move freely under the action of the muscles.

It is preserved from sudden danger by means of the eyelids; and they are fringed with eyelashes, that oppose the entrance of dust.

If dust or any irritable substance enter beneath the lids, tears are copiously produced, and carry it out. The same organs that secrete the tears in this case, are at other times employed in preparing a slight moisture that prevents the eye from being dried by contact of the air, and keeps up the high polish that is essential to the regularity of its refraction. Biot, *Precis*, Vol. II.

677. The external figure of the eye is globular. Its whole surface is opaque, except towards the front, where there is a transparent part that is called the Cornea; this has more convexity than any other part of the surface. If the eye be dissected, there will be found in it three compartments, containing media of different figures, and different refractive powers. The first of these, beginning at the cornea, is of the form of a meniscus, and is filled with a liquid resembling water in appearance,

and which is for this reason called the *Aqueous Humour;* next to this is found a solid transparent body, of the form of a double concave lens—this is called the *Crystalline Humour;* and lastly, the whole of the remainder of the eye is filled with a viscous fluid, resembling melted glass, and hence called the *Vitreous Humour.*

The crystalline humour was found by Mr. John Hunter to be composed of a number of concentric laminæ. These laminæ were by him supposed to be muscular, but this was disproved by Dr. Hosack and Sir E. Home. They are found to increase in density and refractive power as they approach the centre of the lens.

678. The eye is enveloped in several coats, that may be considered as prolongations of the teguments of the optic nerve. The outer of these is called the *Sclerotic Coat,* and the cornea is a prolongation of it. Next to the sclerotic coat we find the *Choroid;* this is covered with a black liquid, and this by its colour is of use in order to prevent any internal reflection. Within this, and spread over its surface, is a nervous membrane of a light gray colour, called the Retina. This appears to be a prolongation of the optic nerve itself, and it is upon this that the rays of light act, to produce the sensation of vision.

679. The manner in which the eye acts will be evident from the description. The rays that proceed from the object fall upon the cornea, pass through the aqueous, crystalline, and vitreous humours, and are, by the refractive powers of these media, concentrated upon the retina, where a small inverted image of the object is formed.

680. This image will, in consequence of the admirable structure of the eye, be perfectly clear, and free from the prismatic colours. For in the eye, the two chief defects of optical instruments are met and obviated. The aqueous, crystalline, and vitreous humours are combined in such a way as to form an achromatic system; and thus, although the rays are refracted, *the colours* are not dispersed. The figures of the several hu-

mours, and the structure of the crystalline, prevent any spherical aberration of the rays. BIOT.

Several other circumstances concur in producing these effects :

(1) There is a diaphragm placed in front of the crystalline lens, that intercepts those rays that make large angles with the axis. This diaphragm, when seen from without, presents the appearance of a coloured ring, that is called the *Iris*. It is pierced by a circular aperture, called the Pupil.

The inner surface of the diaphragm is covered by a prolongation of the choroid coat, and thus absorbs all the rays that may be reflected from the crystalline lens or the retina.

The diaphragm has also the power of dilation and contraction, and thus will admit into the eye a greater or less quantity of light.

In consequence of its situation within the first refracting medium, it will admit a greater quantity of light with less liability to aberration.

(2) The hindermost surface of the crystalline lens is not an exact portion of a sphere, but is flattened towards the axis of the eye. It is consequently less affected by the spherical error of aberration, as oblique pencils fall upon it at less angles of obliquity.

(3) The retina itself has a concave surface, and thus a proper point of reception is prepared for the focus of each pencil of rays. BIOT, *Precis*, Vol. II.

681. The eye possesses the faculty of adapting itself to objects whose distances are very different. It does not do this without effort, nor can we view with distinctness two objects at the same time whose distances are not nearly the same.

Various hypotheses have been laid down to account for the manner in which the eye adapts itself to different distances. Some have supposed that the cornea might at pleasure be rendered more or less convex; others, that the crystalline lens might be moved backwards and forwards; and others, that the crystalline lens was composed of muscles, by which its figure could be changed, and the convexity of its surfaces altered. BIOT, *Precis*, Vol. II.

After summing up these several opinions, Biot concludes, that we are as yet ignorant of the manner in which the eye adapts itself to vision at different distances. This subject, however, appears to be well explained by our countryman Dr. David Hosack, in a paper published in the British Philosophical Transactions for 1794. His theory is as follows, viz.

682. Of the six muscles whose most obvious use is to give motion to the eye, the four straight muscles are of a strength much more than adequate to that purpose. These muscles pass over the ball of the eye, and are inserted by flat and broad tendons into the sclerotic coat at its anterior part. If all these four muscles be contracted at once, they will press upon the ball of the eye, and change its figure; by this means the double effect will be produced of changing, in some degree, the convexity of the cornea, and of lengthening the axis of the eye, the retina being removed to a greater distance from the cornea, and the distance that the ray has to pass through the aqueous and vitreous humours being increased.

These two causes conjoined are sufficient to account for the phenomena. PHILOSOPHICAL TRANSACTIONS ABRIDGED, Vol. XVII.

This subject attracted the attention of Sir Everard Home during the same year, (1794.) His theory is the same as that of Dr. Hosack, and is developed in the Croonian lecture, delivered by him in the year 1795. He made many interesting investigations, and appears to have fully proved a change in the convexity of the cornea, by experiments on the eyes of living persons. He was assisted in his investigations by Ramsden and Skuckburgh. In addition to the facts stated by Dr. Hosack, he discovered that the cornea is itself composed of two pellicles, one of which only is a prolongation of the sclerotic coat, the other formed by the union of the four straight muscles. PHILOSOPHICAL TRANSACTIONS ABRIDGED, Vol. XVII.

This theory of the mode in which the eye adapts itself to vision at different distances, although it derives its fullest confirmation from the reasoning and experiments of these two distinguished philosophers, had been before hinted at by several writers, but never accompanied by evidence sufficient to establish its truth. See HELSHAM, § XXII.

Dr. Thomas Young, on the other hand, conceives that he has established, by experiment, that the cornea does not change its figure; but from the accuracy of Sir E. Home's experiments, and the able assistance with which he was furnished, we should rather incline to trust in his deductions.

683. The image upon the retina is inverted, yet we, notwithstanding, receive from the sensation an impression of the erectness of the object; whether this impression be the result of experience at an early age, or whether it arises from the manner in which the sensation itself is transmitted to the mind, is difficult to determine. We can investigate no more than the mechanical action of the eye; all that we know is, that the impression made upon the retina is conveyed through the intervention of the optic nerve to the brain; nor is it probable that we shall discover more in the present state of our existence.

684. The eye enables us to judge of the distance of objects; this appears to be rather the result of reasoning from experience than an immediate and necessary inference from the perception itself. There are two ways in which we may judge of distances; from the angle subtended by an object of known dimension, and from the inclination of the axes of the two eyes to each other.

685. The organ of vision in animals is constructed upon the same principles with that of man; but the parts are varied in form, and sometimes partially suppressed, according to the circumstances in which the animal is destined to live; while in some cases, other contrivances are adapted that the mode of life appears to require.

A great number of quadrupeds have, during the day, the pupil of the form of a longitudinal slit; at night, the iris contracts, and the pupil becomes circular. Such animals see better at night than man.

This is also the case with birds that seek their prey at night.

In birds, the crystalline humour is less hard than in man, approaching to the consistence of a jelly; the cornea is less, and more convex, and is placed at the summit of a short truncated cone, that projects from the spherical part of the eye. The inner part of the eye of birds contains an organ whose use is not exactly known; it is called the Marsupium, and consists of black membrane, folded like a fan, and floating in the vitreous humour. Biot surmises, that its use may be to change the figure of the eye; and thus, increasing or diminishing the distance between the cornea and the retina, enable it more readily to adapt itself to difference of distance.

Fish live in a medium whose refractive power is nearly equal to that of the aqueous humour of the human eye; its place is, therefore, supplied by a viscous fluid that probably is more refractive; the crystalline lens is spherical, and therefore, under the same bulk, has a shorter focal length than any other lens would have.

The eyes of insects are composed solely of a lenticular body, behind which are spread nervous filaments, that apply themselves to it. These eyes are sometimes separate, at other times the insect has a great number, that are collected in groupes.

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686. Although the eye possesses the power of adapting itself to the view of objects at different distances, this is still confined within certain limits. Those eyes which see distant objects with the greatest distinctness, soonest lose the faculty of distinct vision when the object is brought near; while to those eyes that see best those objects which are near, bodies beyond a certain distance appear confused. When either of these limits is soon attained, it amounts to a positive defect of vision.

In young persons, where the secretions are copious, and the aqueous humour is generated in abundance, the cornea is full and convex; where this convexity is too great, the rays that fall on the cornea from distant objects converge too soon, and the image formed on the retina is indistinct. The images of near objects, on the other hand, are clear. Such persons are called short-sighted, and this defect is generally lessened as they grow older.

In aged persons, the secretions being less copious, the cornea becomes flattened, and loses its power of bringing diverging rays to a focus before they pass the retina. Such persons can, therefore, no longer view near objects.

687. The defect of short-sighted persons may be obviated by the use of a concave lens; for as such a lens will substitute for the object the image that appears to be formed at its focus, it is only necessary to make the lens of a focal length equal to the distance at which the eye sees objects distinctly; when such a lens is placed before the eye, a short-sighted person will see distant objects as distinctly as if they were placed near to him, although he will refer their apparent position to that where he knows they actually are placed.

As the focus of rays from different distances is not in the same point, it will be frequently advantageous to employ different lenses for different purposes. It will also be better to employ lenses whose focal length is a little greater than that at which the eye sees most distinctly, and thus the use of them will tend to improve the power of vision.

It is usual to adapt a lens to each eye, forming the instrument known by the name of spectacles.

688. The defect of long-sighted persons may be remedied by the use of convex lenses. Such persons' eyes cannot bring rays that diverge rapidly to a focus. A convex lens has the power of making diverging rays diverge less, and thus of adapting them to such a state of the visual organ.

689. Lenses being subject to the spherical aberration, only the middle portion of them is of use. The part through which the rays may pass with advantage may be increased by composing a lens of two plano convex lenses, and interposing a diaphragm between the two plane surfaces at which they are joined. These lenses are the invention of Wollaston, and are called, from their properties, Periscopic Lenses. See PHILOSOPHICAL MAGAZINE.

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690. There is a disease called the Cataract that affects the crystalline lens, rendering it opaque. This may be cured by opening the cornea, and extracting the lens. The aqueous humour escapes, but is soon supplied by a fresh secretion. By this operation, (called *Couching*,) the eye does not lose its power of adapting itself to different distances, but merely requires to be aided by a convex lens, of power equal to that of the crystalline humour, placed in front of the eye. See Sir E. HOME, in PHILOSOPHICAL TRANSACTIONS ABRIDGED, Vol. XVII.

The readiness with which the aqueous humour restores itself, is another wise provision of Providence for the preservation of our sight. The cornea, from its situation, and its greater convexity, is more liable to injury than any other part of the eye, but a wound inflicted on it is not irreparable. If, however, the vitreous humour escape, the eye is lost.

OF OPTICAL INSTRUMENTS.

The Microscope.

691. The Microscope is an instrument, by means of which small objects may be viewed under a greater angle than that which they subtend at the nearer limit of distinct vision. This angle is increased by bringing the object nearer to the eye, but in this case it would be very confused, and at last invisible; if now a convex lens be interposed, and the distance between it and the object so adjusted by trial, that the image shall appear to be formed at the distance of the limit of distinct vision, we shall see it as clearly as if it were removed to that distance, but subtending a greater angle.

If d be the limit of distinct vision, and f the focal distance of the lens, p, its magnifying power, may be found by the formula,

$$p = \frac{d}{f}$$

As *d* is a constant quantity in the same eye, an instrument composed of a single lens has a limit that depends upon the size of the lenses that can be conveniently constructed. Small globules, formed by the blowpipe on a thin rod of glass, have been employed for this purpose; water has been dropped into a fine hole pierced in a thin metallic plate; drops of transparent varnish have been proposed for the same purpose; and Mr. Sivwright, of Edinburgh, has proposed to place small fragments of glass in holes pierced in fine platina foil, and melt them there by means of the blowpipe. BIOT, *Precis*, Vol. II.

Microscopes composed of no more than one lens, are called single microscopes; they differ from each other merely in the apparatus by which they are accompanied.

692. A compound microscope has at least two lenses; that which is nearest the eye is called the *eye lens*, the other the *object lens*. In this instrument, the image formed by one lens becomes an object for the other. The operation of this instrument may be illustrated by receiving the first image upon a screen of gauze or ground glass; this may be now viewed by the naked eye, but it is evident that the angle it subtends may be increased by viewing it through a second lens. If this be done, the screen is no longer of use, for each point of the first image will become a radiating focus to the second lens.

The power of this instrument is in the compound ratio of those of the two single lenses. It has a practical limit: in the difficulty of constructing small lenses with accuracy; in the increase of the spherical aberration produced by them; and in the smallness of the quantity of light that minute lenses are capable of receiving and transmitting.

There is also another circumstance that prevents the eye glass from being diminished beyond a certain extent; if the surface of the object lens be very small, each pencil of rays that proceeds from the object, and is refracted through it, will be very slender; for this reason, it cannot cover every part of the eye lens, as would be the case

Dollond, who repeated the Newtonian experiment with a prism made of a different kind of glass, found a result directly contrary, and inferred that Newton's deduction was erroneous; he was hence led to seek for a remedy for the chromatic aberration, and was so fortunate as to discover it.

That such a remedy was to be found, had first suggested itself to Euler, who observed, that it was actually realized in the construction of the eye.

697. If two lenses be taken, one a double concave of flint glass, the other a double convex of crown glass; and if one surface of each glass be ground to the same radius, so that the convexity of the one shall exactly fit the concavity of the other, while their other surfaces are ground to such radii that the focal distance of the concave lens shall be to that of the convex in the ratio of 10 to 7; the two lenses, when placed together, will form a meniscus, and an actual focus will be produced by their joint action, the image formed in which will be devoid of colour.

On the same principle, two convex lenses of crown glass may be united to a concave lens of flint glass.

By the practical application of this principle, John Dollond and his son succeeded in making dioptric telescopes, of wider apertures and higher powers than had before been considered possible, while the length was very much diminished. Few of Dollond's telescopes have object glasses of as much as five feet focal length, while the greater number do not exceed four.

698. The eye lens will itself produce a chromatic aberration, but this may be remedied in a great degree by the introduction of a field glass, as in the compound microscope.

699. In using a telescope for viewing terrestrial objects, the image should be seen in an erect position; this may be effected by interposing two convex lenses between the eye and object glasses.

700. Telescopes are also constructed, in which a convex object lens is combined with a concave eye glass.

The power in this telescope is found as in the astronomic, by dividing the focal length of the object glass by that of the eye lens. This telescope, which was invented by Galileo, is now but little used, except in the form that is called the *Opera Glass*.

701. Sir Isaac Newton, being under the impression that the fault of dioptric telescopes was incurable, proposed the use of catoptric instruments in their stead. Of these instruments, called *Reflecting Telescopes*, there are four different kinds, that are known by the names of their several inventors.

(1) The Newtonian Reflector, which has a concave mirror that closes the tube at the end farthest from the object, and whose axis coincides with that of the tube; a plane speculum is placed within the tube, in the direction of its axis, and inclined to it at an angle of 45°; this intercepts the rays that are about to form an image, and changes the place of this image to the focus of an eye glass fixed in the side of the tube. The plane speculum employed by Newton, was the hindermost surface of a rectangular prism of glass.

(2) In the Gregorian Telescope, the object speculum is perforated, and the image formed by it is in the focus of a smaller concave speculum, that returns it to be viewed through the aperture larger mirror by eye glasses.

(3) Cassegrain substituted a convex speculum for the smaller concave one of Gregory. It is placed nearer to the large mirror than its focal distance, and thus forms the first image at the same place where the second is formed in the Gregorian telescope. This telescope, although less frequently used, appears to be preferable to the Gregorian.

(4) In Herschell's Telescope, the rays are not reflected from a plane speculum, as in Newton's, but continue their course until they reach the common focus of the eye glass and the mirror. In this construction, to prevent the body of the observer from intercepting the view of the object, the axis of the great speculum is inclined to that of the tube of the telescope.

On this principle, Herschell has constructed telescopes of very extraordinary size and power. One of them is forty feet in length, and the speculum five feet in diameter. See ENCYCLOPEDIA BRITANNICA, article *Telescope.* REES, *Telescope.* YOUNG, § XXXVI.

The Camera Obscura.

702. When the image, formed by a lens or mirror, is received upon a level, but unpolished surface, capable of irregular reflection, it is visible in every direction. Several instruments are constructed on this principle; among which are, the Camera Obscura, the Solar Microscope, the Megascope, the Magic Lantern, and the Lucernal Microscope. YOUNG, § XXXVI.

703. The general effect of the camera obscura, is the same as may be often observed in a dark room, where there is a small hole in the window-shutter; the great masses of light and shade before the windows being represented in an inverted position on the opposite walls of the room. A lens, whose focal distance is rather less than the distance of the surface on which the picture is formed, renders the images more distinct; some of them are still unavoidably ill defined, and indeed only those which are at the same distance from the aperture, will be seen to equal advantage in the same position of the lens; by placing the lens in a tube, that may be drawn out or pulled in, objects at every variety of distance may have their images rendered distinct.

The rays that converge to form the image, are sometimes intercepted by a plane speculum placed obliquely; the picture may then be thrown upwards and received on a plate of ground glass; if a sheet of transparent paper be laid upon it, the outline of the several objects in front of the instrument may be traced. This outline will be erect, but the sides of the objects will be reversed.

The lens is sometimes placed in a horizontal position, and an inclined speculum over it; by their joint action, the image is thrown upon a surface placed beneath; on this the objects may be delineated in their natural position. This surface is usually a plane, but it would be better to make it slightly concave, in order that every part of the picture, of objects at equal distances, shall be distinct. YOUNG, § XXXVI.

The Solar Microscope.

704. In the Solar Microscope, an image is formed on a wall or screen, by means of a lens of small focal length; the object is placed very near this lens, and the image is, consequently, much magnified; it is illuminated by the rays of the sun, reflected from a plane mirror, and condensed by means of a convex lens.

The plane mirror must be capable of motion in all angular directions, in order that its position may be accommodated to the changeable place of the sun. This adjustment is sometimes performed by means of the heliostat.

When an opaque object is to be viewed, it may be illuminated either by a plane mirror placed obliquely, or by a perforated concave mirror. YOUNG, § XXXVI.

The Megascope is an instrument contrived by M. Charles, for viewing opaque objects; its principle is the same with that of the solar microscope. BIOT, *Precis*, Vol. II.

The Lucernal Microscope.

705. When the object, instead of being illuminated by the rays of the sun, is enlightened by the condensed rays of a lamp, the instrument becomes the Lucernal Microscope.

The lucernal microscope, is an improvement of an instrument long used for purposes of mere amusement, and known by the name of the magic lantern. The objects, used with this last instrument, are figures painted in transparent colours upon glass sliders. When the ground of the slider is opaque, the lantern of large size and perfect construction, and the image formed on a transparent screen, it constitutes the apparatus once celebrated under the name of the *Phantasmagoria.*

The Camera Lucida.

706. If a quadrangular prism be taken, with its sides inclined in such a manner that the rays, proceeding from a distant object, and falling nearly perpendicular on the anterior surface, shall be twice totally reflected, and afterwards leave the prism in a direction perpendicular to its upper surface, an eye, placed over the prism, will see an erect image of the object, that will appear beneath the prism. If, now, the pupil of the eye be brought near to the prism, and placed in such a way that the reflected rays shall not occupy more than half its surface, it will be manifest, that the spectator will see at the same time and with the same eye, both the object and the paper placed beneath the prism, on which its image will appear to be projected. If, then, he should wish to draw the outline, he may, without difficulty, trace it by means of a pencil, whose point he will see at the same time with the image.

To do this with greater ease, a convex and a concave lens are attached to the prism, and may be used by such persons as have imperfections of vision that require such aid. This instrument was invented by Dr. Wollaston. Biot, *Precis,* Vol. II.

Of Micrometers.

707. When telescopes are used for astronomic purposes, they are frequently furnished with micrometers, or apparatus by which small angles may be measured.

708. Sometimes a scale or wires are introduced in the diaphragm, at the place of the last image; one of the wires is usually movable, and it is necessary, that it be illuminated in

such a manner, as to be distinctly visible. Sometimes a scale is introduced, which, from the apparent magnitude of a known object, enables us at once to ascertain its actual distance. The angular magnitude of an object, seen through a telescope, may also be found, by viewing, with the other eye, a scale, or an object of known dimensions, placed at a given distance. The object glass of the telescope, is sometimes divided into two semicircular portions, one of which slides on the other; each portion acts as a separate lens, and thus two images of every point in the object being formed, the angular distance of any two may be found by bringing their images together, and measuring the displacement of the movable part of the object glass that is required for procuring this coincidence. Sometimes a similar effect is produced by dividing one of the lenses in the eye piece. In reflecting telescopes, one of the mirrors has been divided and applied to the same use. YOUNG, § XXXVI.

709. A double achromatic prism, placed in the tube of the telescope, that will refract the rays in such a way as to form two images, may also be used as a micrometer. The points whose distance is to be determined, are brought into contact by moving the prism backwards and forwards in the direction of the axis of the telescope. Of late years, portions of double refracting substances have been introduced and employed in a similar manner. See BIOT, *Precis*, Vol. II.

OF THE INFLEXION OF LIGHT AND THE COLOURS OF PLATES.

710. The inflexion of light is a modification that its rays undergo when they pass near the edges of bodies. They are then bent from their direct course, and the inflection is not the same in all the colours, but greatest in those which are most refrangible. BIOT, *Precis*, Vol. II.

The inflexion may be rendered visible by intercepting a pencil of solar rays, admitted into a darkened chamber, by a plate of metal, in which a very small hole is pierced. The image of the sun, received upon a screen, will, in this case, be no longer a circular spot of white light, but will be surrounded by a number of concentric coloured rings. If the pencil be first seperated by a prism, and a portion of any one of the coloured rays be transmitted through a similar orifice, the rings will have its colour, and be separated by others, which will be black.

Similar results may be obtained by bringing two sharp rectilineal edges near each other; the light transmitted will not be uniformly white, but will be fringed on each side with the prismatic colours. Biot, Vol. II.

These appearances have been explained by Young, on the hypothesis of undulations; he supposes, that the waves, by whose action light is, on this hypothesis, propagated, interfere with each other.

711. When bodies are reduced to the form of thin plates, they reflect less light from their anterior surface than they did before, while at the second, certain colours are reflected or transmitted in preference to others; this preference appears to depend upon the thickness of the plate, and the nature of the substance of which it is composed.

These appearances may be perceived by pressing two prisms of glass together; a thin plate of air will then be intercepted between them; and, if it be made to reflect light, and viewed, it will be found to present a set of coloured rings, surrounding a black spot; these rings are seen when a convex lens, whose radius of curvature is large, is pressed against a plane of glass; and in thin films of water, in which soap has been dissolved. Biot.

712. The light transmitted through thin plates, is also coloured; in this case, the central spot is white, surrounded by a black ring, this is succeeded by a white circle, and the whole included in a number of coloured rings. Biot.

These several appearances in thin plates, have been accounted for by Newton, on an hypothesis, which is as follows, viz.

713. Every particle of light acquires, in its passage through a refracting surface, a certain transient property, which is reproduced at equal intervals of time, during the whole of its continuance in the refracting medium; hence it results, that at each return of this property, the particle of light would be readily transmitted through the second surface of the refracting medium, if it should at that moment present itself; while in the intervals of this state, it is easily reflected by the second surface. NEWTON. BIOT, *Precis*, Vol. II.

Young has applied his theory of interference, to the explanation of the same phenomena. YOUNG, § XXXIX.

The cause of this phenomenon is evidently the same as that of polarization.

714. The colours of mixed plates constitute a distinct variety of the phenomena of thin plates; they appear, when the interstice between two glasses nearly in contact is filled with a great number of minute portions of two different substances, as water and air, oil and air, or oil and water. Where light is transmitted through such plates, it assumes an appearance similar to that which is produced by reflection from a homogeneous plate.

This appearance was first remarked by Dr. Young. YOUNG, § XXXIX.

715. The colours produced by thick plates, were observed by Newton. He produced them by letting a ray of light fall upon a concave glass speculum, coated with quicksilver, and holding a sheet of white paper in the direction of the reflected ray, he observed four or five coloured rings surrounding the central spots, and resembling the rings transmitted through thin plates. NEWTON, Lib. II. Pars. IV.

POLARIZATION OF LIGHT.

716. From the experiments of Huygens on the phenomena of light refracted by Iceland spar, Newton inferred that the particles of light had each of them four sides; on two of which, they were subject to the usual, and on the other two, to the unusual refraction. This hypothesis is not inconsistent with the phenomena; but that of Malus is better calculated to furnish an explanation of them. He supposes every particle of light to have two poles of contrary properties, in the same manner as a magnet, or as a polar electrical arrangement; that by one of these it is attracted, and by the other repelled, from bodies existing in an appropriate state to produce the phenomena.

717. Some substances act upon the particles of light, in such a way as to turn all the homologous poles towards the same direction. In this case, the ray of light is said to be polarized.

This polarization may be effected in a variety of different manners.

718. When the particles of light traverse crystallized bodies that have the property of double refraction, motions around their respective centres of gravity appear to be impressed upon them; these motions depend on the nature of the force that is exerted upon the ray of light, by the particles of the crystal. It sometimes happens that the forces produce no other effect, than that of arranging all the particles of the same ray parallel to each other, in such a way that their homologous poles shall be turned towards the same portion of space.

Malus called this particular effect, *Polarization;* because he conceived that it resembled the effect produced by a magnet on a system of magnetized needles, all of whose homologous poles would be turned in the same direction. When this phenomenon occurs, the particles preserve the property they have thus acquired, as long as they remain in the double refracting medium, and have no tendency to any farther motion around their centres of gravity. Biot, *Precis*, Vol. II.

719. At other times, the action of the crystal causes the particles to oscillate around their centre with velocities, and during periods that can be calculated; and again, the action is sometimes such as to impress upon them a continuous rotary motion.

720. Phenomena of the same kind may be produced by reflection. This was discovered by Malus, and has been the foundation of a series of experiments and discoveries of the most interesting nature. The persons who have principally distinguished themselves in this investigation, are Malus, Biot, and Arrago, in France, and Brewster, in Great Britain.

Polarization by reflection is complete, or extends to every particle of which the ray is composed, at one certain angle of incidence, only, in each different reflecting body.

Polarization by refraction is not produced instantaneously, but the particles of light acquire it gradually as they pass into the refracting medium; until it is fully developed, the particles appear to oscillate on each side of the direction which they finally attain. This state is called, by Biot, that of movable polarization; when polarization is fully effected, it is said to be permanent or fixed.

Our limits will not permit us to enter more fully into this novel and interesting subject. Those who wish to pursue it, may consult Biot's two works, his *Traite Complet, and Precis Elementaire de Physique*, and Dr. Brewster's several papers on the subject.

Dr. Brewster has proposed the action of crystals upon light in producing polarization, as the basis of a classification of minerals.

721. All the experiments fully confirm the hypothesis of Malus. If it be admitted as true, it furnishes us, at once, with an explanation of the reason why one portion of the same ray of

light is reflected, and another refracted, at the same surface; of the colours of plates, and of various other optical phenomena.

APPLICATION OF OPTICS TO THE EXPLANATION OF NATURAL PHENOMENA.

722. The rainbow is an appearance that is occasionally observed in the heavens; it sometimes consists of one, and sometimes of two circular arcs, tinged with all the colours of the spectrum. The rainbow is only seen in the part of the heavens directly opposite to that occupied by the sun; it is essential that rain should be falling in this direction, and the drops illuminated by the solar ray; the inner bow is never seen when the sun's altitude exceeds 42°, nor the outer, when it is greater than 54°; from all these circumstances, it may be inferred, that the rainbow is produced by the refraction of the white rays of the solar light, aided by one reflection, at least, by means of which they are returned dispersed to the eye of the spectator.

723. The light which falls upon the drop of rain, is refracted on entering it; it passes on in its new direction, until it reaches the second surface; a portion will pass out here, but another portion will be reflected; the reflected ray will proceed to the anterior surface, and a portion pass out, being refracted in a contrary direction to that it at first underwent. If we follow this ray, and suppose an observer placed so as to receive it, he would, if no dispersion took place, see a small image of the sun in the direction of the drop. As the drops are innumerable, he would see innumerable images arranged in the figure of a circular arc, of an angular breadth, equal to the diameter of the sun. In consequence of the unequal refrangibility of the rays of light, the breadth of the circular arc is increased, and it is tinged with the prismatic colours, in the same manner, and on the same principles, that an oblong coloured spectrum is produced, instead of a white circular spot, by refraction through a prism.

The refractive and dispersive powers of water being known, we may calculate the breadth of the bow produced by one reflection, and the angle its semidiameter will subtend. The breadth of the bow, found in this way, will be 1° 45′, the angle subtended by its exterior diameter, 42° 2′; and by its interior semidiameter, 40° 17′.—This is the inner bow, and the red colour is found on its outer, the violet on its inner surface. To the above numbers must be applied a correction for the diameter of the sun.

724. Other drops may be so situated, that light may reach the eye after undergoing two internal reflections from its surfaces. In this way, the light will be rendered more faint, the colours will be inverted, the angle subtended by the bow, and the breadth of its coloured surface greater.

By calculation, the two semidiameters are found to be 50° 57′, and 54° 7′, respectively, and the breadth 3° 10′. This is the mode in which the outer bow is formed. This calculation, like the other, requires a correction for the sun's diameter.

The theory of the rainbow was first partially illustrated by Antonio de Dominis, archbishop of Spalatro; it is also referred by Huygens to the same cause that produces the prismatic colours; but it was reserved for Newton to give a full and satisfactory explanation. See NEWTON, Lib. I. Pars. II. Prop. 9.

725. Rainbows are sometimes formed by the light reflected from the moon, but they occur much less frequently than the solar bow.

Of Halos and Parhelia.

726. In high northern latitudes, where the air is commonly loaded with frozen particles, the sun and moon usually appear surrounded by halos or coloured circles, at distances of 22° and 46° from their centres. This appearance is also sometimes seen around the sun, and frequently around the moon, during the winter of our climate. The halos are often attended, in

high latitudes, by a horizontal white circle, by parhelia or bright spots near their intersection with this circle, and with portions of inverted arches of various curvatures. Anthelia, or bright spots, nearly opposite to the sun, are also sometimes observed on the horizontal circle. The whole of these appearances are accounted for by Young, from a knowledge of the figure which water assumes in passing into the state of ice; he supposes the halo, &c. to be produced by the reflections and refractions that the rays of light undergo from the particles of snow, when in the act of shooting into crystals. YOUNG, § XXXVII.

A similar theory appears to have been given by Mariotte.

Of the Permanent Colours of Bodies.

727. The integral parts of almost all bodies appear to be transparent, and opacity results from the numerous reflections that take place as the light passes from one particle into another, after it has entered the body. NEWTON, Lib. II. Pars. II. Prop. 2.

728. The particles of bodies are probably arranged in groupes, and large spaces intervene between both those of opaque and of coloured bodies; these are either absolutely vacant, or filled with media of a density different from the bodies themselves. So great is the interval between the particles, that it is possible that even in bodies which appear to us as most dense, the intervals may be in magnitude many thousand times as large as the sum of the solid atoms which constitute the body. NEWTON, Lib. II. Pars. II. Prop. 3.

When a pencil of rays enters into a body composed of groupes of particles and intervening spaces, according to the above hypothesis, a certain number of rays may pass by all the groupes, without undergoing either reflection or refraction, and will be transmitted. There will be also another portion of rays that will fall upon the groupes themselves, and pass through them. In entering them, the particles of light will be affected by fits of easy and difficult transmission, exactly as in thin plates, and will reach the second surface of the particles with different dispositions; some for reflection, and others for refraction. Those which are actually reflected, will form the permanent colour of the object, and their quality will depend on the magnitude of the particles of the body, and the manner in which they are grouped. See NEWTON, ubi supra, and BIOT, Vol. II.

That the colour of bodies arises from the manner in which their particles are arranged, may be evident from a number of experiments. Among these is one contrived by Thenard, in which phosphorus may be at pleasure rendered either yellow or black, transparent or opaque.

The tints presented by mother of pearl, is another instance of the same kind, for they depend entirely on the constitution of its surface, and the property may be communicated by impression to sealing wax, or any other ductile body. The action of dilute sulphuric acid on tin plate, produces a similar play of colours on its surface, and a machine has lately been invented, by which *striæ* may be drawn on steel, and a like effect produced.

Of the Blue Colour and Apparent Figure of the Sky.

729. When we view objects through a small extent of intervening air, it will appear to us perfectly diaphanous, but when we examine more distant objects, we shall find their own colours affected by a mixture of a blue tint. We may hence infer, that the air of the atmosphere is so constituted as to transmit all the rays of white light, except the blue, which it reflects. The atmosphere becomes visible to us, for this reason, in the form of a blue vault.

730. The apparent figure of the sky is not a hemisphere, but a less portion; we seem to draw our sensation of this from two other phenomena. The clouds are arranged in a portion of a sphere, whose centre is the centre of the earth; our position is eccentric, and the clouds over our heads are much nearer to us than those which are near the horizon; it is probable that our idea of the heavens is partly deduced from this source. In addition, we conceive the heavenly bodies to be much larger when near the horizon, than when in the meridian; probably because, in consequence of their rays having to pass through a greater portion of the atmosphere, they are less bright, and we are accustomed to find faintness attended with increased distance; we also see many objects intervening between us and the horizon, and can, from their magnitudes, form some estimate of the apparent diameter of the heavenly body; no such means of comparison exist between us and the meridian.

A similar deception is to be seen in relation to the size of all objects seen at an unusual angle of elevation.

OF THE RELATIONS OF LIGHT WITH HEAT AND MAGNETISM.

731. If very sensible thermometers be placed in the several coloured rays, dispersed by refraction through a prism, it will be found that the thermometer placed in the violet ray is least, and that in the red ray most affected. If, however, a thermometer be placed at a little distance from the red ray, and entirely without the visible spectrum, it will be more sensibly affected than if it were placed even in the red ray. Hence, it might be inferred, that heat is composed of particles unequally acted upon by refracting media; that the least refrangible of these is less refrangible than even the red ray.

This experiment was first performed by Herschell.

732. In repeating his experiments, these results were not only *fully* confirmed, but others, still more remarkable, obtained.

The power of the spectrum to effect chemical changes, was found to be greatest in the violet ray, and least in the red; but this power was found to be at its maximum a little without the violet ray, and in a place where there was no perceptible radiation.

The chemical effects of the several rays, were ascertained by their power to produce changes of colour in the salts of silver. This fact was first observed by Ritter, and subsequently confirmed by Wollaston.

733. It has been hence concluded, that the solar beam is composed of three distinct sets of rays; one, which was capable of producing the sensation of light and colours; another, of causing heating; and, the third, of effecting chemical changes.

The third set of rays have, also, the property of inducing magnetism in bars of iron; thus, a needle, exposed to the violet ray of the spectrum, becomes permanently magnetic.

By late experiments of Col. Gibbs, it would appear, that light is of great importance in inducing magnetic properties; when the ore of iron is first dug from the mine, it is not magnetic, but, after a few hours' exposure to the light, it acquires polarity; an artificial magnet, locked up for some months in the dark, lost its powers, which were restored to nearly their original force by the action of the rays of the sun. See SILLIMAN's JOURNAL, Vol. I.

OUTLINES

OF

NATURAL PHILOSOPHY.

ASTRONOMY.

OF THE CELESTIAL SPHERE.

734. When we look at the heavens on a clear night, all the stars appear at the same distance from us, and therefore seem to be situated on the surface of a sphere, having the eye for its centre. Playfair, Vol. II.

To a superficial observer, the Earth itself appears as an extended plane; the intersection of this plane with the sphere is the sensible Horizon.

735. If the stars be observed, and their positions marked, a short time after dusk; and if we view them a second time, in the course of the same night, we shall perceive that some of them which were before seen near one side of the heavens, will have sunk beneath the horizon; that the others have all approached towards this part of the heavens; while some, not visible before, will have risen from beneath the opposite side of the horizon.

The portion of the heavens in which the stars rise, is called the East; that where they disappear, the West.

736. Observations continued for a series of nights will show that all the stars describe circles, having the same point for their pole.

The stars, therefore, appear to move exactly as if they were fixed on the surface of a sphere that revolves upon an axis in a space of time differing but little from a natural day. This motion, which is common to all the heavenly bodies, is called the Diurnal Motion.

737. If we again observe the heavens, at the same hour, some days after our first observation, we shall find that several of the stars that were before observed near the western part of the horizon, will have already set, while those that were on the eastern side have attained a considerable elevation, and those that were before observed to rise, are already risen. We infer from this, that the sun has a motion among the stars in a direction contrary to the Diurnal Motion.

738. The diurnal motion of the sun is apparently in a circle whose axis is the same as that of the circles described by the stars. But he does not constantly describe the same circle; for if his greatest altitude be observed daily, by measuring the length of the shadow cast by a gnomon, he will be found to rise to a much greater altitude in summer than in winter. The whole amount of the variation between these two periods of greatest and least elevation, is about 47°.

The period of this variation is about a year, and nearly the same as that of his motion among the stars, from a given point in the heavens, until he again returns to it. His apparent annual motion is hence determined to be in a circle inclined at an angle of about 23½° to that described by him, on the day when his greatest altitude is at a mean. We shall hereafter give more accurate methods of determining this inclination, and of showing that the apparent annual path of the sun is in reality a circle of the sphere.

739. The Moon is also found to change her position among the stars, advancing towards the east at the rate of about 13° in 24 hours, and completing a circuit of the heavens in 27 days.

740. Five other heavenly bodies, visible to the naked eye, have an apparent motion among the stars, and five more, having

a similar motion, have been discovered by the aid of the telescope.

741. The principal part of the stars have been found, by long observation, to have no sensible motion in relation to each other; they are therefore called *Fixed Stars*. Those which have an apparent motion are called *Planets;* and there is besides another class, whose motions are extremely irregular, that are called *Comets*.

742. The actual figure of the Earth is not an extended plane, but a globe or body bounded by a surface nearly spherical. This may be shown in a variety of ways.

(1) If the Earth were a plane, the circles described by the heavenly bodies would be equally inclined to the horizon, and their pole would have the same altitude at every point on the Earth's surface. But this is not the case, as every change of place towards the North or the South changes the altitude of the pole.

(2) The same heavenly body would in this case be seen to rise, at the the same instant of time, by observers situated at every different point of the Earth's surface.

(3) It may be inferred from the appearance of the sea, and of ships sailing upon it, as viewed from the land; and of the land as approached from the sea.

(4) The globe of the Earth has been circumnavigated.

(5) The section of the Earth's shadow in lunar eclipses is always circular.

743. The diameter of the globe of the Earth is exceedingly small when compared with the distance of the fixed stars.

This may be readily shown, by remarking that the relative positions of the fixed stars do not alter; and that the circles they describe appear the same from every portion of the Earth's surface.

744. Since the diameter of the globe of the Earth is very

small, when compared with the distance of the fixed stars, we may suppose the centre of the Earth to be the centre of the sphere on which the stars appear to be placed. It is to this, called the Celestial Sphere, that we refer the positions of all the heavenly bodies, and in order to do this with more ease, we suppose certain circles to be drawn upon it.

Those circles whose planes pass through the centre of the sphere, are called great circles; and they all mutually bisect each other.

The circumference of each great circle is supposed to be divided into 360 equal parts, that are called degrees.

Those circles whose planes do not pass through the centre of the sphere, are called less circles.

745. The axis of Diurnal Motion is called the Axis of the Heavens, and the points in which it cuts the sphere are called the Poles of the Heavens. In general, the pole of any circle of the sphere is that point in the sphere which is equidistant from every point of the circumference of the circle.

The great circle whose plane is perpendicular to the axis of the heavens, is called the Equator.

The circles of diurnal motion are all parallel to the equator; for this reason they are called *Parallels*.

746. If the direction of gravity be produced upwards, till it intersects the celestial sphere, the point of intersection is called the Zenith. If the same line be produced downwards, it will cut the sphere a second time in a point directly opposite, which is called the Nadir.

The plane of the sensible horizon is perpendicular to this line; and if another plane, perpendicular to it, be supposed to pass through the centre of the earth, it will intersect the celestial sphere in a great circle that is called the rational horizon. As the diameter of the earth bears no sensible ratio to that of the celestial sphere, we may, for the present, consider the sensible and rational horizons as coinciding.

747. The equator being bisected by the horizon, for they are both great circles, the heavenly bodies that describe the equator in their diurnal revolution, are all as long above the horizon as they are beneath it. The sun is in this situation twice annually, and hence the equator has been called the equinoctial line.

The point where the equator intersects the horizon on its eastern side, is said to be due East. The other point of intersection, due West.

If we stand on the surface of the earth, and look due east, the point of the horizon, 90° distant from the east and west points, and which is on our left hand, is called north; the point directly opposite, is called south. These four are called cardinal points. The intervals between them, are each divided into eight equal parts, and the divisions are called points of the compass.

748. A great circle of the sphere that passes through the pole and through the north and south points of the horizon, is called the Meridian.

The axis of the meridian coincides with that of the celestial sphere.

The number of meridians is infinite, for there may be as many as there are points in the circumference of the equator; but at any given place, the term is restricted to that which passes through its zenith, while the others are called hour circles.

The planes of the meridians all cut the equator at right angles.

749. Those circles whose planes pass through the zenith and nadir of any place, are called vertical circles; they are all perpendicular to the horizon.

The meridian of any place passes through its zenith, and is, of course, one of the vertical circles.

The vertical circle that passes through the east and west points of the horizon, is called the *Prime Vertical;* it is perpendicular to the meridian.

The arc of a vertical circle, intercepted between a heavenly body and the horizon, is called its altitude. The arc between it and the zenith, which is the complement of the former, is called the zenith distance. The arc of the horizon, intercepted between the vertical circle, passing through a heavenly body, and the meridian of the place, is called the Azimuth of the heavenly body. Vertical circles are, therefore, sometimes called circles of azimuth.

750. The altitude of all heavenly bodies is greatest when they are in the meridian, and they have equal altitudes when they are at equal distances from the meridian on either side, or when their eastern and western azimuths are equal.

751. If a meridian circle pass through a heavenly body, the arc, intercepted between the body and the equator, is called the Declination of the body; and the arc of the meridian, intercepted between the zenith of a place and the equator, is called the Latitude of the place.

The elevation of the pole at any place, is equal to its latitude. When the position of the pole is on the north side of the prime vertical, the latitude is north; when on the south side, it is south. To an observer, situated at the equator, both poles appear to be in the horizon, and the circles of diurnal rotation are perpendicular thereto.

752. Those stars, whose distance from the pole that is above the horizon of a place is not less than the latitude, never set, while those which are within the same distance of the opposite pole, never rise.

The parallel, whose distance from the pole is equal to the latitude of the place, is hence called the circle of perpetual apparition; and those stars which never set, are called circumpolar stars.

The elevation of the pole at any place, should therefore be found by observing the altitude of one of these stars when it passes the meridian above, and again when it passes the meridian below the pole; that is, by observing its greatest and least altitudes above the horizon; half the sum of these, is the latitude of the place.

When the latitude is known, the declination of a heavenly body may be found by taking its meridian altitude; for the difference between the meridian altitude of the star, and the complement of the latitude, is equal to the declination; when the meridian altitude of the star is less than the complement of the latitude, its declination is of the opposite kind to the latitude of the place; when its altitude is greater than the complement of the latitude, it is of the same kind.

753. The circle which the sun appears to describe annually in the heavens, is, as we have seen, a great circle of the sphere: it intersects and is bisected by the equator. This circle is called the Ecliptic.

The points where the ecliptic intersects the equator, are called the equinoctial points; and those distant from them 90°, the solstitial points.

The moment when the sun is in the equator, is called the Equinox; of these there are two, the autumnal and the vernal.

The moment when the sun is in one of the solstitial points, is called the Solstice; one of these occurs in summer, the other in winter.

The two parallels of declination that touch the ecliptic, are called tropics; and two parallels, distant from the poles as far as the tropics are from the equator, are styled polar circles. The polar circle that is situated in the northern hemisphere, is called the *Arctic*, and that in the southern hemisphere, the *Antarctic* circle.

754. The great circle that passes through the equinoctial points and the pole, is called the Equinoctial Colure; that which passes through the solstitial points and the pole, the Solstitial Colure.

755. If a meridian or hour circle pass through a heavenly body, the arc of the equator intercepted between the meridian and the vernal equinox, is called the right ascension of the heavenly body; this is usually measured in time.

The conversion of degrees, &c. of the meridian into time, is easily done; for the time which elapses between the passage of a heavenly body over the meridian, or any other vertical circle, and its next return to it, is called a day; and astronomers divide the day into twenty-four hours; during each of these an arc of the equator, equal to 15°, will pass over the meridian.

In consequence of the motion of the sun among the stars, the time of his return to the meridian will not be the same as that of a star; hence, days are divided into solar and sidereal.

756. Sidereal days are always of the same length; this is known from the fact, that the more perfect the construction of the instruments by which time is measured has become, the more accurately have they been found to coincide with the sidereal day. An argument has also been drawn by Laplace from the secular equation of the moon's motion; and in this way he has proved incontestably that the length of the sidereal day has not varied since the time of Hipparchus, a period of nearly 2000 years.

The rate of a clock may be compared with the sidereal day, by setting up a telescope in a permanent position, and observing the time at which a known star passes either over a wire fixed in its focus, or past some fixed and distant object. The interval between two successive passages is a sidereal day, or 24 hours; if the interval marked by the clock is greater than this, the clock has gone fast; if the interval be less, the clock has gone slow. The pendulum may then be shortened or lengthened, as may be required to correct the error; and when the rate of the clock has thus been brought near to the truth, the regularity of its motion may be tested by observations of the same kind. See DELAMBRE, *Astronomie Theor. et Prat.* Vol. I.

757. The perpendicular distance between any heavenly body and the ecliptic is called its Latitude; the arc of the ecliptic intercepted between this perpendicular and the equinoctial point is its Longitude.

On the surface of the Earth, longitude means the arc of the equator intercepted between the meridians of two places. It is usual to choose for the first of these, whence all longitudes are reckoned, that which passes through some principal observatory. Greenwich, for this reason, is used by the English, and Paris by the French. It has been usual, on maps published in the United States, to reckon the longitude from Washington; as, however, there is no observatory established there, nor the longitude of any given point ascertained with absolute certainty in reference to that of the places for which the tables of astronomy have been constructed, this practice does not merit commendation.

OF ASTRONOMICAL INSTRUMENTS.

758. An Astronomical Clock must be of the best construction, and furnished with a compensation pendulum. We have already (see § 390) discussed the proper structure of this instrument. Chronometers are also used in fixed observatories, to measure smaller portions of time, and to mark out solar time, in contradistinction to the sidereal time to which the clock is usually regulated.

We have seen the use of a Telescope, fixed in one determinate direction, for the purpose of regulating the clock, and ascertaining its rate. A number of other telescopes are necessary in an observatory, to observe the several phenomena of the heavens, and as parts of the instruments for measuring angles.

759. When a telescope is mounted upon a fixed horizontal axis, so that its motion shall be in a vertical plane, it constitutes the *Transit Instrument*. The suspension is effected by means of a hollow tube fixed at right angles to the tube of the telescope, and of the figure of two truncated cones, whose larger bases are turned towards the telescope, the extremities are cylindrical, and rest upon Ys.

The axis of the hollow tube is rendered horizontal by hanging from it a very sensible spirit level, and raising or lowering one of the Ys by a screw, until the bubble occupies the exact middle of the tube. The axis of the tube is ground to the figure of a circle of large diameter, and parts corresponding to its divisions are marked upon a scale, in order that any accidental variation of the telescope from the vertical plane may be corrected or allowed for.

In the focus of the telescope is a diaphragm furnished with wires; one of these is in the axis of the instrument, and would suffice for the observation; but to compensate any accidental error of observation, it is usual to have at least four others, and the time of passage is taken as the mean of the several passages over the five wires. These wires are illuminated, in nocturnal observations, by means of a lamp, whose light is transmitted through one of the cones on which the instrument hangs.

Instead of wires, Dollond has lately introduced a mode of cutting fine lines on plate glass; this method is for many reasons preferable to the other.

To set the Transit, a meridian line must first be traced; this is done by erecting a gnomon, and marking points occupied by the extremities of its shadow, when of equal lengths, on each side of the meridian; the distance between these points being bisected, a line drawn to the point of bisection from the foot of the gnomon, will be nearly in the meridian. The Transit Instrument being established in this line, may be brought to the true meridian by observations of circumpolar stars; if the interval between the superior and inferior passages of these over the wires of the telescope be not exactly the half of the interval between two successive passages, both either superior or inferior, the instrument is not in the meridian, and must be shifted until this equality takes place.

A meridian line may also be drawn by marking the azimuth of the same star, when it has equal altitudes, on the two sides of the meridian, and bisecting the azimuthal angle.

760. The Astronomical Quadrant, is an arc of a brazen circle included between two radii, at right angles to each other; one of these radii is adjusted to a vertical direction by means of a plumb line; the other will then be parallel to the horizon. A plate of brass is affixed to the centre of the circle, around which

it has a free motion, so that its extremity may apply itself to any portion of the arc, at pleasure; on this plate sights were formerly placed, and directed in observations of altitude to the heavenly body; instead of these, a telescope is now used; it is affixed to the plate of brass, and is carried around with it.

When quadrants are of large size, they are usually permanently fastened to a wall in the plane of the meridian; when in this position, they are styled *Mural.*

Smaller quadrants are used for portable observatories, and in observations out of the meridian; they are mounted upon tripod stands, and levelled by means of screws.

Although the Zenith is not distinguished in the heavens by any particular mark, it is accurately pointed out by the direction of the plumb line; hence this instrument is used to measure zenith distances.

In portable quadrants, it is usual to place their axis in the centre of a horizontal circle, and to affix arms carrying verniers over its divisions; azimuths may, by means of this addition, be observed at the same time with altitudes.

761. The quadrant is liable to errors in its original construction, and may afterwards change its figure, without its having any inherent property of either showing or correcting its errors. For this reason, circular instruments are rapidly superseding the quadrant in the observatories of Europe, and in the estimation of astronomers. It is a property of the circle, that the sum of the two arcs cut from its circumference by any two lines crossing one another at a constant angle, is the same, whether these lines pass through the centre or not; hence, if a zenith distance be observed on two opposite quadrants of the same circle, half the sum of the two arcs will be the truth; but the circle affords an additional compensation of error, for the observation may be read on the two remaining quadrants, or the instrument may have as many as six verniers.

A magnificent instrument of this description, has been constructed by Troughton, for the observatory at Greenwich, and another of equal merit for the observatory at Gottingen, by Reichenbach. The first instrument of the kind was made by Ramsden, for the observatory of Palermo, where it has been used by the celebrated astronomer, Piazzi.

762. In the construction of portable instruments, the circle has also been introduced instead of the quadrant. In some of these, but one observation is made before it is read and registered; such are the altitude and azimuth circles of Troughton. It is, however, better to introduce in portable instruments the principle of repetition first proposed by Mayer; of instruments of this kind, we have examples in the repeating circle of Borda, (see PUISSANT, *Geodesie;* DELAMBRE, Vol. I. cap. VIII. § 28.); in its improvement by Troughton, (see EDINBURGH ENCYCLOPEDIA, and REES, article *Circle,*) and in the instrument lately planned by Professor Playfair, and constructed for the British board of longitude.

763. The Equatorial or Parallactic instrument, has an axis permanently fixed in a direction parallel to that of the heavens; it has three circles, one of which has its plane parallel to the horizon; another is set perpendicular to the axis, and parallel to the plane of the equator; while the third may be made to coincide with any given circle of declination.

The axis of this instrument may be set in the meridian after its equatorial circle has been adjusted to the latitude, by bringing its telescope to a point on the circle of declination that corresponds to the declination of a known star, and fixing it there; the telescope is then to be directed to the star by moving the equatorial and horizontal circles; when the star is bisected by the horizontal wire of the diaphragm, the instrument is properly placed.

In this position, it may be used to ascertain whether the diurnal rotation of the heavenly bodies be uniform or not, for if the telescope be directed, by moving it around the polar axis, to the successive positions of the same body, at known intervals of time; if the arcs described in equal times be equal, and in unequal times proportioned to the times, it may be inferred that the rotation is uniform; and this is the constant result of observation.

The uniformity of the diurnal rotation being thus known, the instrument may be applied to determine the apparent time.

The time being known, the telescope may be directed to any heavenly body whose declination and right ascension are known; and thus with a powerful telescope the stars and planets may be observed in the day time.

When it is wished to follow a heavenly body steadily, with a telescope, it is usual to fix it in the manner of an equatorial, and make it revolve upon a polar axis.

The first equatorial instruments to which telescopes were attached, were not subdivided further than to minutes of a degree; those more recently constructed by Ramsden and Troughton, are of such fine divisions and minute accuracy as to compare with any other astronomical instrument.

764. Instruments that are adjusted by levels or by plumb lines, cannot be employed for nautical purposes, in consequence of the incessant motion of the ship. In this case instruments are employed in which the reflected image of one object, is made to coincide with another viewed directly, by means of a double reflection from two mirrors adjusted for the purpose.

Newton first pointed out the principle on which such instruments might be constructed; they were then constructed in Philadelphia by Godfrey, and subsequently in London by Dr. Hadley, by whose name they go.

765. In Hadley's quadrant, the angle subtended by two objects brought into apparent contact is double that of the inclination of the planes of the two mirrors by which the reflection is performed.

Hadley's quadrant is, for this reason, no more than an eighth of a circle.

766. This principle has been applied to the construction of circular instruments; some of these have the property of repeating the angles that are observed; others read it at once. In the best instruments of the latter kind, there are three indices; a number which is found to afford the most perfect compensation.

When the principle of repetition is employed, very good instruments have been contrived by Borda, by Hassler, and a very neat and portable one by Captain Sabine.

767. When a reflecting instrument is used at sea, to measure altitudes, one of the objects is always the visible horizon, and the reflected image of the other is brought into contact with it. The apparent altitude in this case is always greater than the complement of the zenith distance, as determined by an instrument adjusted by the plumb line, or the spirit level; this arises from the observer's eye being always elevated above the horizontal plane; whence it happens that the line drawn from the eye to the visible horizon is not upon a plane surface parallel to the horizon, but upon the surface of a cone whose vertex is in the eye of the observer. A correction is, therefore, always necessary, the argument of which is the height of the eye; tables of this correction, which is called the Dip of the Horizon, are to be found in all the practical books.

768. When a reflecting instrument is used upon the land, to measure altitudes, the reflected image of the body to be observed is brought into contact with another reflected image seen in a mirror that is parallel to the horizon. The angle observed in this way is double the apparent altitude. The reflecting surface is called an Artificial Horizon. It may be either the surface of a mass of mercury that naturally assumes a horizontal surface, or a plane of glass levelled by a spirit level.

Such observations are not affected by the dip of the horizon.

769. Since telescopes have been affixed to astronomical in-

struments, it is possible to observe angles with great minuteness; and thus divisions of the limbs much smaller than could possibly be drawn on them have been rendered necessary. Each of the smallest divisions that can be drawn upon an instrument may be subdivided into a number of parts, by a contrivance that is called the *Vernier*. Its principle may be illustrated as follows, viz.

A strip of metal, concentric with the graduated limb, and corresponding in length to a number of its divisions equal to $n-1$, is divided into n parts, each of which will be represented by

$$\frac{n-1}{n}$$

The difference between each division of the vernier and one of the limb is equal to

$$\frac{1}{n}$$

If an instrument be divided to minutes of a degree, and if the vernier correspond in length to 59″,

$$\frac{1.}{n}$$

will be 1″; if then the first mark on the vernier correspond to one on the limb, the second will fall 1″ short of the next division, the third 2″, and so on.

If the vernier now be attached to the telescope, and when it is directed to an object, the first mark on the vernier does not correspond with any division on the limb; the division next behind it will show the minutes observed, and the number of the mark of the vernier that first coincides with one on the limb will show the additional seconds.

The division of astronomical instruments has of late years been brought to great perfection, and the lines are so fine and close, that it is necessary to use powerful compound microscopes to read the divisions on the limb, and ascertain their coincidence with those of the vernier.

770. For measuring the diameters of heavenly bodies, and observing the phases of eclipses, Micrometers have been adapted to astronomical telescopes; the construction of several of these has been already spoken of, under the head of Optics.

The value of the divisions of a micrometer are ascertained practically, by observing the time a star situated on the equator takes to pass over the whole field of the instrument; this is then divided into as many equal parts as there are divisions on the micrometer, and converted into parts of a great circle of the sphere.

OF REFRACTION.

771. If we attempt to determine the latitude of a place according to the principle laid down in § 752, we shall find that no two of the circumpolar stars give the same result, but that half the sum of the two altitudes, diminishes as the star's polar distance increases. The cause of this difference is to be found in atmospherical refraction.

The effect of this refraction alters the place of an object only in the vertical plane, and does not deflect it to either side.

772. Refraction produces no effect on an object situated in the zenith, and the error it causes increases as the object approaches the horizon. If the law of refraction be investigated on the hypothesis of a homogeneous atmosphere, abruptly terminating at a well defined surface, we shall obtain an expression for the value of the half of r (the refraction) of the following form :

$$\text{Tan}\ \tfrac{1}{2} r = \text{A} \tan \text{N} + \text{B} \tan^3 \text{N} + \text{C} \tan^5 \text{N} + \&\text{c}.$$

As this expression depends on the tangents, it may be conceived that the increase of refraction will be very rapid as the body approaches the horizon. DELAMBRE, Vol. I. chap. XIII.

The existence of refraction was first pointed out by Ptolemy, who appears to have had clear ideas in relation to it.

Tycho Brahe first attempted to ascertain its quantity, but his theory was less correct than Ptolemy's. Cassini first laid down an hypothesis whence the law could be ascertained. DELAMBRE, Vol. I. chap. XI.

Bradley and Simpson have also aided in improving the methods used in calculating refraction.

The formula now in use, was investigated by Laplace; and by means of it, tables are constructed that are to be found in the several collections.

773. The number given in the tables, as calculated from the formula, is the mean refraction; hence, as it is known that the refractive power of the air is affected by its density, it is necessary to make corrections, the arguments for which are the heights of the barometer and thermometer.

We have seen that the refractive power of the atmosphere is not affected by the moisture it may happen to contain, therefore no hygrometrical correction is requisite. See § 656.

There are various methods in which the constant quantities in the several formulæ may be ascertained; they are given by PLAYFAIR, Vol. II. § 46 and 47, and by DELAMBRE, Vol. I. chap. XIII.

774. There are a variety of other effects produced by atmospherical refraction, among which the following are the most remarkable:

(1) In consequence of the augmentation of refraction with the increase of the zenith distance, the lower limb of the sun will be more elevated by it than the upper; for this reason, the apparent figure of his disk is elliptical; and this is most remarkable when he is near the horizon.

(2) The horizontal semidiameter is also affected by a small increase, that arises from the apparent increase in the convergence of the vertical circles towards the zenith.

(3) The time of rising of the stars is accelerated, and that of their setting retarded. The amount of this acceleration may be found by the formula,

$$d\,P = \frac{R}{\cos D.\ \cos H.\ \sin P} - \frac{R.\ \sin R.\ \cos P}{\cos^2 D.\ \cos^2 H.\ \sin^2 P} + \&c.$$

where R is the horizontal refraction, D the declination, P the hour angle of the star, at the moment it would be in the horizon, if not affected by refraction, and H the latitude.

P may be found by this formula,

$$\cos P = -\tan D.\ \tan H$$

(4) The refraction also changes the azimuth of the heavenly body when in the horizon.

(5) When the body is not in the horizon, its azimuth is not changed by refraction, but its horary angle and polar distance are both affected; a correction must therefore be made for the error that would hence arise.

(6) The apparent paths of the heavenly bodies are not parallel to the celestial equator; hence, when the wires of a telescope are turned till one of them correspond with the apparent course of a star, the vertical wire will not be in the plane of an hour circle, and the difference of the time of two bodies passing over it, will not correspond with their differences of right ascension; for this reason, in determining right ascensions, we avoid observations out of the meridian as much as possible. DELAMBRE, Vol. I. chap. XIII.

775. Atmospherical refraction is also the cause of twilight; it thus prolongs the duration of the day, and prevents a total darkness from succeeding instantly to a brilliant light at the moment of sunset; the break of day is also gradual from the same cause. This last phenomenon is usually styled *Dawn*, but by astronomers the term *Twilight* is applied to both.

The relation, among the several circumstances on which the duration of twilight depends, is given by the formula,

$$\sin \tfrac{1}{2}(P' - P) = \frac{\sin 2\,a}{2 \sin \tfrac{1}{2}(P' + P).\ \cos H.\ \cos D}$$

where P′ is the hour angle, and 2 a the depression of the sun, at the beginning or end of twilight; P the hour angle, at the time of the sun's rising or setting; H and D as before.

If, then, the length of twilight be known from observation, the depression of the sun below the horizon may be calculated; and when the depression is thus found for a given day, the duration of twilight, on any other day, may be obtained.

By the observations of La Caille, the depression of the sun, at which twilight begins and ends, is from 16° to 17°; Lemonnier found it from 17° to 21°; the ancients supposed it to be 18°, which is not far from the mean. DELAMBRE, Vol. I. chap. XIII.

From what has been said, it is evident, that the higher the latitude, the longer the duration of twilight.

The problem of the shortest twilight, has frequently occupied the attention of mathematicians, but it is one of mere curiosity; several solutions are given by DELAMBRE, in his first volume, chap. XIII.

OF PARALLAX.

776. When the eye of the spectator is not situated exactly in the axis of diurnal revolution, the apparent places of the heavenly bodies will not coincide with those they would appear to occupy, to a person situated in this axis, except when the same straight line passes through the point in the axis, the place of the spectator's eye, and the heavenly body, at the same time. For this reason, we consider the true place of a heavenly body as that in which it would appear when seen from the centre of the earth. The difference between the true and the apparent place, is called Parallax.

The relation between the parallax (p), the zenith distance as observed at the surface of the earth (N), the diameter of the earth (r), and the distance of the body (R), is determined by the formula,

$$\sin p = \frac{r}{\mathrm{R}} \sin \mathrm{N}$$

When the angle at the centre (C) is employed,

$$\tan p = \frac{\frac{r}{R}\sin C}{1 - \frac{r}{R}\cos C}$$

When $\frac{r}{R}$ becomes evanescent, or when the semidiameter of the earth bears a very small proportion to the distance of the body, the parallax becomes insensible. This is true of the stars, whose observed zenith distances, when corrected for refraction, always agree with the zenith distances calculated by spherical trigonometry.

When the zenith distance is 90°,

$$\sin p = \frac{r}{R}$$

and is a maximum; the parallax is therefore greatest when the body is in the horizon, and lessens both as the body descends beneath and rises above the horizon; in this last respect, it differs from refraction.

777. The change effected by parallax, is in the altitude and not in the azimuth of the body; and it changes, at the same time, the polar distance and the hour angle.

For the methods of calculating these several effects of parallax, see Delambre, Vol. I. chap. xv.

Parallax tends to depress the bodies that are affected by it; in which respect, also, it acts in opposition to refraction.

778. The apparent longitude and latitude of the heavenly bodies, is also affected by parallax; this is most apparent in the moon, and it is of great importance to determine the quantity of each effect in solar and lunar eclipses, and in occultations of the fixed stars. The method most in use for this purpose, is styled that of the Nonagesimal.

The nonagesimal is that point of the ecliptic which is 90° distant from the horizon; its altitude and longitude may be determined by the formula,

$$\tan N = \cos \omega . \tan M + \frac{\sin \omega . \tan H}{\cos M}$$

$$\sin h = \frac{\cos M . \cos H}{\cos N}$$

When N is the longitude and h the altitude of the nonagesimal, ω the obliquity of the ecliptic, H the latitude, and M the right ascension of the point of the equator that is, at the time for which the calculation is made, upon the meridian.

The parallax in longitude may then be calculated by the formula,

$$\sin \pi = \left(\frac{\sin p \sin h}{\sin d}\right)\left(\frac{\sin L - N}{\sin 1''}\right) + \left(\frac{\sin p \sin h}{\sin d}\right)^2 \left(\frac{\sin L - N}{\sin 2''}\right) + \&c.$$

In this, p is the horizontal parallax, d the polar distance, L the longitude of the heavenly body.

Lastly. The parallax in latitude may be calculated by the formula,

$$\tan x = \frac{\tan h . \cos (L - N + \frac{1}{2} P)}{\cos \frac{1}{2} P}$$

$$\varpi = \left(\frac{\sin p \cos h}{\cos x}\right)\left(\frac{\sin (\Delta - x)}{\sin 1''}\right) + \left(\frac{\sin p \cos h}{\cos x}\right)\left(\frac{\sin^2 (\Delta - x)}{\sin 2''}\right) + \&c.$$

DELAMBRE, Vol. I. chap. xv.

779. There are various methods of determining the horizontal parallax of the heavenly bodies; among which, may be enumerated the following, viz.

(1) The altitude of the body may be observed when in the meridian, and this, when corrected for refraction, may be taken for its true altitude. The altitude is again to be observed when near the horizon, and again corrected for refraction. The altitude, when on the meridian, taken as the true one, will, with the interval of time between the observations and the latitude of the place, furnish data for

calculating what the other altitude should be; the difference between this and that observed, after being corrected for refraction, may be taken as the parallax in altitude; from this an approximate horizontal parallax may be found, for

$$\sin \varpi = \frac{\sin p}{\sin (n + p)}$$

where p is the parallax in altitude, and $n + p$ the observed zenith distance.

(2) The difference between the times at which the body and a star pass the meridian, may be observed for several successive days. In this way, the relative motion of the two will be known, and the right ascension of the body may be determined for any instant of time. From this the true hour angle may be calculated; and if the apparent hour angle be observed by means of an equatorial, the difference will give the horary parallax, whence the parallax in altitude is easily found. In this way Cassini and La Caille observed the parallax of Mars. Delambre, Vol. I. chap. xv.

(3) Two observers, under the same meridian, but at a great distance from each other, may observe the zenith distances of the same body on the same day, at the time of its meridian transit; they can from thence determine its horizontal parallax. See Playfair, Vol. II. § 76.

This method was employed by La Caille, who observed at the Cape of Good Hope, and La Lande, who observed at Berlin, to determine the parallax of the moon.

(4) The same may be done by comparing the position of the heavenly body with that of a fixed star at the time of its passing the meridian, and observing this at two places situated in the same manner as pointed out in (3.) In this manner the parallax of Mars was determined from observations of La Caille, at the Cape of Good Hope, and Wargentein, at Stockholm. Playfair, Vol. II. § 77.

780. From the horizontal parallax of a heavenly body its

distance may be at once known in terms of the semidiameter of the Earth, for

$$R = \frac{r}{\sin p}$$

OF THE FIXED STARS.

781. The relative positions of the Fixed Stars may be determined by observing their zenith distances at the moment of their passing the meridian, (correcting this for refraction,) and the times at which they perform this passage. If the latitude be known, the first observation will give data for calculating the Declination. The last will give the difference in the right ascensions of the several stars that are observed.

The passage over the meridian is observed by means of the Transit Instrument, and the time noted by the clock; the zenith distance by a Mural Quadrant or Circle. For the method of using these instruments for this purpose, see Delambre, Vol. I. chap. xvi.

In this way catalogues have been formed of the fixed stars, in which their position is determined with great precision. Catalogues were also formed by the ancient astronomers, in which the places were less accurately determined, in consequence of the imperfection of their methods and instruments.

Hipparchus (about 130 A. C.) was the first who attempted to make a catalogue of the stars. It appears to have formed the basis of that of Ptolemy, which is still extant. The latter contains 1022 stars.

Ptolemy is suspected of having done little more than reduce the places observed by Hipparchus to those which he supposed the stars ought to have, in conformity with the variation in their positions due to the Precession of the Equinoxes; a variation suspected by Hipparchus, and whose existence was rendered evident by Ptolemy.

There is a work of Eratosthenes on this subject, but it does not merit to be ranked among the catalogues of stars.

Ulag Beg, a Tartarian Prince, constructed a catalogue for the year 1437; the Prince of Hesse formed one in 1594; and Tycho Brahe in 1601.

Hevelius gives the places of 1888 stars, for the year 1661.

Flamstead was the first astronomer who observed the right ascension and declination of the stars by means of the clock, transit, and mural quadrant. Previous to his time, the difference between the right ascension of the stars, and that of the sun, on the day of observation, was determined by the intervention of some one of the planets that are visible while the sun is yet above the horizon. The catalogue of Flamstead contains 2884 stars, and was published in 1725. Our knowledge of the positions of the stars, has been successively improved by Zanotti, Lemounier, La Caille, Mayer, Bradley, Maskelyne, Delambre, and Piazzi.

Flamstead also published a celestial atlas, which for a long time was the only one used by astronomers. It has of late been superseded by that of Bode, published at Berlin, in 1801.

The positions of 36 stars were observed with great accuracy by Bradley and Maskelyne, and their observations have been corrected by their successor, Mr. Pond; the catalogue of these, published in the Nautical Almanac, is sufficient for most of the purposes of astronomers.

782. The stars are considered by astronomers as arranged in certain groups, called *Constellations;* these derive their name from their supposed resemblance to certain figures; the origin and meaning of most of them have been lost in the remote antiquity whence they are derived. The ancients had 48 constellations; to these, Hevelius added 12; Halley, 8 in the southern hemisphere; Bayer, 12; and La Caille, 16. Other astronomers have given us 12 more. See DELAMBRE, Vol. I. chap. XVI.

The ecliptic passes through twelve of the old constellations, whose names and symbols are as follows:

♈ ♉ ♊ ♋ ♌ ♍ ♎ ♏ ♐
Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius,
♑ ♒ ♓
Capricornus, Aquarius, Pisces.

Their names and order may be readily recollected by means of their Latin distich.

Sunt Aries, Taurus, Gemini, Cancer, Leo, Virgo,
Libraque, Scorpius, Arcitenens, Caper, Amphora, Pisces.

783. When the positions of the stars, determined at distant intervals of time, are compared with each other, certain changes, both in their right ascension and polar distance, will be observed.

The cause and law of this change will be mentioned hereafter.

For this reason catalogues of the fixed stars give the place only for a single year, and are always accompanied by a statement of the annual variation; by means of this, the mean place may be calculated for any epoch whatsoever.

784. Many of the stars that appear single when viewed with the naked eye, seem double when seen through the telescope; and others again appear as groupes of three or more stars, very near to each other. Herschell has observed nearly seven hundred of these. Mr. South, of London, is engaged in examining them, and recording their present appearance for the benefit of future astronomers. See QUARTERLY JOURNAL, Vols. XIII. and XIV.

785. There are some stars mentioned by the ancient astronomers, that are now invisible; and others now visible that were unknown to them; there are also a few stars whose brightness varies; and in some this appears to be periodical. PLAYFAIR, Vol. II.

786. Besides the change in right ascension and declination, that has already been mentioned, the stars appear to have a pro-

per, but very slow motion; this has been fully proved in the case of Sirius, and is rendered very probable in relation to several others. PLAYFAIR.

787. In many parts of the heavens, masses of a faint white light are to be seen; these, when examined by a telescope of great power, are found to be composed, each, of a number of stars; the most remarkable of them, is the milky-way. PLAYFAIR.

OF THE APPARENT PATH OF THE SUN.

788. When an observer is furnished with a catalogue of the fixed stars, he has it in his power to ascertain the rate of his clock, daily; and the transit of the sun over the meridian, will give him the right ascension, as compared with some known star. To determine the place of the sun, the declination must likewise be observed; this is done by means of the Mural Instrument, as in the case of the stars.

The Meridian Zenith distance of the Sun, will, in this way, be found to decrease continually from the winter to the summer solstice, and vice versa. If the law of this decrease, and of the increase, be observed, it will be found that the place of the sun is always in the same plane; and consequently that his apparent path, is truly a great circle of the sphere. The inclination of this great circle to the equator, is called the Obliquity of the Ecliptic. It was observed by Hipparchus, to be 23° 51′ it is now 23° 27′ 50″. It is consequently slowly decreasing.

789. The daily motion of the sun in declination, is found, from observation, to be greatest near the time of the Equinox; and least near that of the solstices. The daily motion in right ascension, is greatest about the time of the winter solstice; and least near that of the summer solstice. No error, however, will arise from considering both these motions as uniform for a single day. If the diameter of the sun be measured daily, a *change* will be observed in it that appears to be connected with

the change of the rate of his daily motion in right ascension; the diameter is greatest at the time when this motion is the most rapid, and least when the motion is slowest; this difference in the apparent diameter of the sun, is found to be about $\frac{1}{60}$ part of the whole. DELAMBRE, Vol. I. chap. XVII.

790. On the supposition that the motion, both in right ascension and in declination, is uniform for a single day, the true position of the Equinoctial points may be determined by calculation from pairs of observations; the two separate ones of each set being taken at equal intervals on each side of the day in which the sun is nearest to the equator.

> The position of this point being determined, the absolute right ascension of the stars may be found. See DELAMBRE, Vol. II. chap. XXI.

791. The equinoctial points thus determined are not found to remain in the same place in the equator. Hipparchus detected a motion in them, and his observation was confirmed by Ptolemy; this motion is in a direction contrary to the order of the signs, or retrograde; hence, as the sun reaches the equinox in each year before he has made a full revolution in the heavens, the motion has been called the Precession of the Equinoxes.

> This Precession of the Equinoxes causes no change in the latitudes of the heavenly bodies; but it is attended with a change in their longitude at the rate of 50″ per annum. It is to this change in longitude that the changes in the declination and right ascension of the fixed stars is to be attributed.

> This motion is the same with one that would be produced by a rotary movement of the pole of the axis of the Earth around the pole of the ecliptic, whose period is 25,869 years. DELAMBRE.

792. Those astronomers, with Ptolemy at their head, who hold that the Earth is absolutely at rest in the centre of the universe, are compelled to imagine the existence of two concen-

tric spherical shells, one of which revolves, carrying with it the fixed stars, and makes them appear to revolve with it around the Earth in the space of twenty-four hours, while the other revolves but once in the great length of time above mentioned. Modern astronomers, on the other hand, suppose that the apparent phenomena of diurnal rotation are caused by the motion of the Earth around its axis.

793. The idea of the rotary motion of the Earth is by no means inconsistent with the facts deduced from observation; it may be also inferred from analogy, and draws further confirmation from the explanation itself affords of other phenomena.

(1) The apparent motion of a body in any direction may arise either from its own motion, or from an insensible motion of the spectator in an opposite direction; as is frequently the case with objects on the land, when seen from a vessel in rapid motion.

When a system of bodies is acted upon by a great number of equal and parallel forces, the relative motions of the bodies that compose the system is not affected by their joint action. See § 128.

(2) By observations with the telescope, it appears that the greater part of the planets which compose the solar system revolve upon their axes; those where this rule has not been found good, are placed in such circumstances as would prevent our determining whether they revolve or not. It would from this be a fair inference, according to the Newtonian rules for philosophizing, that the Earth revolves also.

(3) The intensity of gravity decreases as we pass from the equator towards the pole. This decrease is found to follow the same law that would arise from the rotation of the Earth. See § 204.

The surface of the Earth appears, from the portions of meridians that have been measured, to be a spheroid, flattened at the poles, which is the very figure it ought to assume under the action of a centrifugal force caused by a movement of rotation. See § 206.

The reality of the diurnal revolution of the heavens is liable to an objection, because it supposes a common circular motion to be common to a great number of bodies far distant and entirely detached from each other; and this motion would in that case be performed by some of them with a velocity much greater than even that of the rays of light.

From these several modes of reasoning, mutually confirming each other, it may be considered as well established and fully proved, that the theory of the modern astronomers of the rotation of the earth is correct.

The precession of the equinoxes shows, that the pole of the earth is not constantly directed towards the same point in the heavens, but that it is slowly moving around the pole of the ecliptic on the surface of a cone, the semidiameter of whose base is equal to the obliquity of the ecliptic.

794. The time of the equinox, and the position of the equinoctial points, being determined by the method to which we have referred, for a given year, a repetition of similar observations and calculations on the succeeding year, will give the length of the year. To render this determination less liable to error, it is usual to compare together observations fifty or one hundred years apart. The result of a considerable number of comparisons, gives 365 days, 5 hours, 48 minutes, 51 seconds, for the length of the year, reckoned from equinox to equinox.

The ancients employed observations of the solstice for the same purpose; but the instant of the solstice is much less easy to observe and compute than that of the equinox, on which account, all modern astronomers have employed the method of the equinoxes.

From the above data, the mean motion of the sun would appear to be 59′ 8″, 33 per day.

795. The sun's motion in right ascension, is not at this mean rate; but, as has already been remarked, is most rapid a little after the winter solstice, and least so a little after the summer solstice. Observing the solstices and equinoxes of the same year, the intervals will appear to be as follows:

	days.	hours.	min.
From the vernal equinox to the summer solstice,	92	21	36
From the summer solstice to the autumnal equinox,	93	13	44
From the autumnal equinox to the winter solstice,	89	16	56
From the winter solstice to the vernal equinox,	89	1	33

Delambre, Vol. XXII. chap. xxi.

The sun is, therefore, 7 ⅔ days longer in the northern half of the ecliptic than in the southern. The mere difference in the circumstances of motion in the equator and in the ecliptic, is not sufficient to explain this great inequality.

The ancients, who believed that all the motions of the heavenly bodies must be performed in circles, attempted to account for this inequality of motion, by supposing, that the earth was not situated in the centre of the circle described by the sun, but in an eccentric point. See Delambre, Vol. II. chap. xxi.

The instruments of the ancients were rude, and not capable of measuring angles to a greater nicety than 1 minute; this hypothesis had, in consequence, much of the appearance of truth, as errors could not be detected by their means of observation.

The ancients had another hypothesis to explain the appearances. They supposed the earth to be placed in the centre of the ecliptic, but that the sun moved in a small circle, that they called an *Epicycle;* that he made, every year, a complete revolution in this circle, in a retrograde direction, while its centre performed a direct revolution around the circumference of the greater circle, which was called the *Deferent.* This system is called the Ptolemaic.

When astronomic instruments were brought to a greater degree of perfection, it was found that the difference between the true and mean place of the sun, calculated according to this hypothesis, was sometimes twice as great as it should be.

It being manifest, that the apparent orbit of the sun is a re-entering curve, Kepler was induced to substitute the ellipsis, the next curve in simplicity to the circle, for that figure. He then discovered, from a careful collation of the observations of Tycho Brahe, the three laws that go by his name, and that are found to be immediate deductions from the principle of universal gravitation, discovered by Newton. See § 203.

That point of the elliptical orbit of the sun, which is nearest the earth, is called the Perigee; that which is most remote, the Apogee. The line which joins these points, is called the line of the *Apsides*.

796. The direct investigation of the place that would be occupied at any time by a body moving in an elliptic orbit, according to the laws of Kepler, would be difficult. It is rendered easy by a problem invented by him, and that still goes by his name. A fictitious sun is supposed to move uniformly in the equator; its angular distance, at any given time, from the apogee, is called its Mean Anomaly; the apparent angular distance of the sun, at the same time, from the apogee, is styled the True Anomaly. The angular distance of the sun from the apogee, as seen from the centre of the ellipse, is called the Eccentric Anomaly.

If x be the Eccentric Anomaly, e the Eccentricity of the Elliptical Orbit, and z the Mean Anomaly,

$$z = x + e \sin x$$

From this fundamental equation, a variety of others, that enable us to solve the problem of Kepler, have been deduced; they may be seen in DELAMBRE, Vol. II. chap. XXI.

By means of these the Equation of the Centre, or difference between the true and mean anomalies, is calculated, and arranged in the form of tables.

797. Long continued observations, show us that the sun's apogee is not a fixed point, but has a motion in the order of the signs of 11″ or 12″ per annum; and as the equinoctial points fall back 50″ yearly, the distance of the apogee from the equinox increases at the rate of 62″.

For the methods of observing the place of the apogee, and the eccentricity of the apparent orbit of the sun, see DELAMBRE, Vol. II. chap. XXI.

The line of the apsides thus continually moving round, must at one period have connected with the line of the equinoxes. It will appear from calculation that this must have happened about 4000 years before the Christian æra, the very period fixed by Moses as the time of the creation.

798. **By a comparison of distant observations, it would appear that the equation of the centre, and with it the eccentricity of the sun's orbit, are constantly diminishing.**

The rate of diminution is about 18″, 79 per century.

OF THE SYSTEMS OF THE UNIVERSE.

799. **The Ptolemaic System, which was long held by astronomers, supposed the Earth to be immovable and fixed in the centre of the universe, and that all the heavenly bodies moved around it. We have already seen that the phenomena of the diurnal revolution may be as well explained by supposing the Earth to revolve daily on its axis; we shall now find that the assumption of the Earth's moving around the sun in the course of a year, is equally consistent with the appearances presented by the sun himself, and that it is, for a variety of reasons, the most rational. This theory is also the only one that is consistent with the principle of univeral gravitation, and capable of explaining fully the phases and motions of the planets. The system which conceives the sun to be placed in the centre, and the planets, including the Earth, to revolve around him, was, in modern times, first proposed by Copernicus.**

(1) The Earth turning upon its axis in the space of 24 hours, will present its two hemispheres to the sun in succession: one half will be always illuminated; the other will remain in darkness; and day and night will succeed each other with regularity. The Earth revolving in an elliptic orbit around the sun in about 365¼ days, and its poles being inclined to that orbit, the seasons will succeed in the order in which they are found to occur.

(2) The attraction of the sun is a sufficient reason for the Earth's

continuing to describe a re-entering circular orbit, and the same attraction will account for the precession of the equinoxes. Both these phenomena would be unaccounted for on the Ptolemaic hypothesis. The horizontal parallax of the sun is less than 9″; his magnitude, deduced from this, is at least a million of times as much as that of the earth. It would, even at the first glance, appear extraordinary, that so small a body should have so large a one revolving around it; but when we apply the principle of the attraction of gravitation, we see that the thing is impossible.

(3) An objection has been raised to the Copernican system, that the pole of the Earth continually points to the same part of the heavens, notwithstanding the great distance between the opposite points of the Earth's orbit; this objection fails when we consider the immense distance of the fixed stars; and although at one time forcibly urged, it has long been abandoned.

(4) The planets have a motion similar to that ascribed by the Copernican system to the Earth. One of them, Venus, is but little less than the Earth; this planet revolves on its axis, and its motion in its orbit is more rapid than that which the Earth would have; this orbit evidently includes the sun within it; and should the sun revolve around the Earth, Venus must be affected by his motion.

(5) Extending the principle of gravitation throughout the solar system, and adopting the theory of Copernicus, with the addition of the elliptic orbits of Kepler, we find every deduction from the principle fully confirmed by the phenomena.

Conceiving the sun to be in the centre of the system, that point in the elliptic orbit of a planet which is nearest to him is called the Perihelion, and that which is most distant the Aphelion.

OF THE DIFFERENT KINDS OF TIME.

800. It has been stated (§ 756) that clocks are to be regulated by the passage of the fixed stars over the meridian; this species of time is called sidereal; and a sidereal day is the time that elapses between two passages of the same star.

The sidereal day is divided into 24 hours.

801. In consequence of the apparent motion of the sun in the ecliptic, he does not return to the meridian in 24 hours of sidereal time; but describing, at a mean rate, an arc of 59′ 8″ 33 more than the circumference of a circle, is 24 hrs. 3′ 56″ 33‴ 32, of sidereal time, in reaching that vertical circle.

The mean solar day is therefore equal to 24 hrs. 3′ 56″ 33‴ 32, of sidereal time; it is usual to divide it also into 24 hours. The pendulum of a clock beating sidereal time must therefore be lengthened, if it be wished to regulate it to mean solar time.

In observations where the sun is concerned, mean solar time is to be reckoned, but where the stars are to be made use of, sidereal time is to be employed. To facilitate the reduction of one of these to the other, tables have been constructed. The basis of these is to be found in the following equations:

One hour of Sidereal time corresponds with a motion of 15° in the Celestial Sphere;

An hour of mean time is equal to a motion in the same sphere of 15° 02′ 27″, 852;

One minute of mean time corresponds with a motion of 15′ 02″, 4642;

One second of mean time, with a motion of 15″, 04107;

And if m represent the mean, and s the sidereal time,

$$\begin{aligned} m &= s\,(1 - 0.0027379097) + (0.002738)^2 - \&c. \\ &= s - 0.0027304355 \\ \text{if } s &= 1 \text{ hr. of sidereal time} \\ m &= 1 \text{ hr.} - 9'', 82920 \end{aligned}$$

802. The mean time is that which would correspond with the motion of the sun, did he appear to move uniformly in the equator; it differs from apparent time for several reasons:

(1) The apparent motion of the sun, is not in the equator, but in the

ecliptic; the mean and apparent place, would, therefore, did no other cause of difference exist, agree on no other days than those of the equinoxes and solstices.

(2) The sun does not appear to describe equal arcs of the ecliptic in equal times, but moves in an ellipse, where the radius erector describes equal spaces in equal times.

(3) This motion is affected by perturbations arising from the attraction of the other primary planets belonging to the solar system; and,

(4) It is affected by the position of the moon.

The value of these four elements has been investigated, and tables of the difference between true and mean time arising from them, have been calculated; they are published annually in the Nautical Almanac, and in the Connaissances du Tems. This difference is called the *Equation of Time;* it is sometimes additive, and sometimes subtractive; according as the true motion of the sun exceeds or falls short of the mean. DELAMBRE, Vol. II. chap. XXIII.

The largest equation of time is about 16 minutes; and we therefore find, that the mean sun sometimes passes the meridian 16 minutes sooner than the true sun; while, at other times, it passes 13′ 37″ later.

The equation may vary 30″ between one day and that which follows it; this affects the length of the day; about the 23d of March, the true day is 18″, 6 shorter than the mean day; on the 15th of May, the true and mean days are equal; on the 23d of June, the true day is longer by 13″; on the 27th of July, the days are again equal; on the 17th of September, the true day is too short by 21″; on the 3d of November, equality takes for the third time; after which, the true day exceeds the mean day until on the 23d of December, and is 30″ longer than the mean; on the 12th of February, the true and mean day are equal for the fourth time.

These inequalities are such as exactly to counterbalance each other; so that the length of the year is the same as if they did not exist. DELAMBRE.

803. The intervals between the passages of the other planets

over the meridian, differ from the sidereal day for the same reason that the solar day does. Each planet will, however, pass the meridian later than the return of the sidereal day by a quantity that depends upon the rate of its own apparent motion. See DELAMBRE, Vol. II. chap. XXIII.

804. The length of the year is determined by comparing two equinoxes observed at long intervals of time, or by the mean motion of the sun for 100 Julian years. The last method is considered by Delambre as the best. In this way he has ascertained the length of the year to be about 365 days, 5 hours, 48 minutes, 51 seconds. This period of time, determined by the return of the sun to the equinox, is styled the *Tropical Year*, because the ancients made their observations at the solstices when the sun is in one of the tropics.

805. In consequence of the retrocession of the equinoctial points, the sun has no more than 359° 59′ 09″, 9 of the ecliptic to pass over, before he reaches the equinox. The tropical year is, therefore, shorter than the sidereal year. The difference is 20′, 19″, 9, and the sidereal year is 365 days, 6 hours, 9′, 11″, 5.

806. There is another division of time, called the Anomalistic year. In this, the sun returns to the same anomaly or point in his movable orbit. This period is still longer than the sidereal year, being 365 days, 6 hours, 13′, 58″, 8.

807. There are other periods of time, called Synodical years; these bring the several planets back to a conjunction with the sun. The tropical and synodical years coincide in the case of the earth; in the case of the other planets, the length of the synodical year depends upon the length of their own tropical year, compared with that of the earth.

OF THE MOON.

808. The phenomena presented by the Moon are, after those of the Sun, the most interesting. By observations of its transits over the meridian, and taking simultaneous distances from the zenith, the rate of the moon's motion, and the obliquity of its orbit, may be deduced. It will be found, that the lunar is more than 45′ longer than the sidereal day, that the declination changes with great rapidity, and that the form under which the moon appears is continually altering.

809. While the sun constantly presents to us a well defined circular disk, the moon has that form for but a few hours; and in the space of 29 or 30 days, that it takes her to revolve around the earth, she presents herself under every possible variety of shape, from a circular disk to one that is not at all illuminated.

This phenomenon, which is so very obvious, furnished, at a very early period, a mode of estimating time; from this, our months are borrowed, although they no longer correspond with the lunar period.

The several different appearances that the Moon assumes, are called her phases.

810. Every month the Moon is lost to our view for the space of about two days; after this, she is seen in the evening, a little after sunset, of the form of a slender crescent, whose convexity is turned towards the sun; the two points or horns of this crescent are equidistant from the sun, and a great circle of the sphere, perpendicular to the diameter that joins the horns, will pass through the sun's centre; at this period the moon sets a short time after the sun.

The outer circumference of the crescent is semicircular; the interior a semi-ellipse of but little eccentricity.

811. The luminous crescent observed on the next and succeeding nights, will be found to augment gradually in magnitude. The interior curve flattens, and the time of setting becomes later. About the seventh day, the enlightened portion becomes a semicircle, the line which joins the horns being the boundary of the illuminated part of the disk. At this time, the moon is said to be in *Dichotomy;* this period is also called the first quarter.

812. After the eighth day the boundary of the enlightened disk becomes again elliptical, but its convexity is now turned from the sun; the luminous part increases daily in magnitude; the moon is later in passing over the meridian, and enlightens a considerable portion of the night.

When of this figure, the moon is said to be *Gibbous.*

813. About the fourteenth or fifteenth day, the moon appears completely round; she is now said to be *Full;* she rises not far from the time of sunset, and passes the meridian at midnight.

814. On the succeeding day, the moon is again slightly gibbous; the illuminated portion of the disk is lessened; it continues to decrease until the twenty-second day, when the moon is again in dichotomy, which period is called the last quarter. At this time the moon does not rise until long after the sun.

815. After the last quarter, the moon assumes the figure of a crescent; this decreases from day to day, the moon approaches nearer and nearer to the sun, until she precedes him but a short time in his appearance above the eastern horizon; finally, the moon disappears, and is lost to our sight for two or three days, and after this the same set of appearances again recur.

The middle of the interval between the disappearance and re-appearance of the moon, is called the New Moon.

816. At the time of full moon, although the whole disk of the moon is enlightened, every part of it does not appear of equal brilliancy. Some points upon it are much more brilliant than the rest, and others much less so. The last have been supposed to be seas, but when viewed by a telescope, they are not found to have a uniform surface, but round cavities are to be perceived, and these are, in this phase, illuminated to the bottom.

817. Before the first, and previous to the last quarter, the brighter parts are found to cast a deep shadow, showing an elevation above the common level; these shadows decrease up to the time of full moon, when they disappear; after this, they again appear, and increase until the moon is no longer visible.

818. Throughout the whole course of the revolution of the moon, the enlightened part is nearest, and the dark part farthest, from the sun. And if the spots have been noticed with care at the time of full moon, so that they may again be recognised, it will be found that they remain in the same position throughout the lunation; in observing this, we draw our inference not only from the enlightened portion of the disk, but from that which is dark; for it is seldom so much obscured but that it may be seen through a telescope.

From these two facts it may be inferred that the moon constantly presents the same face to the Earth, and that she has no light of her own, but shines by that which she borrows.

It would also appear that the moon is not a simple circular disk, but a body of a globular figure, one hemisphere of which is constantly enlightened, but which enlightened part is not always directed towards us.

The elliptic curve that bounds the enlightened part of the moon, is a great circle of this sphere, and this form is that into which it ought to be projected; the lunar phases being exactly such as would be exhibited by a globe at a very great distance from us, and whose enlightened face is turned in succession at every possible angle of obliquity.

819. If we search for the source whence the moon derives the light by whose reflection she becomes luminous, we can find none other, that is adequate to produce this effect, but the sun. That this light is derived from the sun, may be fully shown by an examination of the phases. When the moon is full, she passes the meridian at midnight; she is therefore in a part of the heavens directly opposite to that occupied by the sun, and will be seen by a spectator on the Earth in the same way as she would be seen by a spectator in the sun, towards which her illuminated hemisphere would be constantly turned, were he the source of the light. Dichotomy is found to take place, not when the moon has described one fourth part of her orbit from the new or full moon, but when a line that joins the centres of the Earth and moon is perpendicular to that which joins the centres of the sun and moon. All the other phases are found to comport exactly with the relative positions of the three bodies. See DELAMBRE, Vol. II. chap. XXIII.

The occurrence of the phenomenon of Dichotomy furnishes a rude manner of determining the comparative distance of the sun and moon from the Earth.

In this way, Aristarchus of Samos found that the sun was more than 18, and less than 20 times as far from us as the moon. DELAMBRE.

820. The Earth, as seen from the moon, must also present phases; the breadth of these is always the complement of the corresponding phase of the moon. The Earth will consequently transmit a reflected light to the moon; and it appears to be owing to this that the dark portion of the disk never disappears entirely.

821. If the diameter of the moon be measured from time to time, it will be found to vary in the same manner as that of the

sun; but the difference in its greatest and least diameters is greater in proportion than in the case of the sun.

The greatest apparent angle subtended by the diameter of the moon, is 33′ 30″; the least 29′ 20″. We might therefore anticipate the discovery of greater inequalities in the motion of the moon than in that of the sun.

The observed diameter of the sun gives 0.0168 for the eccentricity of the Earth's orbit; the moon's orbit has, from similar observations, an eccentricity of 0.0635. DELAMBRE.

From this eccentricity, the equation of the centre may be deduced; it may be as great as 7°. DELAMBRE.

822. The apogee of the moon is not fixed, but is found to make a complete revolution of the heavens in about 9 years, or rather, in 3231 days 8 hours 34′ 57″. DELAMBRE.

823. The longitude of the moon, calculated according to the laws of elliptic motion, does not agree exactly with the true place, but requires a number of corrections; one of these consists in adding an arc, that is proportioned to the sine of twice the angular distance of the body from the sun, minus the mean anomaly.

This inequality is called the *Evection*, and was first discovered by Ptolemy. It appears as if it were an inequality in the equation of the centre, arising from an increase of the eccentricity at the quadratures, and a diminution of it at the syzigies. PLAYFAIR.

824. There is another irregularity in the moon's motion that vanishes at the quadrature and syzigies, and is greatest in the middle, between them. This irregularity in the angular distance is called the Moon's Variation. PLAYFAIR, § 130.

825. The angular motion of the moon is subject to a third irregularity; by this it is diminished when the sun approaches the perigee.

This is called the Annual Equation: it depends on the time of the year, and the mean anomaly of the sun. Tycho Brahe appears to have been acquainted with this inequality.

These are the only inequalities that were discovered by observation alone, and known to exist before the establishment of the Theory of Gravitation. This theory not only explains the cause of these irregularities, but has enabled us to distinguish many more whose united effect was perceived, although their separate action, and the law they severally follow, were not known.

826. From a comparison of the earliest observations with those of modern times, an *Acceleration* may be perceived, by which her velocity appears to be subject to a continual increase, and her period in time to constant diminution. This irregularity has been found by Laplace to be a secular equation, depending on a change in the eccentricity of the Earth's orbit; the moon does not cease to be accelerated for several thousand years; but after the expiration of this long period, it will be retarded again.

827. It was from the moon that the first proof of the principle of universal gravitation was derived. The distance of the moon from the Earth is

$$\frac{1}{\sin 57'}$$

and the effect of gravitation at the distance of the moon would be 0.000692. The actual fall of the moon from a tangent to her orbit is very nearly the same; therefore the moon is attracted towards the Earth, as if it were a gravitating body. A comparison of this effect with that of the sun on the Earth and moon, furnishes the *experimentum crucis* by which the truth of the Newtonian theory can be fully tested. Delambre.

828. The orbit of the moon is inclined to the ecliptic at an

angle of 5°. The points where the orbit cuts the ecliptic are called the moon's *Nodes*. This inclination will modify in some degree the effects of attraction. DELAMBRE.

829. The moon's time of revolution is, like that of the sun, estimated in several different ways. The synodic revolution brings her again into conjunction with the sun; the tropical brings her back to the same longitude; the sidereal to the same fixed star; the anomalistic to the same point in her movable orbit; and the draconitic brings her back to the same node.

	Days.
The revolution, in relation to the node, is performed in	27.21222
In relation to the equinox, in	27.321525
In relation to the stars, in	27.321583
In relation to the perigee, in	27.554570
In relation to the sun, in	29.530589

If these numbers were whole numbers, and were multiplied together, they would give a period at the end of which every phenomenon of the lunar motion would be found to recur a second time in the same order it had previously happened.

Such a period was much sought for by the ancients, in order to regulate their calendar, and serve for the prediction of eclipses. That which was most used by them is the Metonic Cycle of nineteen years.

Calippus quadrupled this period, and subtracted a day, which rendered the cycle more exact.

Hipparchus discovered a period of 345 years, which was well suited for the purposes of the calendar, but not for the prediction of eclipses. The ancients remarked that they returned, nearly the same in appearance, after an interval of 18 years and 10 days. This is called the Chaldean Period.

Modern astronomy furnishes us with more easy and accurate methods of determining the recurrence of eclipses. See DELAMBRE, Vol. II. chap. xxv.

OF ECLIPSES.

830. The Sun, the Earth, and the Moon, are all bodies that are nearly spherical; when, therefore, their three centres are in the same straight line, if the Earth be between the Moon and the Sun, the Moon will enter into the shadow of the Earth, and an inhabitant of the darkened hemisphere of the Earth will see an eclipse of the Moon; in consequence of the greater magnitude of the Sun, this shadow is of a conical figure. DELAMBRE, Vol. II. chap. XXVI.

If the Moon, on the other hand, pass between the Earth and the Sun, the conical shadow of the Moon will most commonly reach the surface of the Earth, and the inhabitants of that portion of its surface over which the shadow passes will lose for a short space of time the sight of the Sun. DELAMBRE.

In a given place, eclipses of the moon are more frequently seen than those of the sun; for when the moon is eclipsed, her light is lost to, and the eclipse is seen from, every point on the hemisphere of the earth that is turned towards the moon; on the other hand, the sun, although he appear eclipsed, loses none of his brilliancy, but is merely hidden from us by the interposition of the body of the moon; and as the moon is much less than the earth, the section of her shadow is but a small spot on a part of the enlightened hemisphere of the earth.

831. Eclipses, either of the sun or the moon, can only happen when the moon is near one of her nodes, that is to say, is either in the plane of the ecliptic, or near it. Those of the sun happen only at new moon, or when the moon is in conjunction with the sun; those of the moon at the full moon, when the sun and moon are in opposition.

832. In investigating the phenomena, and predicting the re-

currence of eclipses, it is necessary to have the means of ascertaining the dimensions of the truncated cone of light whose base is in the sun, and of the conical shadows of the earth and moon. These are found by the following rules:

(1) The sum of the parallaxes of the sun and moon, less the diameter of the sun, will be the angular distance of the moon from the axis of the earth's shadow at the instant the centre of the moon enters into the cone; add to this angle the semidiameter of the moon, and it will give the angular distance of the moon from the axis of the shadow at the moment her limb enters into it.

(2) To the difference of the two parallaxes, add the semidiameter of the sun, for the angular distance between the centre of the moon and the axis of the luminous cone at the moment the centre of the moon is entering into it; add to this angle the semidiameter of the moon, it gives the distance from the axis at the moment the limb of the moon enters the luminous cone. DELAMBRE, Vol. II. chap. XXVI.

Eclipses of the Moon.

833. The length of the earth's shadow varies, with the distance of the sun and earth, between the limits of 212.896 and 220.238, semidiameters of the earth. The mean length being 216.531. PLAYFAIR, Vol. II. § 141.

834. From this and the angle of the cone the apparent semidiameter of the shadow may be calculated; this varies between 37′ 42″ and 45′ 52″. PLAYFAIR, Vol. II. § 142.

As the last of these numbers is more than three times the semidiameter of the moon, a lunar eclipse may continue as long as she takes to describe twice her own diameter; that is, for about two hours. PLAYFAIR, Vol. II. § 142.

835. The phenomena of a lunar eclipse may be understood by supposing two circles of given magnitudes, one representing the disk of the moon, the other a section of the earth's sha-

dow, to move in the same plane with given velocities and given directions: the instants of time, when they touch, when they cease to touch, and when their centres are nearest to one another, determine the beginning, the greatest obscuration, and the end of the eclipse.

The place of the spectator makes no difference in the time of the occurrence of the preceding phenomena; they appear to happen at the same instant of absolute time to all observers, in whatever part of space they are situated. For this reason the phenomena of a lunar eclipse are in themselves simpler, and far more easily calculated, than those of a solar eclipse, in which the body that obscures the other is distant from it so far that their apparent places are much affected by the place and the motion of the observer. PLAYFAIR, Vol. II. § 143.

836. The figure of a section of the shadow is always circular, which circumstance we have already used to prove that the figure of the earth does not differ much from a sphere. But as the arc that is observed to terminate this circular figure is never more than a few degrees of its whole circumference, it is impossible to decide with accuracy, if this curve is truly a circle, or merely an approximation. DELAMBRE.

837. The penumbra that surrounds the shadow, renders observations on the beginning and end of the eclipse very uncertain; it is a better practice to watch the times at which the shadow reaches the several most remarkable spots that are observed on the disk of the moon. The same eclipse will for this reason furnish a hundred or more different observations, that may be compared with others made in distant places, and thus serve to ascertain the difference of longitude between places on the surface of the earth. See DELAMBRE, Vol. II. chap. XXVI.

The ancients, who had no other method to determine terrestrial longitude, might easily be mistaken in their results to the amount of 2° or 8′ of time; this arises from the uncertainty in which we are with respect to the true time of the beginning and end. Indeed, in their geographical tables, we find still greater errors than this.

An eclipse of the moon is visible at all those places where the moon is to be seen; and a complete observation may be made in any place where the moon is above the horizon for its whole duration, which may be from 3 to 3½ hours.

838. In predicting the circumstances of lunar eclipses, and ascertaining whether they are to occur or not, a graphical method is most frequently used; it will give the times of beginning and end within half a minute of time, which is sufficient for the purpose of a prediction.

For this method, see LA CAILLE, *Astronomie*. DELAMBRE, Vol. II. chap. XXVI. FERGUSON, *Astronomy*.

839. When the mean opposition takes place in a point of the lunar orbit that is more than 12° 36′ distant from the node, there can be no eclipse; when this distance is less than 9°, an eclipse must happen; between these two limits the matter is uncertain, and must be decided by calculating the true place of the moon, &c. PLAYFAIR, Vol. II. § 144.

840. When an eclipse is not total, the phase is estimated in digits; these are twelfth parts of the diameter of the body; when the shadow covers any given number of these, it is said to be so many digits eclipsed.

The ancients used digits to signify an aliquot part, not of the diameter, but of the whole surface; this is perhaps a more precise mode of estimating the phase, but it is more difficult to calculate.

Solar Eclipses.

841. The length of the moon's shadow is less than that of the earth, in the same ratio that the diameter of the moon is

less than the diameter of the earth; that is, in the ratio of 1 to 3.562. Playfair.

Hence, when the earth is in the aphelion, the length of the moon's shadow is 59.730; and if the moon is in perigee, its distance from the earth is only 55.902; so that the shadow may reach the earth, and a total eclipse take place. But if the moon were in her apogee, where her distance is 63.862, the shadow could not reach the earth, and the eclipse could not be total any where.

When the earth is in the perihelion, the length of the moon's shadow is 57.76, and if the moon be at the same time in perigee, a total eclipse may happen.

The duration of a total eclipse, in any given place, cannot exceed 7′ 58″.

An annular eclipse, or one where the sun's disk appears like a ring all round the moon, may last 12′ 24″.

To have an eclipse, it is not necessary that the shadow should reach the earth; it is sufficient that the apparent distance of the centres of the sun and moon be less than the sum of their apparent semidiameters. Playfair, Vol. II. § 147.

It is in this case that central eclipses are annular.

842. When an opaque body is opposed to one that is luminous, there is a certain space behind the former from which the latter is only partially visible. This space is called the Penumbra of the opaque body.

If the bodies are spherical, the penumbra is a cone, having for its axis the line joining the centres of the two bodies; and for its vertex the point in that line where tangents to the opposite sides of the two bodies intersect each other.

Half the angle of the conical penumbra is equal to the apparent semidiameter of the sun, *plus* the angle which the moon's semidiameter subtends at the sun. Playfair, Vol. II. § 148.

843. To conceive the phenomena of a solar eclipse, we may

consider the section of the moon's penumbra as advancing on the earth from the west, and as being viewed by an observer in the moon in the same manner that an eclipse of the moon is viewed by an observer on the earth. PLAYFAIR, Vol. II. § 149.

844. The penumbra in a solar eclipse, if viewed from the moon, would subtend an angle equal to the sum of the diameters of the sun and moon, as seen from the earth.

When the penumbra just touches the disk of the earth, the distance of their centres is equal to half the angle subtended by the section of the penumbra, *plus* the moon's horizontal parallax.

If the least distance of the centres of the sun and moon be less than this, there can be no eclipse. The greatest value of this angle is 1° 34′ 27″. If this be the distance at the time of ecliptic conjunction, the distance from the node is 17° 21′ 27″. If the conjunction happen nearer to the node than this, there may be an eclipse; if it be more distant, there can be none.

The limits of solar eclipses are therefore greater than those of lunar; hence there are more eclipses of the sun than of the moon by nearly one half. But in any given place, the number of eclipses that are observed are many fewer of the sun than of the moon; for a lunar eclipse is, as has been stated, visible to a whole hemisphere, but a solar eclipse only to that part over which the penumbra passes. PLAYFAIR, Vol. II. § 150.

845. To calculate the beginning, end, and other phenomena of an eclipse at a given place, there are a variety of methods, but that one which is most used is that of the Nonagesimal. It is thus performed. For the probable time of beginning, calculate the right ascension of the mid-heaven, the longitude and altitude of the nonagesimal degree; from this determine the parallaxes in latitude and longitude, and apply them to the true latitude and longitude, according to their algebraic signs; in this way we obtain the elongation E, and the apparent latitude G; if $E^2 + G^2 = s^2$, s being the sum of the apparent semi-

diameters, the instant chosen is that of the beginning of the eclipse; if $E^2 + G^2 > s^2$ the eclipse has not yet begun; if $E^2 + G^2 < s^2$ it has commenced. If either of these be the case, make a calculation for an instant a few minutes earlier or later; a simple statement in proportion between the two values of $E^2 + G^2$, and the interval of time will give the precise instant of contact.

Similar methods are to be employed to calculate the end of the eclipse, and to find the shortest distance of the centres, and the time of the middle of the eclipse.

The parallax in altitude may be employed instead of the method of the nonagesimal, and also the parallaxes in right ascension and declination. See DELAMBRE, Vol. II. chap. XXVI.

846. Eclipses, when carefully observed, may be applied to determine terrestrial longitude, and to correct the lunar tables. In both these cases it is best, and in the latter absolutely necessary, to observe the phase from time to time by means of a micrometer.

847. In predicting solar eclipses, a geometrical construction is, as in lunar eclipses, sufficiently accurate, and may be true within 1′ of time. The most simple is that which supposes the observer to be placed in the sun, and to see the path which any place on the earth's surface describes, in consequence of the diurnal motion, projected into an ellipsis on the plane of the earth's disk, while the path of the moon's shadow is projected into a right line on the same disk.

This method has been improved by LA CAILLE, who proposes to give a geographical projection of the path of the shadow, the times of obscuration, &c. for all places on the earth's surface. PLAYFAIR, Vol. II. § 151.

Of the Number of Eclipses.

848. The number of eclipses in the Chaldean period is about 70, of which 29 are of the moon, and 41 of the sun. These recur in different numbers in each year, but according to certain laws, which are as follows, viz.

(1) There cannot be more than seven, nor less than two eclipses in the same year.

(2) If there are seven in the same year, five must be of the sun, and two of the moon. If there are only two, they are both solar.

(3) There are in every year at least two eclipses of the sun.

(4) There can never be more than three eclipses of the moon in a year; and in some years there are none.

Occultations of the Stars.

849. The methods that are used in the calculation of solar eclipses may be applied to the investigation of the circumstances of the occultation of a fixed star by the moon. When, however, the moon is distant from the ecliptic, the base of the right angled triangle, used in the former case, cannot be supposed to be equal to the difference of longitude; its value may be found by multiplying that difference by the cosine of the latitude. PLAYFAIR, Vol. II. § 152.

All the stars whose latitude is less than 4° 32′ may suffer occultations. Occultations may be employed for the same purposes as eclipses of the sun; say, to determine terrestrial longitude, and to correct the lunar tables.

OF THE PLANETS.

850. Beside the sun and moon, there are, as has already been stated, other bodies that have a proper motion among the fixed stars. The general direction of this motion is from west to east, and the bodies are called Planets. Five of these, besides the sun, moon, and earth, were known to the ancients; the moderns have discovered five others.

Ancient Planets.	*Modern Planets.*
Mercury,	Herschell,
Venus,	Ceres,
Mars,	Pallas,
Jupiter,	Juno,
Saturn.	Vesta.

The ancient planets have been known from the most remote antiquity; they are visible to the naked eye, and the observations of a few days will enable us to perceive their motion. The modern planets are only visible through the aid of the telescope. DELAMBRE, Vol. II. chap. XXVII.

851. Venus is the most beautiful of all the planets, and was probably the first known; her phenomena are also the most readily observed. We shall therefore begin with them; they are as follows, viz.

(1) Venus always accompanies the sun, never receding from him more than about 46°; she is sometimes seen on his east side, setting after him; at other times, on his west side, preceding him in rising.

In the first case she goes by the name of the *Evening*, and in the second, of the *Morning Star*.

These were at first supposed to be different stars.

(2) Venus, when an evening star, and at her greatest distance from the sun, appears through a telescope to have a semicircular disk, like

the moon in her first quarter, the convexity being, as in that case, turned towards the sun. As her distance from the sun diminishes, the quantity of the illuminated disk diminishes, like that of the moon in her wane, but the diameter increases rapidly, so much so, as for a time to more than counterbalance the loss of brilliancy occasioned by the lessening of the portion of the disk that is illuminated.

At the time of her greatest elongation, Venus is for a time stationary in relation to the sun, or has the same motion with him in longitude; and at a certain point in her approach, from this stationary position, towards the sun, she becomes stationary in relation to the stars, or has no motion in longitude. After this, her motion becomes retrograde in relation to the stars, or towards the west.

(3) Venus at last approaches so near to the sun as to be lost in his rays, and is for some time entirely invisible, except when she is occasionally seen to cross the disk of the sun like a round black spot. This last phenomenon is called the Transit of Venus.

(4) After this conjunction, Venus continues retrograde, and is after a time again seen in the form of a thin crescent; as her elongation from the sun increases, the portion of disk that is illuminated and her apparent diameter increase also, until she attain the distance of 46° on the other side of the sun, when her disk is semicircular.

(5) After the period of greatest elongation, the motion of Venus again becomes direct, and she approaches towards the sun; as she does this, her diameter gradually diminishes, her disk becomes gibbous, and at last nearly round; she is then again lost in the sun's rays, being in conjunction with him. This conjunction is called the Superior, to distinguish it from that already mentioned, which is called Inferior.

(6) After the superior conjunction, Venus again becomes visible on the east side of the sun; her disk is nearly round, and of the same diameter it appeared to have before the conjunction; her elongation from the sun then increases, her disk becomes a less portion of a circle, and her diameter greater, until she attain the distance of 46°; after which the phenomena again recur, in similar order.

(7) The period which includes all these changes, or that which elapses between an inferior or superior conjunction, until the recurrence of another of the same kind, is 584 days.

The time of greatest elongation is 69 days before, and as long after, the inferior conjunction.

(8) The greatest apparent diameter of Venus is near one minute; the least does not exceed 10″. See DELAMBRE, chap. XXVII. PLAYFAIR, Vol. II. § 154, 155, 156, &c.

852. From the comparison of these several phenomena it would appear, that the light of Venus is derived from the sun; that the orbit near the inferior conjunction is convex, and near the superior conjunction concave, towards the earth; and that there is every probability that the orbit of Venus includes the sun within it.

853. If the radius of the earth's orbit be unity, and that of the orbit of Venus r, the distance of this planet from the earth will vary between the limits $1 + r$ and $1 - r$. These distances have to one another the ratio of the apparent diameters, hence,

$$1 + r : 1 - r :: 60'' : 10''$$

$$2 : 2r :: 70'' : 50''$$

$$r = \frac{50}{70} = 0.72$$

DELAMBRE, chap. XXVII.

854. The orbit of Venus is inclined to the ecliptic, and she is found to pass from one node to another in 112 or 113 days, and by a series of observations her periodic time is found to be 224 days 16 hrs. 42′ 27″, 5. DELAMBRE.

The same result may be derived from a comparison of the period of 584 days, within which the phases are comprised, with the annual revolution of the earth, for if v be the angular velocity of the planet in her orbit, or her mean diurnal motion, that of the earth being

59′ 8″ 3; then the arc described by the sun during a revolution of Venus is (59′ 8″, 3) × 584, and that described by Venus in the same time is 360° + (59′ 8″, 3 × 584;) therefore the diurnal motion of Venus is

$$\frac{360 + (59' \, 8'', 3 \times 584)}{584} = 59' \, 8'', 3 \times \frac{360}{584} = 1^\circ \, 35' \, 56''$$

and the periodic time,

$$\frac{360}{1^\circ \, 35' \, 56''} = 224 \text{ days nearly.}$$

From the periodic time the distance may be concluded by the third law of Kepler, for

$$T^2 : t^2 :: R^3 : r^3 \text{ and}$$

$$r = \frac{R \, t^{\frac{2}{3}}}{T} = 0.7233$$

thus the two modes of calculation mutually confirm each other.

855. The inclination of the orbit of Venus to that of the earth is 3° 23′ 25″, and her apparent latitude may be as great as 12°. Delambre, chap. xxvii.

856. The apparent semidiameter of Venus, when her distance is 0.72, is 12″; when her distance is unity, or equal to the distance of the sun from the earth, it will be 8″, 4; the horizontal parallax of the sun is not more than 8″, 7; hence the magnitude of Venus is but little less than that of the earth.

857. The conjunctions of Venus return to the same points of the heavens in about 8 years, and nearly all the phenomena of her appearance occur in the same order at the end of this cycle. If a conjunction take place at the moment the planet is in one of her nodes, Venus will pass over the disk of the sun in the form of a black spot. This spot is too small to be seen by the naked

eye, and for this reason the transit was never observed by any ancient astronomer.

858. As Venus, after the lapse of 8 years, returns to a conjunction at the same part of the heavens, we might, after observing one transit, expect another at the end of this period; and indeed it sometimes does take place; but a third can never follow at the end of a second period of the same length, for the latitude of Venus changes from 20 to 24″ within the 8 years. For this reason, a passage over the northern part of the sun succeeds to one over the southern part, and vice versa; but at the third conjunction, the planet's latitude will have become greater than the semidiameter of the sun. It will then be 121½ years from the first transit before another happens.

HORROX was the first who had the good fortune to see Venus pass over the disk of the sun.

The two transits of 1761 and 1769 were observed in many parts of the earth, and none will again occur until the month of December, 1874.

859. The circumstances of the transit of Venus may be calculated in the same manner as those of a solar eclipse. Venus here holds the place the moon does in the other; but her diameter is much less, her relative motion is retrograde, the parallaxes are less in quantity, and more easy of calculation.

860. The duration of the transit, as seen from the centre of the earth, and from a point on its surface, will not be the same, in consequence of the parallax; this difference may be calculated, and it will vary in different countries, and more particularly between places situated in different hemispheres; if the parallax increases the duration of the transit in one hemisphere, it will lessen it in the other. For this reason, the observed duration in two places, remote from each other, may serve to determine the parallax that is the cause of the difference.

It was long before this advantage, presented by the transit of Venus, was noticed. Kepler, and Horrox after him, thought of it only as a curious and rare phenomenon. Halley first pointed out the important use that might be made of it in determining the dimensions of the solar system.

It is important to choose proper stations for the observation of the transit; but when the time at which it is to happen has been once ascertained, the places whence it may be seen may be determined by the mere inspection of a globe. See Delambre, chap. xxvii.

In general, it is necessary to seek for one station in high southern, and another in high northern latitudes.

Observations of the transit of 1769 were made, at Wardhus in Lapland, at which place, being above the parallel of 67½° N., the sun did not set;

At Cajaneburg, in Sweden, where the immersion was visible a short time before sunset, and the emersion a short time before sunrise;

In California, where the sun was on the meridian when the planet entered on his disk;

In Hudson's Bay;

In the southern hemisphere no inhabited country presented itself at any high latitude, but it was observed by Captain Cook, at Otaheite.

861. From a comparison of these several observations, Delambre infers, that the horizontal parallax of the sun cannot be more than 8″, 64 nor less than 8″, 498; and that no sensible error can arise in taking it at 8″, 6.

The horizontal parallax being given, the distance of the sun may be calculated in terms of the semidiameter of the earth by the formula,

$$R = \frac{r}{\sin p} = \frac{1}{\sin 8'', 6} = 23935$$

If, then, the semidiameter of the earth be taken at 4000 miles, the distance of the sun is 95,000000 miles.

This estimation of the distance of the sun in miles is a subject of curiosity merely; it enters into no astronomical calculation; and as to the parallax, it will be of little importance, even in the most delicate investigations, whether it be taken at 8″, 5 or at 8″, 7. DELAMBRE, chap. XXVII.

862. Mercury presents phenomena precisely analogous to those of Venus. Like that planet, he accompanies the sun, and is never seen farther from him than 23°.

His greatest diameter is not more than 11″, 275; his least, than 5″, 025; his mean distance is 0.7887.

The synodic revolution of Mercury is 115.877 days.

The diurnal motion in his orbit, 4° 5′ 32″.

The periodic time, 87.97 days.

His bulk, compared with the earth taken as 1, is 0.3838. DELAMBRE.

863. Mercury, like Venus, is occasionally seen to pass over the disk of the sun, and his transits are more frequent than those of Venus. Their periods are, 6, 7, 13, 46, and 263 years. DELAMBRE.

The transits of Mercury are chiefly of use in determining the elements of his own orbit; his parallax is too small to permit his being used to advantage in determining that of the sun.

864. There are but two hypotheses that can explain the motions and phases of this planet and of Venus; the first is, that they turn around the sun, and that the earth revolves around him also; this is the system of Copernicus. The second hypothesis supposes that the sun revolves around the earth, and carries with him the orbits of Venus and Mercury, exactly as the earth, in the Copernican system, carries with it the moon;

this is the system of Tycho Brahe. The system of Ptolemy gives us no explanation of these phenomena.

> The Egyptians appear to have discovered, at an early period, the propriety of considering two planets, that never quitted the sun, as revolving around him. These planets are seen in the morning, before sunrise, or in the evening, just before or after sunset; when in their nodes at the time of conjunction, they are seen to pass over the sun's disk, but this always takes place at the inferior, and never at the superior conjunction; at the latter period, then, they are more distant than the sun; they therefore turn around him. This fact is more fully established by the appearance of their phases, and a knowledge of the variation in their apparent diameters.

> The errors of the school of Ptolemy appear to have been in some measure owing to the want of the telescope; but even before its introduction, the insufficiency of the Ptolemaic system had been pointed out, and was admitted even by the most obstinate opponents of Copernicus. DELAMBRE, chap. XXVII.

865. The planets, Mercury and Venus, that accompany the sun, and have an inferior conjunction with him, are called Inferior Planets.

866. There are other planets, that are seen in opposition to the sun, and which are called Superior. Among these is Mars, whose appearances are as follows, viz.

(1) His apparent diameter varies between 18″ and 6″.

(2) His light is of a reddish hue, and he has no sensible phases, never showing less than four fifths of his disk.

(3) His mean distance from the sun is 1.52; his distance from the earth varies between 2.52 and 0.52, being 2.52 when Mars is in conjunction with the sun, and 0.52 when he is in opposition. At this last period, he passes the meridian, or culminates, at midnight, rises when the sun sets, and sets when he rises; and is, according to the mode of expression used by the Greek Astronomers, Acronical. When near conjunction, and in opposition, his disk is perfectly round.

(4) In consequence of the great difference in the diameter of Mars, there is a great difference in the brilliancy of his appearance at different times.

(5) The parallax of Mars is to that of the sun as 10 : 4; and was used, before transits of Venus were observed, to discover the dimensions of the solar system.

(6) Mercury and Venus, as seen from the earth, are retrograde towards the inferior conjunction. If the earth be an inferior planet to Mars, it must appear, to an observer on his surface, to be retrograde towards the same part of the orbit. If the earth appear retrograde towards Mars, the latter should appear so to the earth. This is actually the fact, furnishing additional evidence that the earth revolves around the sun, and that the orbits of the inferior planets are included within that of the earth.

(7) The greatest elongation of the earth, as seen from Mars, would be 53°; this planet is therefore stationary at 136° of elongation. DELAMBRE, chap. XXVII.

867. Jupiter is the most important planet of the solar system, both from his great size, and the four satellites with which he is attended. His orbit is next to that of Mars in distance from the sun.

(1) The oppositions of Jupiter return in 399 days; and each time his longitude increases about a sign.

(2) His tropical revolution occupies 11 years 318 days.

(3) The diameter of Jupiter, when at his mean distance from the earth, is 37″. His disk is usually covered by two, or sometimes three dark belts. He is flattened at the poles, his equatorial being to his polar axis as 15 : 14.

(4) His phases are still less sensible than those of Mars; and like him, he may be observed at all angular distances from the sun. He is, in consequence, a superior planet to Mars, and to the others that have been mentioned.

(5) His annual parallax is not more than 12°. To an observer on

his surface the earth never appears farther than that distance from the sun, nor Mars more than 17°; while Venus and Mercury, having no greater elongations than 8° and 4¼, are probably invisible, unless they should be seen performing transits over the solar disk.

(6) The distance of Jupiter from the sun, in terms of the semidiameter of the earth's orbit, is 5.2028. DELAMBRE, chap. XXVII.

868. Saturn, although smaller in size, less brilliant, and farther from us, than Jupiter, is still distinctly visible; his mean diameter is 18″, and he is surrounded with a ring whose apparent mean diameter is 42″. His periodic revolution is performed in 29 years 12 days, and his distance is 9.53877.

869. The planet Herschell, or Uranus, was discovered by Herschell, in 1781. He is the most remote from the sun of any planet in the solar system. His periodic time is 84 years; his distance, 19.183305.

870. Jupiter, Saturn, and Herschell, are all affected in the same manner as Mars, being for the greater part of the year direct in motion; but when near the opposition, they become stationary for a time, and are then retrograde.

871. Besides these, there are four small bodies that revolve around the sun between the orbits of Mars and Jupiter. Kepler, observing the sudden increase of distance in passing from the orbit of Mars to that of Jupiter, conceived that a planet was wanting between these two. His idea was recalled to the minds of astronomers by the discovery of the planet Ceres, which was made by Piazzi, in 1801. Placing this in the system, there is a most remarkable progression to be observed in the distances of the planetary bodies.

Mercury is distant,	$4 = 4$
Venus,	$7 = 4 + 3 \times 2^0$
The Earth,	$10 = 4 + 3 \times 2^1$
Mars,	$16 = 4 + 3 \times 2^2$

Ceres,	$28 = 4 + 3 \times 2^3$
Jupiter,	$52 = 4 + 3 \times 2^4$
Saturn,	$100 = 4 + 3 \times 2^5$
Uranus,	$196 = 4 + 3 \times 2^6$
———	$388 = 4 + 3 \times 2^7$

If the planet Ceres be removed, the progression is incomplete. If this law be actually founded in nature, we can hardly expect to discover any new planet in the solar system.

The other three planets were discovered in succession.

Pallas, by Olbers, in 1802;

Juno, by Harding, in 1804; and

Vesta, by Olbers, in 1807.

They are supposed, from the near coincidence of their mean distances and periodic times, to be the fragments of a larger planet that once existed in that part of the system. They are so small that our best micrometers hardly suffice to measure their diameters.

872. The phenomena of the superior planets all tend to confirm the fact that the earth is a planet revolving in an elliptic orbit around the sun; that his apparent motion is due to the motion of the earth; that the orbit of the earth includes the orbits of Venus and Mercury, and is included by the orbits of the remaining planets.

Hence little doubt can remain of the truth of the Copernican system of the universe.

873. The orbits of the planets formerly known were determined originally by the result of long observations of their motions, and particularly, of their returns to their nodes, whence the times of their revolutions were calculated. From these observations, the following laws were deduced, in relation to their motions, which fully agree with those deduced *a priori*, by means of the theory of universal gravitation.

(1) The orbits of the planets are ellipses, having the sun in their common focus.

(2) The velocities of the angular motions of a planet are inversely as the squares of the distances; and the sector described by the radius of vector of each planet is proportioned to the time of description.

(3) The squares of the periodic times are as the cubes of the distances.

874. These laws, having the double confirmation of theory and observation, may be taken as general laws of all planetary bodies whatsoever, and hence we may apply them to determine the distance, periodic time, &c. of a newly discovered planet.

The methods used for this purpose have been brought to perfection by Gauss, an eminent German astronomer, and three geocentric observations of any new planet are now sufficient to determine the elements of its orbit.

875. The several most important circumstances in relation to the planets are comprised in the following table :

	Periodic times. Days.	Mean distances, Earth's being unity.	Eccentricities, the mean distance being unity.	Inclination of the orbit.
Mercury,	87.969	.3788787	0.2055	7° 0′ 0″
Venus,	224.700	.7233323	0.0068	3° 23′ 35″
Earth,	365.256	1.0000000	0.0168	
Mars,	686.979	1.5236935	0.0931	1° 51′ 0″
Jupiter,	4332.596	5.2027911	0.0482	1° 18′ 52″
Saturn,	10758.970	9.5387705	0.0562	2° 29′ 38″
Uranus,	30688.713	19.1833050	0.0467	0° 46′ 25″

876. The several planets of the solar system, by their mutual action, are constantly disturbing the regularity of each other's motions. To enable us to calculate the true place of a planet, for any given time, it is necessary to introduce among the elements allowances for these irregularities; the investigation of the tables by which they are computed is the province

of Physical Astronomy. It is sufficient here to state, that every perturbation of the planets that has been investigated has been found to be periodical.

877. When planets are attended with satellites, their masses may be determined in relation to that of the sun; the masses of the other planets, although less easily ascertained, may be found from investigating the action they produce on the other bodies in the system. As the actual density of the earth is known, from the experiments of Cavendish, and the observations at Schehallion, we may hence ascertain the actual weight of all the bodies of the solar system.

The following table exhibits their apparent diameters, if seen at the distance of the sun; their diameters compared with the earth; their magnitudes; and their masses.

	Apparent diameters.	Diameters compared with the Earth.	Volumes.	Masses compared with the Earth.
Mercury,	6″, 6	0.3838	0.0565	0.1627
Venus,	16, 5	0.9593	0.8828	0.9243
Earth,	17, 2	1.0000	1.0000	1.0000
Mars,	8, 9	0.5174	0.1386	0.1294
Jupiter,	186, 8	10.8600	1280.9	308.94
Saturn,	171, 7	9.9825	974.78	93.271
Herschell,	74, 5	4.3314	81.20	1.6904
Sun,	32′. 02, 0	111.74	1395324.	329630.0
Moon,	4, 7	0.2730	0.20351	0.0146

For the several inequalities of the planets' motions that have not been mentioned, see Laplace, *Mechanique Celeste*, Vol. III.

878. The Sun, the Moon, Mercury, Venus, Mars, Jupiter, and Saturn, have been found, by observation, to revolve upon their axes, and we might hence infer, in addition to the other proofs already alleged, that the circumstances of diurnal motion were produced by the rotation of the Earth; and also, that Uranus, Ceres, Vesta, Pallas, and Juno, revolve in like manner,

although the great distance of the first, and the very small size of the others, prevent us from perceiving it.

879. The rotation of the planets is determined, and its period ascertained, by watching the position of points on their surface. If these remained nearly stationary, it might be inferred, that the planet did not revolve; but in all that have been mentioned above, they are found to have regular motions, of greater or less rapidity.

The spots of the Moon are visible to the naked eye; those of the Sun are very conspicuous to the telescope, and were discovered by the very first observers; Mercury is so small and bright, and Saturn is so distant, that it requires very powerful instruments to perceive and determine the motion of their spots; Venus, from her brilliancy, is also difficult to observe; while the rotation of Mars and Jupiter can be readily ascertained. DELAMBRE.

880. Their times of diurnal rotation are as follows, viz.

Mercury,	24 hrs. 05′ 30″
Venus,	23 hrs. 21′
Mars,	24 hrs. 40′
Jupiter,	9 hrs. 56′
Saturn,	10 hrs.
Sun,	24 days 18 hrs.
Moon,	27 days 7 hrs. 43′ 3″

DELAMBRE.

The rotation of the moon on her axis is exactly equal to the time of her revolution around the earth, and this coincidence has probably some physical cause; if the moon had been originally a fluid mass, the attraction of the earth would have elongated it, and one side becoming thus heavier than the other, would, by the force of gravity, always fall towards the earth. LAGRANGE.

The motion of the moon around the earth, as we have seen, is not uniform; she also moves in an orbit inclined to the ecliptic; hence it happens, that a spot which appears to us exactly in the centre of the lunar disk when she is in the ecliptic, is higher than the centre when her latitude is south, and lower than the centre when her latitude is north; that it is seen to the eastward of the centre, when her longitude is greater than that which she would have by a mean and equable motion; and to the westward of it when her longitude is less. These slight changes in the surface, which the moon presents to us, go by the name of Libration; and are distinguished as libration in longitude and latitude, in right ascension and declination; these librations are also affected by the parallax, which produces a daily change in the quantity of each of them. DELAMBRE.

881. Jupiter, Saturn, and Mars, are compressed at the poles, in the same manner as the Earth. The compression of Jupiter is $\frac{1}{13}$, of Saturn $\frac{1}{11}$, and of Mars $\frac{1}{16}$.

Were Venus flattened at the poles as much as the earth is, the quantity could not be measured by our micrometers. DELAMBRE.

882. The axes of several of the planets are inclined towards their orbits, in the same way as that of the Earth's is; the inclination of the axis of Venus is 75°; of Mars, 30° 18′; of Jupiter, 2° to 3°. DELAMBRE.

Of the Satellites.

883. Jupiter is accompanied by four Satellites; they revolve around him in orbits that are but little inclined to his equator. Their motions have been carefully observed ever since they were first discovered, and tolerably accurate tables of their motions have at last been constructed. By means of these, their eclipses by the shadow of the planet are predicted, and employed in determining terrestrial longitudes.

884. Saturn is distinguished from the other planets, by a plane ring with which he is surrounded. He has seven satellites, six of which revolve in orbits that nearly coincide with the

plane of his equator, while the orbit of the seventh is inclined at an angle of about 30°.

> The ring of Saturn presents a variety of phases, and sometimes disappears altogether. This last occurrence happens either when the plane of the ring passes through the earth, or when it passes through the sun. The ring, by the best observations, appears to be double, and it has a rotary motion around the planet. DELAMBRE.

885. Uranus has six satellites, whose orbits are nearly perpendicular to the ecliptic, and Herschell conceives that he has seen a ring surrounding the planet like that of Saturn. DELAMBRE.

886. The phenomena of these planets, with their secondaries, resemble, in miniature, those of the solar system itself; their motions are found to follow the laws of Kepler, and they are liable to perturbations, arising from their own mutual action, in the same manner as the primaries. DELAMBRE.

OF THE ABERRATION AND ANNUAL PARALLAX OF THE STARS, AND OF THE NUTATION OF THE EARTH'S AXIS.

Aberration.

887. The motion of light from one point on the surface of the earth to the other is so rapid as to be performed in a space of time that is absolutely insensible. Were it a sensible quantity at the distance of the lunar orbit, it would affect the time of the beginning of both solar and lunar eclipses; but it would also equally affect the end, and the difference between the errors it produced in different eclipses would be imperceptible; as a constant error cannot be either observed or corrected, these celestial phenomena would afford us no measure of the velocity of light. It is otherwise with the eclipses of Jupiter's satellites; his distance from us varies by twice the ra-

dius of the earth's orbit; and it is found, that if the tables be calculated for the mean distance of Jupiter, the eclipses of his satellites happen 8 minutes sooner, when he is in opposition to the sun, and 8 minutes later, when he is near conjunction.

Hence we infer that light traverses the semidiameter of the earth's orbit in 8 minutes, or has a velocity of nearly 12 millions of miles per minute.

888. The phenomenon, called the aberration of the fixed stars, arises from the motion of the rays of light, combined with the motion of the earth in its orbit. For if the velocity of the earth have any determinate ratio to the velocity of light, it will be evident from the mechanical theorem of the composition and revolution of forces, that we shall see a star, not in the direction in which it actually is, but in the direction of the diagonal of a parallelogram, one side of which represents in magnitude and direction the velocity of light, the other that of the earth in its annual orbit.

The variation in the apparent plane of the fixed stars, which arises from this cause, was first observed by Bradley, and he also pointed out the cause. DELAMBRE.

The earth moves through an arc of 20″, 232, while light traverses the semidiameter of its orbit; 20″, 232 should therefore be the maximum aberration, and this is found by observation to be true. Every star appears to describe a small ellipsis in the heavens, of which the true place of the star is the centre; whose semitransverse axis is 20″, 232, in the direction of a tangent to the parallel of the star's latitude; and whose semiconjugate axis is 20″, 232 × sin lat. of the star.

If the star is in the pole of the ecliptic, it appears to describe a circle whose radius is 20″, 232; if the star is in the ecliptic, it describes a straight line in the plane of that circle, whose length is 20″, 232. PLAYFAIR, Vol. II. § 210 and 211.

The error arising from aberration is computed, in the case of the most important stars, and their corrected places given, in the *Nautical Almanac* and *Connaisance du Tems.*

889. The motion of the earth in its orbit is fully established by the aberration of the fixed stars; nearly all the other phenomena of the heavens might be explained by the system of Tycho Brahe; but this last can arise from no other cause than the motion of the earth around the sun in the course of a year. DELAMBRE.

890. The moon and sun have aberrations, arising from the same cause as that of the fixed stars, but these are nearly constant quantities; when, however, the true place of the sun is to be known, as in the calculations of the geocentric places of the planets, we add 20″, 25 to the place taken from the tables of an ephemeris. DELAMBRE.

Annual Parallax of the Stars.

891. It being demonstrated by the aberration of the fixed stars, that the earth revolves around the sun, those bodies should have an annual parallax. But it is so small that it has never been measured by astronomers. Bradley found no sensible parallax in the stars he observed in order to determine the aberration; a parallax of from 3″ to 5″ has however been suspected, although not proved to exist, in Lyra, and in some of the stars of Cassiopeia.

Nutation.

892. A small inequality that has been observed in the precession of the equinoxes, and in the mean obliquity of the ecliptic, is known by the name of the Nutation.

Bradley, in making observations to determine the truth of his theory of aberration, discovered the stars to be affected by another inequality; the maximum of this he found to be 9″, and its period 18 years. This period being the same as that of the revolution of the moon's nodes, he was induced to compare the inequality with that motion, and he found that the quantity of the nutation depended on the position of the lunar nodes. PLAYFAIR, Vol. II. § 217.

The phenomena of nutation may be represented by supposing, that, while a point that may be considered as defining the mean place of the pole of the equator, describes a circle in the heavens around the pole of the ecliptic, at a distance equal to the mean obliquity of the ecliptic, and with a retrograde motion of 50″ annually; another point, representing the actual pole of the equator, moves around the former at the distance of 9″, so as to be always 90° more easterly than the moon's ascending node. The inequality thus produced in the precession of the equinoxes, and in the obliquity of the ecliptic, will exactly agree with the appearances as observed. PLAYFAIR, Vol. II. § 218.

OF THE COMETS.

893. Comets are bodies belonging to the solar system, that move in very eccentric orbits, and are only visible to us near the time of their passing their perihelion. Such is the rapidity of their motion that they soon retire to so great a distance as to render it impossible to see them. Their magnitudes also are for the most part small, and they are surrounded by nebulous matter, which unfits them for reflecting much light.

They are called comets, from the circumstance of their being most commonly attended with a tail, or, as it was called by the ancients, hair. DELAMBRE.

894. From the great brilliancy that some of the comets occasionally exhibit, from the rapidity of their motion, and the short time for which they are visible, it may readily be inferred that they must move in very eccentric ellipses. The mode of calculating the several equations for determining the place, and

predicting the return, of a body revolving in a re-entering orbit, is rendered more intricate, as the ellipse becomes more eccentric; hence, in the case of comets, the modes used in reference to the planets can no longer be employed.

The several methods that have been successively employed in determining the elements of the orbits of comets, may be seen by referring to DELAMBRE, Vol. III. chap. XXXIII.

Their general principle is, to take three observed geocentric places, and three radii vectores; from these three points in a parabolic orbit, the place of the focus, of the perihelion, and three anomalies, are deduced; the place of the node, and the inclination of the orbit, may be calculated from two latitudes and the difference of longitude on the days when they are observed. If these agree with the deduction from the latitude of the third observation, the nearest parabolic orbit may be conceived to be discovered; and this is reduced to an ellipsis by methods that may be found in DELAMBRE, ubi supra.

Should the first parabola not fulfil the requisite conditions, a second must be calculated, or, rather, the first corrected, and so on, until one is found that represents all the phenomena. DELAMBRE.

Of all the comets that have been observed, there is but one, the elements of whose orbit have been very accurately ascertained; this is the comet of Halley; its period is about 75 years. DELAMBRE.

895. Comets being bodies that describe elliptical orbits around the sun, placed in one of the foci, are therefore of the same general class with the planets; the most remarkable difference that has yet been observed between them, is in the great eccentricity and in the inclination of their orbits; the latter, in the case of the planets, is never greater than 35°, and it only amounts to that in the case of one of the new small planets; while, in the case of the comets, it is sometimes as great as 180°.

The comets are much smaller than any of the old planets;

but when they are compared with Vesta, Ceres, &c., this can no longer be considered an essential difference.

The comets are surrounded with nebulæ, or masses of faint light; but in this respect, also, the new planets form an intermediate link.

To strengthen the analogy between comets and the planets, a body has lately been discovered, whose orbit is of a nature intermediate between the very eccentric curves described by the former, and those nearly circular, described by the latter bodies.

Comets are probably spherical bodies, but their nebulæ for the most part prevent us from observing the phases to which their sphericity renders them liable; phases were, however, observed in the comet of 1682.

> The nebula appears to produce an effect similar to that which globes of ground glass do on the Argand lamp, and we see, instead of a well-defined sphere, or its proper phase, merely a mass of denser light in the centre. DELAMBRE.

896. The comets shine, not by their own light, but by reflecting that of the sun; this fact, however, has lately been called in question by some, who imagine that all the bodies of the system have a small and faint light inherent in themselves.

897. The nebula that surrounds a comet is by some considered as its atmosphere; this is supposed to be much more extensive in proportion than that of the planets, and to vary in bulk with the distance of the comet from the sun; when the comet approaches the sun, its atmosphere is much heated, and expands, while its attraction for the comet being thus lessened, it is left forming a stream of light behind it, that is called the tail.

This atmosphere, however, is found to precede the comet, after it has passed its perihelion; to account for this phenomenon, it has been supposed that an impulse is given it by the impact of the rays of light.

The tails of comets subtend angles of various dimensions; some have been seen, where the tail appeared 90° in length.

The apparent velocity of a comet varies not only with its distance from the sun, but in consequence of its position in respect to the earth. One was observed in 1472, that appeared to describe an arc of 120° in 24 hours.

898. The ignorant have been accustomed to consider comets as the messengers and forerunners of divine wrath; even the learned have supposed that some one or other might impinge against the earth, and change its external appearance, or, coming near it, cause deluges. Some have supposed that the earth was originally a comet; others, that the moon is one that has been arrested in its course by the attraction of the earth.

None of these hypotheses, however, will bear the test of accurate examination. DELAMBRE.

This is most remarkable in relation to the supposition that a comet might impinge against the earth; for of 117, that have been observed, there is not one that can ever come into contact.

899. It has been disputed whether the comets were as ancient as the other bodies of the system; but of this we have no means of judging. Herschell has supposed that they might occasionally be formed by the collection of floating particles of matter; and Newton thought that they may sometimes fall into the sun, to repair the waste caused by the continual emission of his light.

OF THE FIGURE AND MAGNITUDE OF THE EARTH.

900. We have already stated reasons for supposing that the earth is a body which does not differ much from a sphere; see § 742. Its curvature is even evident to the eye, and appears to be equal in all directions. Humboldt, on the Peak of Teneriffe, saw the horizon depressed on every side of him, to the distance of 92° from his zenith. This being, therefore, well established, it becomes a matter of curiosity to ascertain what its dimensions are, and whether the sphere is a perfect one.

The oblateness of the earth was inferred from the theory of gravitation; it has more recently been measured, as we shall presently see.

Aristotle, in his treatise *de Cœlo*, states, that the circumference was supposed, in his day, to be 400,000 stadia; as we have no means of determining the value of the stadium intended by him, and as we do not know of any actual measurement before his day, it is difficult to say how near this estimate was to the truth.

Eratosthenes, having remarked that the sun shone at Syene, in Egypt, at noon on the day of the summer solstice, to the bottom of a well, while on the same day its zenith distance was 7° 12′ at Alexandria, inferred that the distance between the two places was $\frac{1}{50}$ part of the circumference of the earth; he found this to be about 5000 stadia; he thence assumed the circumference to be 252,000 stadia, and each degree to contain 700. In this calculation, he omitted the effects of refraction and parallax, and seems to have considered the course of the stream of the Nile as a straight line coinciding with a meridian. See Delambre, Vol. III. chap. xxxv.

The detail of this operation has been handed down to us by Cleomedes.

Posidonius pointed out another method of measuring the magnitude of the earth. Canopus, a bright star in the constellation Argo, is invisible in Greece, but at Rhodes it was observed just to appear at the southern verge of the horizon, and immediately to set. The same star, at Alexandria, has an elevation, when on the meridian, of 7° 30′. Assuming the distance to be 5000 stadia, and this arc the 48th part of the whole circumference, he made the degree 666.66 stadia. In this estimate, he neglected the refraction, and supposed the two places to be on one meridian.

PTOLEMY pointed out the correction that would be necessary when the two places are not in the same meridian.

A degree was measured in the plains of Mesopotamia, by order of the Caliph Almamoun, and found to be 56⅔ Arabic miles; but the magnitude of this measure is lost, as well as that of the stadium. Bailly, in his History of Astronomy, supposes that these several lineal measures are deduced from some one accurate measurement of the earth, the memory of which is lost. Of this there is no proof whatever; and all our knowledge of the proficiency of the ancients in these departments of science would rather tend to invalidate his reasoning.

FERNEL, a Frenchman, supposing that Paris and Amiens were under the same meridian, measured the distance between them by counting the number of revolutions made by the wheels of his carriage. He thus made the length of the degree, in the latitude of Paris, 57070 French toises. LA CAILLE afterwards found it 57074.

The first trigonometrical measure of a degree was performed in Holland, between Bergen op Zoom and Alcmaer, by SNELLIUS. His calculations made it 55020 toises, but a recalculation from his observations give a length of 57115.

In England, in 1635, NORWOOD, by the union of a variety of methods, measured the distance between York and London, and estimated the degree at 57424 French toises. This measure was not known to Newton, and, in consequence, he took 60 English miles for the length of a degree; as this value does not correspond with the law of Kepler, it was near inducing him to abandon his investigation of the theory of universal gravitation.

PICARD, in 1670, measured the arc between Amiens and Paris, using greater precautions than had hitherto been applied. His horizontal angles were measured by means of a quadrant furnished with telescopic sights, and his latitudes determined by a zenith sector, constructed for the express purpose. His measure was inaccurate, in consequence of an error in the toise he made use of, and of his being unacquainted with the change in the position of the fixed stars, caused by nutation and aberration. La Caille, afterwards, applied corrections for these several causes of error.

This arc was, afterwards, in the year 1718, extended to Dunkirk, on the one hand, by Lahire, and to Perpignan, on the other, by Cassini. From these measures it was inferred, that the earth was a spheroid, elongated towards the poles. This is inconsistent both with theory, and the fact as now established.

Richer, who was sent to Cayenne in 1671; to observe the refractions in that part of the world, and to correct the theory of the sun, found that his clock, which had been regulated at Paris, went too slow. He was, therefore, compelled to shorten his pendulum. From this, he inferred, that gravity diminishes as we approach the equator. This may be caused either by an increase in the distance from the centre of the earth, or by the centrifugal force arising from the revolution of the earth on its axis. The first cause would not produce a change in the length of the mean day, of more than 0″, 9, while the second would cause one of 1″, 46. The actual effect he found to be 1″, 53. Both causes were, therefore, combined.

Newton, considering this subject in a more general manner, and with reference to his theory of universal gravitation, determined the figure of the earth to be an oblate spheriod. Supposing every particle to have an equal density, he estimated the flattening at the poles as $\frac{1}{230}$ part of the whole diameter. As this deduction differs from that drawn from the actual measurement of a portion of the meridian in France, it was proposed to settle the question, by measuring two degrees so distant from each other, that the probable error of observation would be much less than the difference in the length of the degrees. For this purpose, Codin, Bouguer, and La Condamine, set out for Peru in 1736, and, after ten years of labour, succeeded in measuring three degrees in the southern hemisphere. The difference between the greatest calculated length of one of these, and the degree measured in France, was 302 toises, the degree near the equator being so much shorter than the other. Maupertius, Clairaut, Camus, Lemonnier, and Orthier, were, at the same time, sent to Lapland, where they measured an arc of 57′ under the polar circle, and found the degree to be 349 toises more than that of Paris. From these measures it was deduced, that the compression of the earth was $\frac{1}{178}$.

The whole of the arc in France was again measured by **La Caille** in 1736; other arcs were measured subsequently in different places:

By **Mason** and **Dixon**, in Pennsylvania, in latitude 39° 12′;

By **La Caille**, at the Cape of Good Hope;

By **Maire** and **Boscovich**, between Rome and Rimini; and

By **Liesganig**, in Hungary.

This subject again attracted attention about the year 1787, when simultaneous trigonometrical surveys were instituted in France and England. In France, the arc between Dunkirk and Perpignan has since been measured for the third time, and extended through Spain to the Balcaric islands, by **Mechain** and **Delambre**. In England, the whole extent of the British dominions, from Dunnose, in the Isle of Wight, to Unst, in Shetland, has been measured by Gens. **Roy** and **Mudge**.

The observatories of London and Paris, were, at an early period of these surveys, connected by a series of triangles, and it was intended that the work should have been carried on in concert by the two nations. This was prevented by the long war that soon broke out between them.

In addition to the above, it may be mentioned, that an arc of nearly 10° has been measured in India by Colonel Lambton, and that a great survey of the coast of the United States, was commenced in 1816 by Professor Hassler. This last, begun under the best auspices, and with the most perfect instruments ever constructed, has, from some unaccountable cause, and to the disgrace of the scientific character of our country, been suspended. For a more particular history of geodetic surveys, see **Delambre**, Vol. III. chap. xxxv.

901. An arc of the meridian is measured by ascertaining the distance in some known measure between two points under the same meridian, and whose latitude is known. To determine the latitude, the French astronomers employed observations of the two meridian altitudes of circumpolar stars, taken by means of the repeating circle. The English, the altitudes of stars

observed near the zenith by the zenith sector. The French instrument has great advantages over the other, but their method appears to have been in fault, inasmuch as they did not employ meridian altitudes of southern stars, by which the errors of the instrument might have been compensated.

With the best instruments, latitudes cannot be ascertained to a greater accuracy than about 2″. BRINKLEY.

902. As it would, generally speaking, be impossible to find a district of country where a degree, or larger arc, could be measured in one straight line, the operation is performed by measuring a base, and thence determining the value of the sides of a series of triangles, whose angles are measured. The mensuration of a base is now performed with very great accuracy; the following are the principal methods that have been used:

The French academicians used, in Lapland and Peru, rods of iron.

Mechain and Delambre employed bars of platina.

General Roy used cylinders of glass; and when General Mudge repeated the measure of the base of Hounslow heath, he employed a steel chain, joined like the chain of a watch.

In using all these several instruments, a correction for the variation in length, by change of temperature, is indispensable.

A base should be of considerable length; that measured in England, on Hounslow heath, was five; and Hassler's, in the English neighbourhood in New-Jersey, seven miles.

903. In choosing the stations, it should be kept in view, that an equilateral triangle affords the best chance of accuracy in the calculated results. At each station, a signal that can be seen at a great distance is set up. In the earlier observations cones of wood were used, and it is often necessary to employ artificial objects that exist in the country. Angles are, also, frequently observed at night; Argand lamps, with parabolic reflectors, whose axis is turned towards the observer, may be used for

this purpose; blue lights are employed by the English, but they do not burn long enough to permit the repeating circle to be used.

Delambre, in his observations by day, made use of truncated pyramids of wood, and Hassler employed truncated cones of planished tin plate.

The angles in these operations being measured on the curved surface of the earth, there is an excess in the sum of the three angles of each triangle over three right angles. The triangle cannot, therefore, be considered as plane without a correction for this spherical excess.

For the methods by which this may be done, see DELAMBRE, PLAYFAIR, in Edinburgh Transactions, and HUTTON'S Course.

904. The direction of the meridian is ascertained by taking the horizontal angle contained between the vertical circle passing through a heavenly body, and that passing through one of the stations, as seen from the adjacent one. The azimuth of the heavenly body is calculated for the time, and the sum or difference of this and the observed angle, gives the angle which the direction of the station makes with the true meridian. The terrestrial angles and azimuth, were observed in the French surveys by the repeating circle, whose plane was inclined until it coincided with that of the two objects; in this mode a correction was necessary to reduce the angle to the horizon. The English had the theodolite, where no such correction was needed, but where the advantages of the principle of repetition did not apply. Repeating instruments are now constructed with both vertical and horizontal circles. Besides the application of these operations to the discovery of the true figure of the earth, the length of a quadrant of a meridian has been proposed as a standard of universal measure, and has been actually used by the French for this purpose. See VOL. I. § 181.

905. Delambre states, as the result of his observations and

calculations, that the compression at the pole is $\frac{1}{308}$ part of the equatorial diameter of the earth. Dr. Adrain makes it $\frac{1}{321}$.

Supposing the earth to be an ellipsoid of revolution, the mean diameter would be found at the latitude of 35° 16′.

The mean diameter of the earth, deduced from these measurements, is about 7920 English miles.

906. Observations on the length of the pendulum have always accompanied the operations for measuring arcs of the meridian; the most complete coincidence has been found in the oblateness of the terrestrial spheroid, as determined by these two methods; hence the very expensive and tedious processes used to measure degrees, may be considered as superfluous, except when other objects are to be attained at the same time, as where the coast of a country is to be surveyed, or its geography to be delineated.

The subject of the measurement of arcs of the meridian, is to be found fully treated of in DELAMBRE, *Base du Systime Metrique.* A synopsis of his methods is given in his Astronomie, Vol. 3. Chap. xxxv. The detail of the English operations is to be found in the several volumes of the Philosophical Transactions. It is with regret that we state, that Mr. Hassler's papers have not been given to the world; it is to be hoped that they will soon appear, as they have been communicated, more than four years since, to the American Philosophical Society, at Philadelphia.

OF THE CALENDAR.

907. Among the different divisions of time, the civil year is the most important. It must necessarily consist of an exact number of days, and thus cannot agree with the tropical year, which consists of 365 days and a fractional part.

As the tropical year includes and corresponds with the vicissitudes of the seasons, it is essential that some method be contrived, by means of which the two modes of computation shall be brought to agree from time to time.

We borrow the artifice, by means of which this is effected, from the Romans. Their calendar was reformed by Julius Cæsar, who, conceiving that the length of the year was 365¼ days, directed, that every fourth year should consist of 366 days. This year, from the circumstance of the intercalary day falling on the sixth day, before the kalends of March, was called *Bissextile;* in English, *Leap-year.* The mean year of 365¼ days, assumed as the foundation of the Julian calendar, being between 11′ and 12′ too long, an error arose, amounting to about a day in 75 years. The Council of Nice, in the year 325, finding that the equinox fell on the 21st of March, determined, that Easter should, thereafter, be held on the first Sunday after the full moon that happened next after that day of the month. Previous to this time, the celebration of Easter had been regulated by astronomical phenomena. In 1582, the equinox, instead of falling on the 21st, happened on the 11th of March. Pope Gregory XIII., wishing to correct this error, and to prevent its again occurring, suppressed 10 days, in the month of October in that year, and directed, that from that time, three of the four centurial years, in each four centuries, should not be reckoned as bissextile, but as common years.

So near is this to the truth, that it will require no more than the suppression of one day in 3,000 years to make the civil years of the Gregorian calendar correspond with the tropical year.

The Julian calendar is still used in Russia, and wherever the rites of the Greek church prevail; their days are said to be reckoned in *Old Style.* The computation, by the Gregorian calendar, is called *New Style.* The difference between these, is now 12 days; for a day has been suppressed since the introduction of the latter in each of the years 1700 and 1800.

908. The year is divided into 12 months; these have certain arbitrary and unequal lengths, borrowed from the Roman calendar; the intercalation is effected by adding a day to the month of February. A better method than this, would certainly be, to make each month of the equal length of 30 days, and add, at the end of each year, five or six intercalary days, according as it were, or were not, leap year.

909. A year, composed of 365 days, is equal to 52 weeks and one day more, while a leap year is two days more than this number of weeks; in composing a perpetual calendar or almanac, the days cannot be distinguished by their name, nor can any two contiguous years begin with the same day. The several days are, therefore, marked by the first seven letters of the alphabet. The letter A always corresponds to the first day of the year; if the year begin on Sunday, all the days in the calendar marked with the letter A are Sundays, and the dominical letter is A. If the year commence on Monday, B is the dominical letter, and so on. Leap year adds a day to the year, but not to the week. There are, therefore, two dominical letters for each leap year, one of which serves until 28th of February, after which time the other must be used. In the Julian calendar the succession of dominical letters is completed in 7 × 4 = 28 years.

This period is called the solar cycle; it is no longer used except in calculating the moveable feasts of the Catholic and Episcopal churches.

The other elements that are used in these calculations are the indiction, the epact, and the golden number; the method of using these is given in the book of Common Prayer of the Episcopal church. It is needless to explain them here, for they are no longer employed by astronomers, and the deductions obtained from them are sometimes erroneous. Thus, in the year 1798, the first of April should have been Easter Sunday, whereas it was not observed until the eighth; and in 1818 it should have fallen on the 29th of March, whereas it was celebrated on the 22d.

Were it essential that the celebration of Easter should be determined in strict conformity with astronomical phenomena, it would be necessary to calculate the time of its recurrence from the accurate tables of solar and lunar motion with which the improvement of the science of astronomy has furnished us. See DELAMBRE, Vol. III. chap. XXXVI.

OF PROJECTIONS OF THE SPHERE.

910. It is impossible to delineate on a plane any figure that can accurately resemble one that is extended in three dimensions. A certain degree of resemblance may, however, be obtained, and in the construction of maps this has been sought for in two ways : (1) By the *projection* of the spherical surface on a plane, such as it would be seen by an eye situated in some particular point; and (2) By the *developement* of a spherical on a plane surface. PLAYFAIR, Vol. II. § 70.

911. The STEREOGRAPHIC PROJECTION is a representation of the surface of the sphere on the plane of a great circle, such as it would appear to an eye situated in its pole. This circle is called the primitive.

(1) The stereographic projection is usually limited to the representation of the hemisphere opposite to the eye, and whose projection falls within the primitive; but it may be extended to a portion of the hemisphere in which the eye is placed, and whose projection falls without the primitive.

(2) Great circles that pass through the poles of the primitive, are called *right* circles, and are projected into right lines, passing through the centre of the primitive; they are measured on a scale of semitangents, beginning at the centre of the primitive.

(3) Circles not passing through the poles of the primitive are called *oblique ;* and whether great or less, are projected into circles.

(4) The distance between the centre of an oblique circle and the centre of the primitive, is equal to the tangent of its elevation, or of the angle which its plane makes with the primitive; and the radius is equal to the secant of its elevation.

(5) The distance of the circumference of an oblique great circle, from the centre of the primitive, measured on a line passing through the centre of both, is equal to the semitangent of the complement of its elevation. This line, which passes through the centre of any oblique

circle, and of the primitive, is called the line of measures of the former.

(6) The distance of the pole of an oblique great circle, from the centre of the primitive, on the line of measures, is equal to the semitangent of its elevation.

(7) The distance of the centre of a less circle, whose pole is in the circumference of the primitive, from the centre of the primitive, is equal to the secant of its elevation; and its radius is equal to the tangent of its elevation.

(8) Less circles, whose poles are not in the primitive, are projected by setting off, on their line of measures, the distance of their nearest and remotest points from the centre of the primitive, taken from the scale of semitangents. Their centre bisects the interval between these two points.

(9) Arcs of oblique great circles are measured by drawing straight lines through their poles; if one of these pass through the centre of the primitive, and another be drawn through any other point, they will intercept equal arcs of the oblique circle and the primitive.

(10) Angles, at the circumference of the primitive, are measured by the arc cut from a right circle that passes through the centres of the two circles that form the angle.

(11) Angles made by right circles at the centre of the primitive, are measured on its circumference.

(12) To measure angles within the primitive, lines are drawn from the angular point to the poles of the circles that form the angle; these cut off the measure from the primitive.

912. The stereographic projection is easy of construction, as it admits of no other lines than straight and circular; and it has this additional advantage, that the angle made by any two circles in the projection, is equal to that at which they cut each other on the surface of the sphere; thus, the parallels of latitude in this projection cut the meridians at right angles. On

the other hand, it has the disadvantage, that equal areas on the surface of the sphere occupy very unequal spaces, according as they are situated near the centre or the circumference of the primitive.

The use of this projection is, therefore, limited to the construction of the problems of spherical trigonometry, to the representation of an entire hemisphere of the earth, and to planispheres of the heavens.

913. In the ORTHOGRAPHIC PROJECTION, the eye is supposed to be placed in the axis of the circle on whose plane the projection is made, but at an infinite distance. The rules for this projection are as follows, viz.:

(1) Right circles are projected into straight lines, and measured on the scale of sines.

(2) Less circles, perpendicular to the primitive, are also projected into straight lines, and measured on scales of sines, adapted to their respective lengths.

(3) Oblique circles are projected into ellipses. The greater axis of the ellipse, into which an oblique great circle is projected, is the diameter of the primitive, and its lesser axis is equal to the cosine of the elevation.

(4) Perpendiculars, drawn from the greater axes, cut off equal portions from the oblique circle and from the primitive.

The use of the orthographic projection, is confined to the graphical construction of the phenomena of eclipses, and the transits of the inferior planets.

914. In the representation of portions of the earth's surface, less than hemispheres, the projection of Flamstead is, for the most part, employed.

(1) In this construction, a straight line is drawn for the meridian of the middle of the map; on this equal distances are marked off, to show the degrees of latitude. From a point in this line, produced

as a centre, and with a radius, that is to the length of one of the spaces intended to mark a degree of latitude as the cotangent of the middle latitude to an arc of one degree, an arc of a circle is described to represent the degree of latitude. From the same centre, are described other arcs through the different points marked off on the meridian of the middle of the map; these represent the several parallels of latitude. On each of these parallels, equal distances are set off on each side of the middle of the map, which are to the assumed degree of latitude as its cosine is to radius. The degrees of longitude, being thus marked on the parallels, the curves that pass through corresponding points in the different parallels are meridians. They are curved more and more as they retire from the middle of the map. But when the extent to be represented is not great, they afford a pretty good representation of the convex surface. This construction has a very remarkable property; the quadrilaterals on the map, included between the same parallels, have nearly the same ratio to each other with the quadrilaterals which they represent on the surface of the globe. PLAYFAIR, Vol. II. § 72.

915. The projection called MERCATOR's, from the name of its inventor, is chiefly used for nautical charts. The meridians are all parallel; the degrees of longitude are all equal; the parallels of latitude are parallel lines; and the degrees of latitude increase on the chart, in the same ratio that the degrees of longitude diminish on the sphere.

In this construction the bearings of places from each other are the same as on the surface of a sphere, while the lines of azimuth, or rhumbs, which, on a sphere, are spirals, are here straight lines; hence its value for the purposes of navigation.

The method of constructing this projection was first fully explained by Edward Wright in 1599, who showed, that the parts into which the meridian is divided, are inversely as the cosines, or directly [illegible] the secants of the latitudes.

It was, afterwards, observed, that the meridian line thus divided was analogous to a scale of logarithmic tangents of the half complements of the latitudes; this was first demonstrated by James Gregory, and, afterwards, in a more concise manner, by Halley. Playfair, Vol. II. § 73.

OF THE CALCULATIONS OF PRACTICAL ASTRONOMY.

Of Spherical Trigonometry.

916. The calculations of practical astronomy are chiefly performed by means of spherical trigonometry. This branch of mathematical science, which was rendered much more easy in practice by the introduction of logarithmic sines, &c. has been still further improved by the introduction of the analytic method of proceeding, for which we are indebted to Euler.

917. Of the relations that subsist between the several trigonometric lines, the more important are as follows :

Value of sin a	Value of cos a	Value of tan a
(1) $\cos a \tan a$	(1) $\sin a \cot a$	(1) $\dfrac{\sin a}{\cos a}$
(2) $\dfrac{\cos a}{\cot a}$	(2) $\dfrac{\sin a}{\tan a}$	(2) $\dfrac{1}{\cot a}$
(3) $\surd\,(1-\cos^2 a)$	(3) $\surd\ 1-\sin^2 a)$	(3) $\surd\left(\dfrac{1}{\cos^2 a}-1\right)$
(4) $\dfrac{1}{\surd\,(1+\cot^2 a)}$	(4) $\dfrac{1}{\surd\,(1+\tan^2 a)}$	(4) $\dfrac{\sin a}{\surd\,(1-\sin^2 a)}$
(5) $\dfrac{\tan a}{\surd(1+\tan^2 a)}$	(5) $\dfrac{\cot a}{\surd\,(1+\cot^2 a)}$	(5) $\dfrac{\surd\,(1-\cos^2 a)}{\cos a}$
(6) $\dfrac{1}{\text{cosec.}\, a}$	(6) $\dfrac{1}{\sec a}$	(6) $\dfrac{2\tan\frac{1}{2}a}{1-\tan^2\frac{1}{2}a}$

In the first and second quadrants, the algebraic sign of the sine is +, in the third and fourth, −. In the first and fourth quadrants, the sign of the cosines is +, in the second and third, −. In the first and third quadrants, the sign of the tangents is +, in the second and fourth −. In the first and third quadrants, the sign of the cotangents is +, in the second and fourth, −.

218. The whole of analytic spherical trigonometry is included in a single formula, whence three other fundamental theorems are deduced. If a, b, c, denote the three sides, A and B and C the three angles, of which A is opposite to a, B to b, and C to c, the four formulæ are as follows, viz.

$$\left.\begin{array}{l}\cos a = \cos A \sin b \sin c + \cos b \cos c \\ \cos b = \cos B \sin a \sin c + \cos a \cos c \\ \cos c = \cos C \sin a \sin b + \cos a \cos b\end{array}\right\} \quad A$$

$$\left.\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}\right\} \ldots\ldots\ldots\ldots B$$

$$\left.\begin{array}{l}\cos a \cos B = \cot c \sin a - \sin B \cot C \\ \cos b \cos C = \cot a \sin b - \sin C \cot A \\ \cos c \cos A = \cot b \sin c - \sin A \cot B\end{array}\right\} \quad C$$

$$\left.\begin{array}{l}\cos C = \cos c \sin A \sin B - \cos A \cos B \\ \cos A = \cos a \sin B \sin C - \cos B \cos C \\ \cos B = \cos c \sin C \sin A - \cos C \cos A\end{array}\right\} \quad D$$

These formulæ are not well adapted to calculations, when performed by means of logarithms; for which reason, others that may be deduced from them are chiefly employed. These may be divided into two classes, one for the resolution of right, the other of oblique, angled triangles.

It sometimes happens, however, that the processes deduced from the four fundamental formulæ, are much more concise than the others.

919. The formulæ by which the several cases of right angled spherical trigonometry may be solved, are as follows; c representing the hypothenuse, and C the right angle:

$$
\begin{aligned}
&(1)\ \cos c = \cos a \cos b\\
&(2)\ \cos c = \cot A \cot b\\
&(3)\ \sin a = \sin c \sin A\\
&(4)\ \tan a = \sin b \tan A\\
&(5)\ \tan a = \cos B \tan c\\
&(6)\ \cos A = \sin B \cos a
\end{aligned}
$$

920. The several cases of oblique angled spherical trigonometry may be solved in the following manner:

1st. Given the three sides, (a, b, c,) to find an angle, (A.)

$$(1)\ \tan \tfrac{1}{2} A = \sqrt{\frac{\sin \frac{1}{2}(a+b-c) \sin \frac{1}{2}(a+c-b)}{\sin \frac{1}{2}(b+c-a) \sin \frac{1}{2}(a+b+c)}}$$

Or,

$$(2)\ \sin \tfrac{1}{2} A = \sqrt{\frac{\sin \frac{1}{2}(a+b-c) \sin \frac{1}{2}(a+c-b)}{\sin b \sin c}}$$

Or,

$$(3)\ \sin \tfrac{1}{2} A = \sqrt{\frac{\sin\left(\frac{a+b+c}{2} - b\right) \sin\left(\frac{a+b+c}{2} - c\right)}{\sin b \sin c}}$$

DELAMBRE, Vol. I. chap. x.

2d. Given the three angles, (A, B, C,) to find a side, (a.)

$$(4)\ \tan \tfrac{1}{2} a = \sqrt{\frac{-\cos \frac{1}{2}(B+C-A) \cos \frac{1}{2}(A+B+C)}{\cos \frac{1}{2}(A+B-C) \cos \frac{1}{2}(A+C-B)}}$$

Or,

$$(5)\ \sin \tfrac{1}{2} a = \sqrt{\frac{-\cos \frac{1}{2}(A+B-C) \cos \frac{1}{2}(A+C-B)}{\sin B \sin C}}$$

3d. Given two sides, (b and c,) and the contained angle, (A,) to find the other angles:

$$(6)\ \tan \tfrac{1}{2}(B + C) = \frac{\cos \frac{1}{2}(b - c)}{\cos \frac{1}{2}(b + c)} \cot \tfrac{1}{2} A$$

$$(7)\ \tan \tfrac{1}{2}(B - C) = \frac{\sin \frac{1}{2}(b - c)}{\sin \frac{1}{2}(b + c)} \cot \tfrac{1}{2} A$$

$$B = \tfrac{1}{2}(B + C) + \tfrac{1}{2}(B - C) \qquad C = \tfrac{1}{2}(B + C) - \tfrac{1}{2}(B - C)$$

To find the third side:

$$(8)\ \sin a = \frac{\sin b \sin A}{\sin B}$$

4th. Given two angles, (B and C,) and the contained side, (a,) to find the other sides, (b and c):

$$(9)\ \tan \tfrac{1}{2}(b + c) = \frac{\cos \frac{1}{2}(B - C)}{\cos \frac{1}{2}(B + C)} \tan \tfrac{1}{2} a$$

$$(10)\ \tan \tfrac{1}{2}(b - c) = \frac{\sin \frac{1}{2}(B - C)}{\sin \frac{1}{2}(B + C)} \tan \tfrac{1}{2} a$$

$$\tfrac{1}{2}(b + c) + \tfrac{1}{2}(b - c) = b \qquad \tfrac{1}{2}(b + c) - \tfrac{1}{2}(b - c) = c$$

To find the third angle (A):

$$(11)\ \sin A = \frac{\sin C \sin a}{\sin c}$$

Or,

$$(12)\ \sin A = \frac{\sin B \sin a}{\sin b}$$

5th. Given two sides, (a and c,) and an angle (C) opposite to one of them, to find the angle opposite to the other, (A):

$$(11)\ \sin A = \frac{\sin a \sin C}{\sin c}$$

6th. Given two angles, (A and C,) and a side opposite to one of them, (c,) to find the side opposite to the other, (a):

$$(8)\ \sin a = \frac{\sin c \sin A}{\sin C}$$

LACROIX, § 57.

These several cases may also be solved by letting fall a perpendicular from one of the angles on the opposite side; using the same notation as before, and with the addition of S and S′ to signify the segments of the base c, V and V′ to denote the portions into which the perpendicular divides the vertical angle, we shall have the following formulæ:

Case 1st. a, b, c, given:

$(1)\ \tan \tfrac{1}{2}(S - S') = \tan \tfrac{1}{2}(b - a) \tan \tfrac{1}{2}(b + a) \cot \tfrac{1}{2} c$

$(2)\ S = \tfrac{1}{2} c + \tfrac{1}{2}(S - S')$, and $S' = \tfrac{1}{2} c - \tfrac{1}{2}(S - S')$

$(3)\ \cos A = \tan S \cot b$, $(4)\ \cos B = \tan S \cot a$

$$(5)\ \sin C = \frac{\sin c \sin A}{\sin a} = \frac{\sin c \sin B}{\sin b}$$

Case 2d. A, B, and C, given:

$(6)\ \tan \tfrac{1}{2}(V - V') = \tan \tfrac{1}{2}(B - A) \tan \tfrac{1}{2}(B + A) \tan \tfrac{1}{2} C$

$(7)\ V = \tfrac{1}{2} C + \tfrac{1}{2}(V - V')$, $V' = \tfrac{1}{2} C - \tfrac{1}{2}(V - V')$

$(8)\ \cos b = \cot A \cot V$, $(9)\ \cos a = \cot B \cot V'$

$$(10)\ \sin c = \frac{\sin C \sin a}{\sin A} = \frac{\sin C \sin b}{\sin B}$$

Case 3d. *b*, *c*, and A, given:

(11) $\tan S = \cos A \tan b$

(12) $S' = c - S$

(13) $\cos a = \dfrac{\cos b \cos S'}{\cos S}$

(14) $\tan B = \dfrac{\tan A \sin S}{\sin S'}$

(15) $\sin C = \dfrac{\sin c \sin A}{\sin a} = \dfrac{\sin c \sin B}{\sin b}$

Case 4th. A, C, and *b*, given:

(16) $\cot V = \tan A \cos b$

(17) $V' = C - V$

(18) $\cos B = \dfrac{\cos A \sin V'}{\sin V}$

(19) $\tan a = \dfrac{\tan b \cos V}{\cos V'}$

(20) $\sin c = \dfrac{\sin C \sin a}{\sin A} = \dfrac{\sin C \sin b}{\sin B}$

In the first four cases, the solution admits of no doubt. The other two cases are doubtful, and frequently admit of two conclusions. The mode of proceeding is also double. In practice, however, it most frequently happens, that but one of the conclusions is within the limits of possibility.

Case 5th. Given a, b, and A:

Solution 1st.

$$\tan S = \cos A \tan b \quad (11)$$

$$\cos S' = \frac{\cos a \cos S}{\cos b} \quad (13)$$

$$c = S \pm S'$$

$$\tan B = \frac{\tan A \sin S}{\sin S'} \quad (14)$$

$$\sin C = \frac{\sin c \sin A}{\sin a}$$

$$= \frac{\sin c \sin B}{\sin b}$$

Solution 2d.

$$\cot V = \tan A \cos b \quad (16)$$

$$\cos V' = \cos V \tan b \cot a \quad (19)$$

$$C = V \pm V'$$

$$\cos B = \frac{\cos A \sin V'}{\sin V} \quad (18)$$

$$\sin c = \frac{\sin C \sin a}{\sin A}$$

$$= \frac{\sin C \sin b}{\sin B}$$

Case 6th. Given A, B, and b:

Solution 1st.

$$\tan S = \cos A \tan b \quad (11)$$

$$\sin S' = \sin S \tan A \cot B \quad (14)$$

$$c = S \pm S'$$

$$\cos a = \frac{\cos b \cos S'}{\cos S} \quad (13)$$

$$\sin C = \frac{\sin c \sin A}{\sin a}$$

$$= \frac{\sin c \sin B}{\sin b}$$

Solution 2d.

$$\cot V = \tan A \cos c \quad (16)$$

$$\sin V' = \frac{\cos b \sin V}{\cos A} \quad (18)$$

$$C = V \pm V'$$

$$\tan a = \frac{\tan b \cos V}{\cos V'} \quad (19)$$

$$\sin c = \frac{\sin C \sin a}{\sin A}$$

$$= \frac{\sin C \sin b}{\sin B}$$

DELAMBRE, Vol. I. chap. x.

921. By the above methods all the cases in spherical trigonometry may be solved. It however frequently happens, that shorter processes, depending upon occasional artifices, may be employed to advantage. This is more particularly the case when rules are laid down for the use of persons unacquainted with the principles on which the methods are founded. In such cases, tables are frequently constructed, whence the results of the calculation of several logarithms may be taken out by mere inspection. In almost every case, however, the trigonometrical method has the advantage.

In applying logarithms to the calculation of the above formulæ, it is usual to make all the quantities additive by taking the arithmetic complements of the logarithms of those quantities that stand in the denominators of fractions. These correspond in the case of the sines with the logarithm of the cosecant, and in that of the cosines with the logarithm of the secant; for this reason we introduce those lines in the practical examples.

Of Ephemerides.

922. The labour of the practical astronomer may be much abridged by the use of the accurate ephemerides that are published by the British and French boards of longitude; the former by the name of the "*Nautical Almanac,*" the latter of the "*Connaissance du Tems.*" In these publications, the places and phenomena of the heavenly bodies are given as calculated for the meridian of the respective observatories of Greenwich and Paris. As the Nautical Almanac is regularly republished in this country, we shall confine ourselves to the explanation of the use of the several tables which it contains.

923. All the calculations of the Nautical Almanac, except of the eclipses of Jupiter's satellites, are made for the apparent time of the meridian of Greenwich. Astronomers, in reckoning time, begin at noon, or 12 hours later than the beginning

of the civil day of the same denomination, and count 24 hours up to the succeeding noon.

924. In using the Nautical Almanac for any other meridian, it is necessary to reduce the apparent time of the place to that of Greenwich. This is done by subtracting the difference of longitude, in time, from the apparent time at the place, when this is situated west of Greenwich; and adding it when it is to the east.

Degrees of the Equator may be converted into time by the formula,

$$T = \frac{D}{15} = \frac{4D}{60}$$

EXAMPLE.—Convert 74° 56′ into time:

$$\begin{array}{r} 74^\circ\ 56' \\ 4 \\ \hline 60\,)\,299\ 44 \\ \hline \text{hrs. } 4\ 59'\ 44'' \end{array}$$

Time may be converted into degrees of the Equator, by the formula,

$$D = 10\,T + \frac{10\,T}{2}$$

EXAMPLE.—Convert 4 hrs. 56 min. into Degrees:

$$\begin{array}{r} 4\ 56 \\ 10 \\ \hline 2\,)\,49\ 20 \\ 24\ 40 \\ \hline \text{deg. } 74\ 00 \end{array}$$

925. In the Nautical Almanac, the sun's declination, longitude, and right ascension, are calculated for the time of apparent noon at Greenwich, and are, therefore, given for intervals

of 24 hours. The sun's motion may be considered as equable during this space of time, and the place of the sun may, therefore, be calculated for any given time by a simple proportion. Where the time given is that of any other meridian, it must be reduced to the time at Greenwich before the proportion is stated, and this is a general rule, applicable to every case whatsoever. On the same page with the declination, right ascension, and longitude of the sun, the equation of time is to be found. This is used in regulating clocks and watches, where the time has been deduced from transits of the sun over the meridian, or from observations of his altitude.

926. The semidiameter, and hourly motion of the sun, are laid down for intervals of six days. The amount of either of these quantities may be found for any intermediate time, by a statement in proportion.

927. The motion of the moon being much more rapid than that of the sun, her place is given for intervals of no more than 12 hours; and her longitude, latitude, declination, and right ascension, are calculated for noon and midnight on the meridian of Greenwich. So irregular is the motion of the moon, that a simple proportion will not suffice to find either of these quantities for a given intermediate time; but, for this purpose, a table of a quantity called the equation of second differences has been constructed. It was formerly to be sought in the collection of requisite tables, but is now bound up with the Nautical Almanac. In using this table, the declination, or latitude, &c. of the moon, as the case may be, is to be taken for the two nearest periods preceding and following the time for which the calculation is to be made. Their difference is to be used in a proportion, as in the case of the sun, to find the proximate place. The correction is then to be found by taking the two places next preceding, and the two next following the given time; these are to be written beneath each other, and their three differences taken; the two differences of these differences, or the second differences,

are next to be taken, and their mean forms one of the arguments with which the table is to be entered. The other argument is the time from the nearest period for which the place of the moon is given.

This subject will be best illustrated by the following example, taken from the Nautical Almanac:

Required, the Moon's Longitude for July 16th, 1767, 16 hrs. 22′ 16″.

For the mean Longitude:

Longitude, July 16th, at 12 hrs.	0s 6° 40′ 25″
July 17th, noon,	0s 13° 47′ 48″
Difference,	0s 7° 7′ 23″

12 hrs. : 4 hrs. 22′ 16″ :: 7° 7′ 23″ : 2° 35′ 41″

being the moon's motion during the interval, at a mean rate. For the correction, take

	Moon's Longitude.	*1st dif.*	*2d dif.*
July 16. noon,	11s 29° 29′ 24″		
		7° 10′ 51″	
12 hrs.	0s 6° 40′ 25″		+ 3′ 28″
		7° 7′ 53″	
17, noon,	0s 13° 47′ 48″		+ 3′ 44″
		7° 3′ 39″	
12 hrs.	0s 20° 51′ 27″		

The mean of the second differences is 3′ 36″. With this, and 4 hrs. 22′ as arguments, enter the table of second differences; the longitude of the moon is then found as follows:

Longitude, 16th, at 12 hours, ...	0s 6° 40′ 25″
Difference for 4 hrs. 22′ at a mean rate,	0s 2° 35′ 41″
Equation of second difference, ...	0s 0° 00′ 24″
Moon's true Longitude,	0s 9° 16′ 30″

When the difference of the moon's longitude for intervals of 12 hours decreases, the sign of the second difference is +, and the correction is additive. When the difference increases, the sign of the second difference is —. If the difference first increase, and then decrease, or vice versa, the second differences have contrary signs, and are to be subtracted from each other, and the half taken for the mean; the equation is to be added or subtracted, according to the algebraic sign which the mean has.

The same method applies to the calculation of the latitude, declination, and right ascension, with this remark, that when the declination or latitude change their denomination during either of the intervals, their difference is to be taken as if they had contrary algebraic signs.

928. To facilitate the method of finding the longitude by lunar observations, the distances of the moon from stars, or from the sun, are calculated for every three hours. The time at Greenwich, at which the observed distance happens, is ascertained by a simple proportion; and this is generally solved by the use of proportional logarithms, constructed for the purpose, and contained in the several collections of Requisite Tables.

929. The other useful calculations given in the Nautical Almanac, are: the time of the sun's semidiameter passing the meridian, which serves to reduce the transit of either limb of the sun to that of his centre, when the passage of but one limb could be observed; the place of the moon's node; the times of the eclipses of Jupiter's satellites; the places of the planets; the semidiameter and horizontal parallax of the moon; and the conjunctions of the moon with fixed stars.

Modes of Finding the Latitude.

930. The simplest mode of finding the latitude, is that used by nautical men, where the meridian altitude of the sun is observed by a reflecting instrument; his declination is then calculated for the time at Greenwich, by means of the nearest estimated longitude; the sum, or difference of this and the observed altitude, is the complement of the latitude.

The same method is also applicable to the fixed stars and the planets; in the first of which the declination is known with greater certainty than that of the sun; and to the moon, where, however, a small error in the estimate of the longitude of the place may cause an erroneous result.

An instrument of reflection affords, to a practised observer, a mode of ascertaining very nearly the precise instant at which a heavy body culminates; hence the advantage of this method in preliminary observations even on land, where the meridian has not been marked out, nor the time ascertained with any approach to accuracy.

931. When a meridian line has been drawn, the meridian altitudes of heavenly bodies may be ascertained by means of the astronomical quadrant, or mural circle; and in portable observatories, by the altitude and azimuth circle of Troughton.

932. When the apparent time is known nearly, altitudes of the heavenly bodies may be taken a short time before, and a short time after their passing the meridian, by means of the repeating circle. The reduction of the observed altitudes to the meridian, is readily effected, and tables have been constructed for the purpose. See BREWSTER, *Cyclopedia*, article *Circle.*

933. Stars that never set have a peculiar advantage in observations for the latitude, as they are twice in the 24 hours on the meridian. Their altitudes at the times of their transits, may be observed by instruments of reflection, by the altitude and azimuth circle, and by mural instruments. They may, also, be calculated from observations taken with repeating instruments near the meridian.

To correct this last kind of observation, the French astronomers have used observations of the same stars, taken near the time of their greatest eastern and western azimuths. Observations of the meridian altitudes of bodies that pass the meridian to the south of the zenith, appear better fitted to diminish the chance of error.

934. The Latitude may also be calculated from simultaneous

observations of the altitudes of two known heavenly bodies. If h and h' be their several altitudes, t the difference of their right ascensions, D and D′ their north polar distances, P the hour angle of that body which is farthest from the meridian, L the complement of the latitude, the formulæ will be as follows, viz.

$$\tan x = \tan D' \cos t$$

$$\tan V = \tan t \sin x \operatorname{cosec} (D - x)$$

V is the angle at the star that is farthest from the meridian, and whose polar distance and altitude are h and D.

$$\cos C = \cos D' \cos (D - x) \sec x$$

$$\sin \tfrac{1}{2} W = \sqrt{\cos \left(\frac{h + h' + C}{2}\right) \sin \left(\frac{h + h' + C}{2} - h'\right) \sec h \operatorname{cosec}}$$

$$u = V \pm W$$

$$\tan z = \cos u \cot h$$

$$\tan P = \tan u \sin z \operatorname{cosec} (D - z)$$

$$\tan L = \sin h \cos (D - z) \sec z$$

It is not absolutely necessary that the observations be simultaneous; in any other event, the interval in time, between the observations, must be applied to the angle t.

On the same principle, the latitude may be calculated from two altitudes of the same heavenly body, and the time elapsed between them; in this case, D = D′, and the first triangle is isosceles, by which the calculation is somewhat shortened. The observations are frequently made when the body has equal altitudes on opposite sides of the meridian; the calculation is then much simpler, for

$$P = \tfrac{1}{2} t$$

and the formulæ become

$$\tan S = \cos \tfrac{1}{2} P \tan D$$

$$\cos S' = \sin h \cos S \sec D$$

$$L = S \mp S'$$

EXAMPLE.

t =	4 hrs. 10′ 30″	D = 72° 15′ 20″	h = 57° 50′ 10″
2)	62° 39′ 00″		
P =	31° 19′ 30″	cos 9.93158	
	72° 15′ 20″	tan 0.49485	sec 0.51603
S =	69° 53′ 38″	tan 0.43643	cos 9.53625
	57° 50′ 10″		sin 9.92764
S′ =	17° 17′ 10″		cos 9.97992
L =	52° 36′ 28″		
l =	37° 23′ 32″	Latitude of the place.	

This admits, in practice, of no uncertainty, for the difference of the segments is to be taken, when the latitude of the place, and the polar distance, are of the same denomination; their sum, when those quantities are of different denominations.

When the stars are used in the last method, it is liable to no inaccuracy, except what may arise from unequal refraction; in the case of the sun, the declination varies, and an allowance must be made for the change.

These methods are of great value at sea, where it may frequently happen, that the observation of meridian altitudes is prevented by clouds; but, in this case, allowance is to be made, where there is an interval of time, for the motion of the vessel, between the two observations.

Methods of Finding the Apparent Time.

935. The hour of noon may be determined within a few seconds from the observation of the meridian altitude of the sun, taken by a reflecting instrument.

936. The passage of the sun over the wires of a transit instrument, accurately placed in the meridian, gives the time of apparent noon with absolute certainty. Transits of stars, whose right ascension is known, give the hour with equal accuracy.

937. When the transit instrument is not placed truly in the meridian, the observation is made on another vertical circle, and the hour angle can thence be calculated, when the latitude is known; for in the triangle, whose angles are at the pole P, and at the two stars A and B, there will be given the angle t = the difference of right ascension ± the interval of time, and the two polar distances, D and D′, to calculate the angle A; and in the triangle A P Z will be given the angle A, the side P Z = co-latitude, and the side A P = D, to find P = Z P A, the hour angle of the star A.

The formulæ are,

$$\tan S = \cos t \tan D$$

$$S' = D' - S$$

$$\tan A = \tan t \sin S \operatorname{cosec} S'$$

$$\tan S'' = \cos A \tan D$$

$$\cos S''' = \cos L \cos S'' \sec D$$

$$H = S'' \pm S'''$$

$$\sin P = \sin H \sin A \operatorname{cosec} L$$

938. The time may be calculated in a place whose latitude is known from a single altitude of the sun, or of some known star.

The formula for this purpose is,

$$\sin \tfrac{1}{2} \mathrm{P} = \sqrt{\cos\left(\frac{h+l+\mathrm{D}}{2}\right) \sin\left(\frac{h+l+\mathrm{D}}{2} - h\right) \operatorname{cosec} \mathrm{D} \sec l}$$

In which P is the hour angle, h the sun's altitude, l the latitude of the place, and D the sun's polar distance.

When the latitude is not known with absolute certainty, as will frequently happen at sea, the altitude should be taken near the prime vertical, for, in this position, a variation in the latitude produces the least change in the hour angle. **DELAMBRE.**

Example of the Calculation :

h =	6° 10′ 20″		
l =	40° 42′ 45″	sec	0.12034
D =	85° 30′ 20″	cosec	0.00134
	132° 23′ 25″		
$\frac{1}{2}$ sum =	61° 11′ 42″	cos	9.68229
$\frac{1}{2}$ sum − h	55° 01′ 22″	sin	9.91348
		2)	9.71805
$\frac{1}{2}$ P =	43° 48′ 40″	sin	9.85952
P =	87° 17′ 20″ = 5 hrs. 49′ 09″		

939. When the latitude is calculated from two altitudes of the same or different heavenly bodies, the hour angle P, corresponding with one of the altitudes, is ascertained in the course of the process, and thus the same observation gives both the latitude and the time, as may be seen by the following example :

$t = 65° 20' 30''$ $D' = 69° 47' 50''$ $D = 51° 23' 30''$ $h' = 47° 30' 10''$
$h = 18° 45' 20''$

t =	65° 20′ 30″	cos 9.62035	tan 0.33812	
D′ =	69° 47′ 50″	tan 0.43417		cos 9.53825
x =	48° 35′ 15″	tan 0.05452	sin 9.87504	sec 0.17949
D =	51° 23′ 30″			
D−x =	−2° 58′ 15″		cosec −0.28545	cos 9.99941
V =	107° 53′ 04″		tan −0.49861	
C =	62° 43′ 59″			cos 9.71215
h =	18° 45′ 20″			
h' =	47° 30′ 10″			
C =	62° 43′ 59″			
	2) 128° 59′ 29″			
½ sum =	64° 29′ 44″	cos 9.63406		
h' =	47° 30′ 10″	cosec 0.05116		
	16° 59′ 34″	sin 9.44014		
	18° 45′ 20″	sec 0.02370		
		2) 9.14906		
½ W =	22° 03′ 04″	sin 9.57453		
W =	44° 06′ 08″			
u =	62° 26′ 56″	cos 9.66516	tan 0.28257	
	18° 45′ 20″	cot 0.46908		sin 9.50722
z =	53° 43′ 03″	tan 0.13424	sin 9.90638	sec 0.22385
D−z =	−2° 19′ 34″		cosec −1.39162	cos 9.99964
P =	91° 30′ 17″ = 6 hrs. 6′ 1″		−tan −1.58057	
L =	32° 32′ 30″			cos 9.73071
l =	57° 27′ 30″ Latitude of the place.			

940. Dials are frequently constructed for the purpose of showing the hour by means of the shadow projected from a gnomon. The gnomon usually points to the pole of the heavens, while the surface of the dial is plane; this plane may be horizontal, vertical, or inclined. When a gnomon points to the pole, the shadow will always fall on the same line, at the same apparent time, whatever be the declination of the sun.

> The angle which the line that bounds the shadow of the gnomon makes, at any given hour, with the base of the gnomon, may, in the case of the horizontal dial, be found by the formula,
>
> $$\tan a = \sin l \tan P,$$
>
> Where a is the angle, l the latitude of the place, and P the hour angle of the sun on the celestial sphere.
>
> When a vertical dial faces due north or due south, its angles may be found by this formula,
>
> $$\tan a = \cos l \tan P.$$
>
> When a dial is vertical, and inclined at any angle to the meridian, or when it is inclined to the horizon, the position of its lines, and the elevation of its gnomon, may readily be found by the application of the formulæ of spherical trigonometry. As these constructions are of little real value, it is unnecessary to give them. See FERGUSON, *Select Exercises*. ENCYCLOPEDIA BRITANNICA, article *Dialling*.

Mode of Calculating Altitudes.

941. When the latitude of the place of observation, and the hour angle of a heavenly body are known, its altitude may be calculated by the following formula:

$$\tan x = \cos P \cot l$$

$$\sin h = \sin l \cos (D - x) \sec x$$

EXAMPLE.

$$P = 52^\circ\ 50' \qquad l = 40^\circ\ 42'\ 45'' \qquad D = 66^\circ\ 32'\ 30'$$

$$\begin{array}{rcl|l|l}
 & & 52^\circ\ 30'\ 00'' & \cos\ 9.78113 & \\
 & & 40^\circ\ 42'\ 45'' & \cot\ 0.08702 & \sin\ 9.81442 \\
 \cline{4-4}
x & = & 35^\circ\ 04'\ 16'' & \tan\ 9.86815 & \sec\ 0.08702 \\
D - x & = & 31^\circ\ 23'\ 14'' & & \cos\ 9.93089 \\
 \cline{3-3}\cline{5-5}
h & = & 42^\circ\ 48'\ 35'' & & \sin\ 9.83233 \\
 \cline{3-3}\cline{5-5}
\end{array}$$

This problem is of use in taking lunar observations on the land, with the reflecting repeating circle, where it is usual to employ the time and latitude by estimate, and calculate the altitudes of the bodies whose distance is observed.

Modes of Calculating Azimuths.

942. The azimuth of a known heavenly body, may be calculated when the latitude is known, and its altitude is observed; or, when the latitude and the hour are given. The first of these is performed by the formula,

$$\sin \tfrac{1}{2} Z = \sqrt{\sin\left(\frac{D+L+H}{2} - H\right) \sin\left(\frac{D+L+H}{2} - L\right) \operatorname{cosec} H \operatorname{cosec} L}$$

The second by the formula,

$$\cot Z = \sin L \operatorname{cosec} P \cot D - \cos L \cot P$$

DELAMBRE, chap. XVIII.

Where D, L, and H, are the complements of the declination, latitude, and altitude, and P the hour angle.

The calculation of the first of these formulæ is so similar to those of § 939, that no example is necessary; we shall, therefore, give one of the second only.

EXAMPLE.

L = 49° 17′ 15″ D = 72° 15′ 45″ P = 87° 15′ 30″

L = 49° 17′ 15″	sin 9.87956		cos 9.81442
P = 87° 15′ 30″	cosec 0.00050		cot 8.68022
D = 72° 17′ 45″	cot 9.48298	.031235	log 8.49464
	9.36304	log .230700	
Z = 74° 48′ 52″	cot 9.41819	log .261935	

Modes of Finding the Longitude of a Place.

943. An eclipse of the moon being seen to an entire hemisphere of the earth, and beginning at the same instant of absolute time, difference of longitude may be determined by the difference of the apparent time of beginning under two different meridians.

As the shadow of the moon is not well defined, some uncertainty takes place in the observation; this amounts, as has been observed, §837, to eight minutes in time. In making use of this method, the apparent time of beginning is compared with that calculated for the first meridian.

944. On the same principle, the eclipses of Jupiter's satellites may be employed. These may be compared with the prediction as given in the Nautical Almanac, or, which is better, with actual observations made under some known meridian. The difference in the time of immersion, or emersion, is the difference of longitude in time.

This method cannot be used at sea, where the motion of the vessel will prevent the use of the telescope.

945. The longitude may be found by means of a chronome-

ter; if the rate of this be well ascertained, it will furnish, by mere inspection, the mean time at the place where it was set; the apparent time at the place of observation, obtained by some of the methods that have been pointed out, is then reduced by means of the equation of time to mean time; the difference of the two is the difference of longitude.

This method is much used at sea; it is also applicable to observations on the land; but chronometers have not yet generally attained that perfection which would enable them to sustain, without changing their rate, the rapid motion of carriages, or of the human body in walking, while the more equable oscillation of a ship does not affect them. Some of Parkinson and Frodsham's chronometers are said to be fitted for this purpose.

Useful as chronometers are in navigation, it would be unsafe to trust to them altogether, for their mechanism is liable to accidents that cannot be guarded against. It is, therefore, absolutely necessary, on long voyages, to make use of the method called the lunar observation.

946. Of all the heavenly bodies, the moon has the greatest geocentric motion. This is so rapid as to be distinctly perceptible in the transits of the moon over different meridians. An interval in the apparent time of this passage, equal to $\frac{1}{5}$ of a second, corresponds with 6″ of longitude in time. And this accuracy may be readily obtained from the mean passages over several wires.

This will, probably, form the usual mode of ascertaining the difference of longitude between fixed observatories; it has been put in practice at Greenwich and Paris, but the result was not estimated by Delambre, in 1814, to have reached the above degree of accuracy.

947. At sea the same method cannot be used, nor is equal accuracy to be expected; here the distances between the moon and fixed stars, or the sun, may be measured. These distances are to be found in the Nautical Almanac, calculated for intervals of three hours. The motion of the moon for three hours, is so nearly uniform, that a simple proportion is sufficient to ascer-

tain the distance for any intermediate instant, or to determine the time at Greenwich to which any intermediate distance corresponds; the last, compared with the apparent time at the place of observation, gives the longitude. **Delambre**, chap. xxxvi.

948. Although nothing can be simpler in theory, or at the same time more accurate, and although it is more than 200 years since this plan was first proposed, it has but lately succeeded tolerably in practice. To predict the distances of the moon from the sun, and other heavenly bodies, with sufficient nicety, requires very accurate tables. Even with the best tables now in use, and with the best reflecting instruments, the error may amount to one third or one fourth of a degree of longitude. **Delambre.**

949. The calculation of the lunar observations is itself long, and, to persons unacquainted with the principles of spherical trigonometry, abstruse. The observed distance is affected by parallax and refraction, in a degree that varies with the position of the observer, and cannot be calculated beforehand. When, therefore, the observer has taken the apparent distance of the objects, and marked the time of his observation, he must correct the apparent distance for the effects of these two causes of error. The elements, whence they are calculated, are the altitudes of the bodies at the time of observation. **Delambre.**

950. A lunar observation consists, therefore, of three parts, and it is usually made by three observers, one of whom observes the distance, and the other two the altitudes. Several sets of these are commonly taken, the mean of which furnishes the data for calculation. A single observer may, however, take lunar observations; in this case he will observe altitudes and the distance alternately; or, the true altitudes may be calculated by the method in § 941.

These observations give, then, P, the apparent distance, H, the altitude of the sun, and H′ that of the moon; h and h', the true corresponding altitudes, may be thence ascertained; for calling r and r', p and p', the refractions and parallaxes of the two bodies,

$$h = H - r + p \text{ and } h' = H' - r' + p'$$

It may be remarked here, that the apparent altitude of the sun is always greater than the truth, for the sun is more raised by refraction than he is depressed by parallax; on the other hand, the moon's apparent place is always lower than the true one, for the moon's parallax exceeds her refraction.

It is also to be noted, that it is the distance between the centres of the bodies that is to be used, for which reason the semidiameter of the moon, taken from the Nautical Almanac, and corrected for her altitude, is always to be applied; and, also, the semidiameter of the sun, when he is used. With these data, the calculation may be performed, and d, the true distance, found by this theorem.

$$\cos d = \sin h \sin h' + \frac{\cos h \cos h'}{\cos H \cos H'} (\cos D - \sin H \sin H')$$

This formula is not adapted to general use, being liable to difficulty in consequence of the algebraic signs of $\cos d$ and $\cos D$. It has been converted into the following form by Borda:

$$\sin \tfrac{1}{2} d = \cos \tfrac{1}{2} (h + h') \cos x$$

$$S = \frac{H + H' + D}{2}$$

$$\sin x = \sqrt{\frac{\cos S \cos (S - D) \dfrac{\cos h \cos h'}{\cos H \cos H'}}{\cos \frac{1}{2} (h + h')}}$$

We shall give an example of each of these two methods, taken from Delambre.

$D = 101^\circ\ 46'\ 43''$
$H = 56^\circ\ 16'\ 50''$
$H' = 17^\circ\ 47'\ 27''$
$h = 56^\circ\ 16'\ 18''$
$h' = 18^\circ\ 38'\ 43''$

$\sec H \quad 0.25561$
$\sec H' \quad 0.02128$
$\cos h \quad 9.74449$
$\cos h' \quad 9.97658$
$2 \cos M \quad 9.99796$

Thus far the process is common to both methods.

$(1)\ -\ 0.20318$
$(2)\ -\ 0.25295$
$-\ 0.45613$
$(3)\quad 0.26589$
$-\ 0.19024$
whose log $9.27929 = \cos d$
$d = 100^\circ\ 58'\ 00''$

$\cos D = -\ 9.30991$
$(1)\ -\ 9.30787$
$2 \cos M -\ 9.99796$
$\sin H \quad 9.92000$
$\sin H' \quad 9.48508$
$(2)\ -\ 9.40304$
$\sin h = 9.91995$
$\sin h' = 9.50475$
$(3) \quad 9.42470$

By the formula of Borda:

$D = 101^\circ\ 46'\ 43''$
$H = 56^\circ\ 16'\ 50''$
$H' = 17^\circ\ 47'\ 27''$
$175^\circ\ 51'\ 00''$
$S = 87^\circ\ 55'\ 30''$
$S - D = -15^\circ\ 51'\ 13''$
$h = 56^\circ\ 16'\ 18''$
$h' = 18^\circ\ 38'\ 43''$
$\frac{1}{2}(h+h') = 37^\circ\ 27'\ 30''\ 5$

$2 \cos M \quad 9.99796$
$\cos S \quad 8.55880$
$\cos (S - D) + 9.98718$
$2\ )\ 8.54394$
9.27197
$\sec \frac{1}{2}(h + h') \quad 0.10029$
$\sin x \quad 9.37226$
$\cos x \quad 9.98759$
$\cos \frac{1}{2}(h + h') \quad 9.89971$
9.88730

$\sin \frac{1}{2} d = 50^\circ\ 29'\ 00''$
$d = 100^\circ\ 58'\ 00''$

These direct methods are but little used in practice; other formulæ have been investigated, the several members of which have been formed into tables, and thus the labour of calculation has been much lessened. Among these may be mentioned, the Cambridge tables, those of Mendoza de Rios, and Stansbury of New-York; other methods are given in the requisite tables, by Maskelyne; and, more recently, Lax has published a new set of very portable tables for this purpose. It may, however, be stated, that if the methods given above be longer, they are more accurate than any others.

A variety of other formulæ may be seen in DELAMBRE, Vol. III. chap. XXXVI.

951. No notice is usually taken, in freeing the apparent distance of the effects of parallax, of the variation produced in that element, in consequence of the spheroidal figure of the earth. In taking this into account, (which is but seldom necessary in lunar observations,) the parallax for the latitude of the place must be employed, and the parallax in altitude calculated in reference to the geocentric zenith. The refraction, on the other hand, has reference only to the zenith pointed out by the plumb-line. See DELAMBRE, chap. XV. and XXXIV.

952. Observations of solar eclipses, and occultations of fixed stars by the moon, furnish very excellent data for calculating the longitude. The apparent time of beginning and end are to be observed by means of a good telescope; for each of these, with a time at the first meridian, deduced from the longitude by estimate, the apparent distance of the centres is to be calculated; should this agree with the sum of the semidiameters of the two bodies, the estimated longitude is correct; should it not, the distance of the centres is to be calculated, for an instant sufficiently distant from that which has been before used, to include the apparent distance of the centres. The motion of the moon from the sun, for the interval, on her apparent orbit, may be taken as uniform, and the correction in time to be applied to the estimated longitude ascertained by a simple proportion. Each

eclipse and occultation, if the whole duration be visible, furnishes two such observations, whose mean may be taken as the truth; a total eclipse, and a central annular eclipse, furnish four. DELAMBRE, chap. XXVI.

953. When the beginning and end of the eclipse are both observed, the time of geocentric conjunction, for the place of observation, may be calculated; this, compared with the time of geocentric conjunction, for the first meridian, gives the difference of longitude. See GARNETT, *Requisite Tables*, and MACKAY, *on the Longitude.*

It has been customary to deduct 3″ from the semidiameter of the sun, given in the Nautical Almanac, for his irradiation; and 2″ from that of the moon, for an inflexion, supposed to be produced by her atmosphere. Strong doubts, however, may be entertained of the correctness of this method.

954. The elements that are employed in the calculation of the parallaxes in longitude and latitude are as follows, viz.

ω the obliquity of the ecliptic, corrected to the time assumed for the calculation for the effects of mutation.
M the right ascension of the meridian.
l the geocentric latitude.
p the differences of the horizontal parallaxes of the sun and moon.
Δ the polar distance of the moon.
☽ the longitude of the moon.

955. The formula for calculating the longitude of the nonagesimal degree is,

$$\tan N = \cos \omega \tan M + \frac{\sin \omega \tan l}{\cos M}$$

DELAMBRE, chap. XV.

EXAMPLE.

$$\begin{array}{rlll}
\omega = 23^\circ\ 27'\ 46'' & \cos\ 9.9625312 & & \sin\ 9.6000502 \\
M = 90^\circ\ 09'\ 22'' & \tan\ -2.5646957 & & \sec\ -2.5646894 \\
l = 40^\circ\ 37'\ 27'' & & & \tan\ 9.9318698 \\
\cline{1-2} \cline{4-4}
(1)\ -336.69 & \log\ -2.5272269 & (2)\ -124.92 & \log\ -2.0966094 \\
(2)\ -124.92 & & & \\
\cline{1-2}
-461.61 & \log\ -2.6642752 & \tan 90^\circ\ 07'\ 29'' &
\end{array}$$

The formula for the altitude of the nonagesimal is,

$$\sin h = \cos M\ \cos l\ \sec N$$

Delambre, chap. xv.

EXAMPLE.

$$\begin{array}{lll}
M & = 90^\circ\ 09'\ 22'' & \cos\ -\ 7.4353106 \\
l & = 40^\circ\ 37'\ 27'' & \cos\ +\ 9.8808890 \\
N & = 90^\circ\ 07'\ 29'' & \sec\ -\ 2.6621789 \\
\cline{3-3}
h & = 72^\circ\ 04'\ 12'' & \sin\ +\ 9.9783785
\end{array}$$

956. The formula for the parallax in longitude is,

$$\pi = \left(\frac{\sin p \sin h}{\sin \Delta}\right)\left(\frac{\sin\ \text{☽} - N}{\sin 1''}\right) + \left(\frac{\sin p \sin h}{\sin \Delta}\right)^2 \left(\frac{\sin 2(\text{☽} - N)}{\sin 2''}\right) + \&c.$$

EXAMPLE.

p	= 00° 55′ 08″ 58	sin 8.2051960		
h	= 72° 04′ 12″	sin 9.9783785		
Δ	= 89° 48′ 29″	cosec 0.0000026		
		8.1835771	× 2	6.3671542
☽ − N	= 62° 26′ 29″	sin 9.9476794	☽ − N × 2	sin 9.9139559
	1″	cosec 5.3144251	2″	cosec 5.0133951
(1)	2790″ 5	log 3.4456816	(2) 19″ 7	log 1.2945052
(2)	19″ 7			
	2810″ 2			
	46′ 50″ 2			

957. The parallax in latitude may be calculated by the following formulæ :

$$= \varpi \left(\frac{\sin p \cos h}{\cos x}\right)\left(\frac{\sin \Delta - x}{\sin 1''}\right) + \left(\frac{\sin p \cos h}{\cos x}\right)^2 \left(\frac{\sin 2\,(\Delta - x)}{\sin 2''}\right) + \&c.$$

$$\tan x = \tan h \cos (☽ - \mathrm{N} + \tfrac{1}{2}\varpi) \sec \tfrac{1}{2}\varpi$$

DELAMBRE, chap. xv.

EXAMPLE.

h =	72° 04′ 12″	tan 0.4900324
☽ − N =	62° 26′ 29″	
$\frac{1}{2}\varpi$ =	23′ 25″	
	62° 49′ 54″	cos 9.6596567
	23′ 25″	sec 0.0000101
x =	54° 41′ 05″	tan 0.1496992

$p =$	55′ 08″ 58	sin	8.2051960		
$h =$	72° 04′ 12″	cos	9.4883722		
$x =$	54° 41′ 05″	sec	0.2379414		
			7.9315096	× 2	5.8630192
$\Delta - x =$	36° 07′ 24″	sin	9.7599234	$2(\Delta - x)$ sin	9.9736618
	1″	cosec	5.3144251	2″ cosec	5.0133951
(1)	1013″ 5	log	3.0058581	(2) 7″ 1 log	0.8500761
	7″ 1				
	1020″ 6				
	17′ 00″, 6				

The parallax, in latitude and longitude, being applied to those elements, give the moon's apparent place, in relation to the sun; and hence the distances of the centres may be calculated. The triangle, employed in this case, being small, is usually considered as a plane triangle.

The calculation for occultations is, in every respect, the same, except that, instead of p, ϖ the moon's horizontal parallax is taken; and the distance of the centres is equal to the moon's semidiameter only.

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THE END.

Corrigenda

IN VOL. I.

Page 13. line 12. *for* "R," *read* "R^2."
18. *for* "when three forces," *read* "when three oblique forces."
14. 15. *for* "R," *read* "R^2."
20. *for* "arcs," *read* "axes."
21. 3 from bottom, and p. 22. line 3 from top, *for* "frustrum," *read* "frustum."
28. 2 from bottom, *for* "as the thickness," *read* "as half the thickness."
58. 21. *for* "Equally," *read* "Equably."
76. 1. *for* "twice," *read* "four times."
94. 6. *for* "might," *read* "weight."
95. last line, *for* "Nievellement," *read* "Nivellement."
115. line 11. *for* "terniers," *read* "Verniers."
29. *for* "Sharpsburgh," *read* "Shuckburgh."
30. *for* "gembals," *read* "gimbals."
117. 14. *for* "qq'," *read* "$q - q'$"
119. 16. *for* "Geodotic," *read* "Geodetic."
128. 1. *for* "y," *read* "q."
143. 16. *for* "the given quantity," *read* "Q the given quantity."
144. 23. *for* "the section, the stream," *read* "the section of the stream."
148. 10. *for* "26000," *read* "27600."
153. 13. *for* "Elesian," *read* "Etesian."
164. 6. *for* "concavity," *read* "convexity."
165. 14. *for* "of interval," *read* "of the interval."
167. 1. *for* "T, H, *t*, H, T, *t*, T," *read* "T, H, T, *t*, H, T, *t*."
175. 1. *for* "act," *read* "acts;" line 4, *for* "instance," *read* "instant."
179. 23. *for* "84600," *read* "86400."
195. 3 from bottom, *dele* "and third."
213. 7. *for* "half," *read* "twice."
216. 5. *for* "greater," *read* "less."

Corrigenda

IN VOL. II.

Page 57. line 2. *for* "N. E. and S. W." *read* "N. W. and S. E."
59. 7. *for* "Graphomentor," *read* "Graphometer."
79. 14. *for* "effected," *read* "affected."
80. 6. *for* "convex," *read* "concave."
7. *for* "Meniscas," *read* "Meniscus."
81. 8. and l. 10. *for* "convex," *read* "concave."
94. 3. *for* "concave," *read* "convex."
100. 5. *after* "this operation," *insert* "or by depressing the lens into the bottom of the eye."
122. 24. *for* "determined," *read* "conceived."
127. 2 from bottom, *for* "meridian," *read* "equator."
145. 26. *for* "decreases," *read* "increases."
27. *for* "decrease," *read* "increase."
last line, *dele* "common."
152. line 1. *for* "connected," *read* "coincided."
222. 11. *after* "cosec," *insert* "h'."
223. 17. *for* "61°," *read* "66°."